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Erratum: the author's version (including its final form hereafter) is correct, but the official published version contains the following "minor errors":

- On p. 10 of the official version, replace "a matrix H whose columns are the unit-norm Eigenvectors of $E\{\tilde{x}(t)\tilde{x}(t)\}$" by "a matrix H whose columns are the unit-norm Eigenvectors of $E\{\tilde{x}(t)\tilde{x}^T(t)\}$".
- On p. 14 of the official version, replace "The most "natural" representation of a signal (u)" by "The most "natural" representation of a signal u".
- On p. 16 of the official version, replace "and the observation with index $i = 1$, provided $a_{1_{j_0}} \neq 0$" by "and the observation with index $i = 1$, provided $a_{1j_0} \neq 0$".
- Same as above for "up to the scale factor $1/a_{1_{j_0}}$" on p. 17 of the official version.
- On p. 17 of the official version, replace "$i = 2$ $i = N$" by "$i = 2$ to N".

Blind source separation and blind mixture identification methods

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Blind Source Separation (BSS) is a generic signal processing problem, which may be briefly defined as follows:

BSS methods aim at estimating a set of "source signals" (which have unknown values but some known properties), using a set of available signals which are "mixtures" of the source signals to be restored, without knowing (or with very limited knowledge about) the "mixing transform", i.e. the transform of source signals which yields their mixtures.

The term "signal" is here to be understood in a broad sense: the considered problem not only concerns monodimensional functions (especially time-dependent functions), but also images and various types of data.

BSS methods appeared in the 1980s and then quickly expanded. Various books, especially [1, 2, 3, 4, 5, 6], provide a detailed description of BSS methods, or at least of some classes of such methods defined hereafter, such as Independent Component Analysis, Sparse Component Analysis or Nonnegative Matrix Factorization (related topics are also discussed in the book [7]).

Moreover, the BSS problem, focused on *signal* restoration, is closely linked to the estimation of (the parameters of) the mixing *transform*, and thus to the problem often referred to as Blind Mixture Identification, or BMI (see e.g. [1] pp. 65-66 or [8, 9, 10, 11, 12]).

In this chapter, we provide an overview of the fields of BSS and BMI, i.e. we first define in more detail the considered goal (see Section 1) and conditions of investigation (Section 2), and we then introduce the major classes of methods which make it possible to solve the considered problems. The presentation of BSS/BMI methods themselves, and of typical applications, is split in successive sections (Sections 3 to 7), where we progress from standard to more advanced configurations, in terms of properties of source signals and/or class of mixing transform. Some additional topics are eventually outlined in Section 8, which also contains general conclusions about BSS and BMI.

Contents

Keywords: Independent Component Analysis, Sparse Component Analysis, Nonnegative Matrix Factorization, Geometrical blind unmixing methods, Statistical data and signal processing, Information theory, Blind system identification and inversion, Quantum Information Processing.

List of abbreviations:
AD-TiFCorr: Attenuation and Delay - TIme-Frequency CORRelation
AD-TiFROM: Attenuation and Delay - TIme-Frequency Ratios Of Mixtures
AMUSE: Algorithm for Multiple Unknown Signals Extraction
ANC: Adaptive Noise Cancelling
BMI: Blind Mixture Identification
BQPT: Blind Quantum Process Tomography
BQSS: Blind Quantum Source Separation
BSS: Blind Source Separation
COM2: COntrast Maximization 2
DUET: Degenerate Unmixing Estimation Technique
fMRI: Functional Magnetic Resonance Imaging
ICA: Independent component analysis

i.i.d.: independent and identically distributed
IP: Information Processing
ISA: Independent Subspace Analysis
IVA: Independent Vector Analysis
JADE: Joint Approximate Diagonalization of Eigen-matrices
LI-TiFCorr: Linear Instantaneous - TIme-Frequency CORRelation
LI-TiFROM: Linear Instantaneous - TIme-Frequency Ratios Of Mixtures
MICA: Multidimensional Independent Component Analysis
MIMO: Multiple-Input Multiple-Output
MISO: Multiple-Input Single-Output
NMF: Nonnegative Matrix Factorization
NNICA: Nonnegative Independent Component Analysis
QIP: Quantum Information Processing
QP: Quantum Physics
QPT: Quantum Process Tomography
QSICA: Quantum-Source Independent Component Analysis
QSS: Quantum Source Separation
SCA: Sparse Component Analysis
SOBI: Second-Order Blind Identification
STFT: Short-Time Fourier Transform

List of notations:

a_{ij}	mixing coefficient (scale factor) associated with i^{th} observed mixed signal and j^{th} source signal
$a_{ij}(t)$	impulse response of mixing filter associated with i^{th} observed mixed signal and j^{th} source signal
A	mixing matrix
C	separating matrix
$E\{.\}$	$E\{X\}$ is the expectation (i.e. statistical mean) of random variable X
$F(.)$	mixing function
N	number of source signals
P	number of observed mixed signals ($P = N$ for determined mixtures)
$s_j(t)$	value of j^{th} source signal at time t
$s(t)$	value of vector of source signals at time t
S	matrix containing the values of all source signals at all times
$S_j(t, \omega)$	value of STFT of source signal $s_j(t')$ at time t and angular frequency ω
$S(t, \omega)$	value of vector of STFTs of source signals at time t and angular frequency ω
t_{ij}	time shift associated with i^{th} observed mixed signal and j^{th} source signal in mixing model
^{T}	M^T is the transpose of matrix M
$\tilde{U}(\omega)$	value of Fourier transform of (arbitrary) signal $u(t)$ at angular frequency ω
$U(t, \omega)$	value of STFT of (arbitrary) signal $u(t')$ at time t and angular frequency ω

$x_i(t)$	value of i^{th} observed mixed signal at time t
$x(t)$	value of vector of observed mixed signals at time t
X	matrix containing the values of all observed mixed signals at all times
$X_i(t,\omega)$	value of STFT of observed mixed signal $x_i(t')$ at time t and angular frequency ω
$X(t,\omega)$	value of vector of STFTs of observed mixed signals at time t and angular frequency ω
$y_j(t)$	value of j^{th} output signal of separating system at time t
$y(t)$	value of vector of output signals of separating system at time t (this vector is restricted to a single signal in the core of MISO BSS methods)
$z(t)$	value of vector of sphered observations at time t

1 Goal of Blind Source Separation (BSS)

We consider the situation when a set of signals $x_i(t)$ are available and they result from a set of unknown source signals $s_j(t)$ (often simply called "sources $s_j(t)$"), due to a "mixing" phenomenon, as shown in Fig. 1. This notion of "mixing" will be detailed further in this chapter, but we can already clarify it at this stage, by stating that the simplest class of mixture corresponds to the case when the signals $x_i(t)$ are linear combinations of the signals $s_j(t)$. In particular, in the basic case when two mixtures $x_1(t)$ and $x_2(t)$ of two source signals $s_1(t)$ and $s_2(t)$ are available, these mixed signals read, at any time t,

$$x_1(t) = a_{11}s_1(t) + a_{12}s_2(t) \quad (1)$$
$$x_2(t) = a_{21}s_1(t) + a_{22}s_2(t) \quad (2)$$

where a_{ij} are scalar coefficients. The mixed signals $x_i(t)$ are also called "observations" or "observed signals". They are e.g. provided by sensors, such as radio-frequency antennas, microphones... Mixing between source signals then occurs during their simultaneous propagation to the sensors.

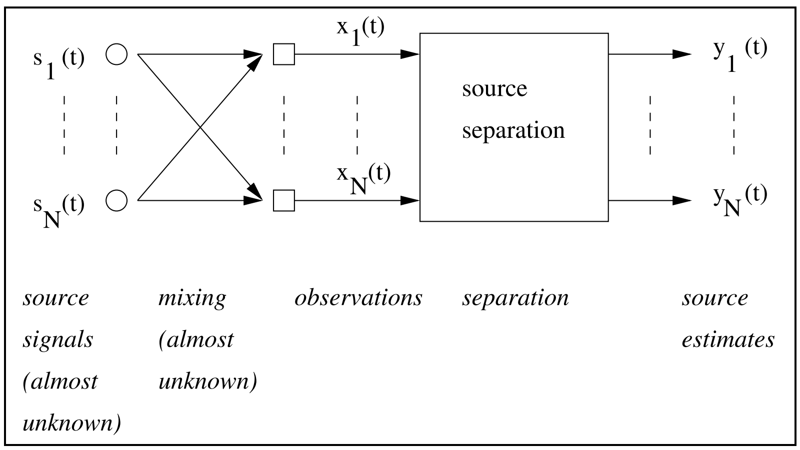

Figure 1: General configuration for the source separation problem.

BSS aims at creating a processing method (or a set of methods) which only receives the mixed signals $x_i(t)$ as inputs and which "extracts" all source signals $s_j(t)$ from these signals $x_i(t)$. This "extraction" or "separation" of source signals may be defined as follows:

- Ideally, BSS methods should aim at creating output signals $y_j(t)$ which are respectively equal to the source signals $s_j(t)$.
- However, one has to restrict oneself to a somewhat simpler goal, because the considered problem intrinsically entails some limitations: unless additional information is available, one can only aim at making the output signals of a BSS method equal to the source signals up to a set of unavoidable modifications, called "indeterminacies". The nature of these indeterminacies depends upon the constraints set on the source signals, on the considered class of mixtures and on the selected type of BSS method. For the sake of clarity, we can already state at this stage that, in the most standard case (namely determined linear instantaneous mixtures, defined further in this chapter), these indeterminacies are restricted to a possible permutation concerning the order of the output signals and to scale factors applied to these signals.

Whatever type of signals is studied, the general considered configuration is shown in Fig. 1. More explicit versions may then be derived from it, for each particular application domain. For instance, Fig. 2 illustrates the case when the considered source signals have an acoustic nature, the sensors are microphones and the separating system aims at extracting one or several speech signals of interest, from the microphones signals consisting of noisy mixtures of these speech signals. A specific version of this configuration is obtained when the microphone signals contain no noise components (or when noise has a negligible magnitude). The latter problem is often referred to as the "cocktail party problem", in connection with the situation when a set of microphones are placed in a cocktail room where a group of people are talking, and one aims at extracting each of these speech signals.

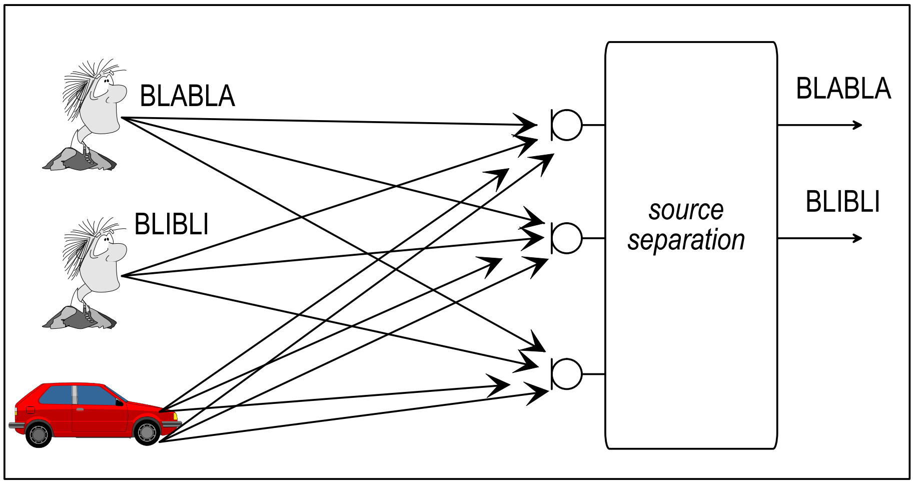

Figure 2: Application of source separation methods to acoustic signals.

2 General conditions of investigation

We again consider the general BSS configuration illustrated in Fig. 1. We here review the quantities involved in this configuration and the information assumed, or not, to be available about these quantities.

2.1 Observed signals

The mixed signals $x_i(t)$ are known, both from the point of view of their number and of their values: they are the measured signals from which BSS is carried out.

In many practical applications, the number of mixed signals is equal to the number of sensors which respectively provide each of these observations $x_i(t)$.

2.2 Source signals

Concerning the source signals $s_j(t)$:

1. The values of these signals are assumed to be unknown, otherwise the BSS problem would already be solved.

2. In the simplest configuration, the number N of source signals $s_j(t)$ is assumed to be equal to the number P of observed signals $x_i(t)$. The mixture is then said to be "determined". In this case, the number of source signals is known, since the number of observed signals itself is known. In this chapter, we consider this case of determined mixtures, unless otherwise stated. We then denote as N the number of source and observed signals. In Fig. 1, we already focused on this configuration.

   Besides, it is implicitly assumed that these observations are not "redundant": for instance, it would e.g. be an illusion to pretend that N mixtures $x_i(t)$ of the considered source signals were available if some of these signal $x_i(t)$ were in fact strictly identical ! This notion of non-redundant observations may be expressed in a way which depends on the nature of the mixing which occurs between source signals. When the mixture is defined by a mixing matrix A (see e.g. Section 3.1.1), this non-redundancy corresponds to assuming that this matrix A is invertible.

Apart from determined mixtures, the other two possible situations are as follows:

(a) The mixtures are "overdetermined" when the number of observed signals is higher than the number of source signals. One may imagine that this case does not entail major difficulties as compared with the determined case, since the information concerning the source signals and provided by the measured signals is then as rich as in the case of determined mixtures, or even richer.

Starting from an overdetermined mixture, one may try to get back to a determined mixture. A brute-force approach to this end would consist in totally ignoring some of the observations, by only taking into account a number of observations equal to the number of source signals (again assuming that the observations thus kept are not redundant). A more interesting approach, especially for the noisy version of the linear instantaneous mixtures defined further, consists in applying Principal Component Analysis to the observations. This yields signals whose number can be made equal to the number N of source signals and which are equal to the N first principal components of these observations (see e.g. Chapter 6 of [2] for more details).

(b) On the contrary, the mixtures are "underdetermined" when the number of observed signals is lower than the number of source signals. One may expect that this situation is more difficult to handle, using the same analysis as in the overdetermined case considered above. Indeed, even for the simplest class of mixtures, if the mixture is underdetermined, the source signals can be estimated only if they have specific properties (this phenomenon is explained in more detail in Section 4.1.2, where it is especially shown that, for underdetermined mixtures, one should distinguish between the BMI and BSS tasks).

3. The source signals are constrained to meet some properties, which then makes it possible to build methods that exploit these properties to separate these signals. Several properties have thus been used in the literature. For the simplest class of mixtures, the most used property applies to random source signals and consists in assuming that they are statistically independent, as explained in Section 3, whereas the other classical properties are detailed in Section 4.

2.3 Mixing model

The other quantities and assumptions concern the model which defines the mixing phenomenon between sources, i.e. the multidimensional deterministic function F which makes it possible to express the observed signals $x_i(t)$ with respect to the source signals $s_j(t)$ as

$$x(t) = F(s(t)) \tag{3}$$

where the vectors of source and observed signals are defined as

$$s(t) = [s_1(t), \ldots, s_N(t)]^T \tag{4}$$

$$x(t) = [x_1(t), \ldots, x_N(t)]^T \tag{5}$$

where ^{T} stands for transpose.

The model given by eq. (3) corresponds to so-called "noiseless mixtures". On the contrary, in the case of noisy mixtures, each observation $x_i(t)$ not only contains the components associated with the source signals, defined by eq. (3), but also a random noise component, often assumed to be additive and independent both from the source signals and from the noise components associated with the other observations. The noiseless model is considered in many investigations although the noisy model (more general but more difficult to handle) is better suited to various applications. In this chapter, we consider the noiseless model, unless otherwise stated.

The most studied situation then corresponds to the following conditions:

- The nature of the mixing model, i.e. the class of functions containing function F, is assumed to be known.

- The values of the parameters which appear in this model are unknown.

- The parameters of the mixing model (or of the inverse of this model) are estimated only from the observations $x_i(t)$.

This problem is then called "blind source separation" because: (i) generally speaking, "source separation" consists in restoring source signals from their mixtures and (ii) this separation is here performed "blindly", i.e. without knowing the values of the source signals $s_j(t)$ nor the mixing function F that was imposed on these source signals to create the available observations $x_i(t)$, so that the parameters of this function F (or of its inverse) are only estimated from the observations. It should be noted that the mixing function F is however assumed to be partly known, since its functional form is fixed. Observations are therefore not processed in a completely blind way, so that some authors state that this processing problem is "myopic", rather than blind.

On the contrary, the case of *non-blind* source separation includes two situations: (i) the ideal case when the mixing function F (or its inverse) is a priori known and (ii) the situation when F is unknown and this function (or its inverse) is estimated by using not only values of observations $x_i(t)$ but also corresponding values of source signals $s_j(t)$. The latter version of non-blind source separation is linked to non-blind system identification: in that non-blind identification task, one aims at determining the function F which defines the behavior of a system, from known values of inputs $s_j(t)$ and outputs $x_i(t)$ of that system, but without focusing on the subsequent restoration of unknown input values $s_j(t)$ from known output values $x_i(t)$, unlike in the source separation task. For more details about non-blind system identification, the reader may e.g. refer to [13] in this encyclopedia.

The same type of link as above exists in the blind case, i.e. between blind source separation (BSS) and blind mixture identification (BMI), where BMI may be seen as a blind extension of the above-mentioned non-blind system identification: in BMI, one determines the function F defining the behavior of the considered system, but blindly, i.e. only from known output values $x_i(t)$ (and from some properties imposed on its inputs $s_j(t)$).

Different sub-fields may then be distinguished within the overall field of BSS/BMI, depending on the considered mixing model, which has a strong influence on the separation/identification methods that may be developed. In the subsequent sections, we progress from simple to more complex mixing models and we describe corresponding BSS/BMI methods, which often consist of various classes of methods, depending on the considered source properties. We thus investigate linear instantaneous mixtures in Sections 3 and 4, anechoic and general convolutive mixtures in Section 5 and nonlinear mixtures in Section 6 and 7.

3 Methods for linear instantaneous mixtures of independent sources

The main two classes of mixing models that may first be distinguished are linear and nonlinear mixtures. Linear mixtures, especially instantaneous ones, often yield simpler BSS/BMI methods than nonlinear ones and are sufficient for addressing a large number of application fields. Linear mixtures have therefore been much more studied. Among them, three sub-classes may be distinguished. The first one is defined hereafter.

for i.i.d. sources (see [100] for an extension to sources which are filtered versions of i.i.d. processes). As shown by eqs. (70) and (72), any single source signal s_j yields an "'overall contribution" (i.e. taking all successive values $s_j(n-k)$ of that source signal s_j into account), in any observed signal, which may be reinterpreted as follows. One considers the $(M+1)$ values $s_j(n-k)$ as the values of an extended set of $(M+1)$ source signals, which are mutually statistically independent since the original source signals are assumed to be i.i.d. The convolutive mixing model defined by eqs. (70) and (72) is then reinterpreted as a *linear instantaneous* mixture of $(M+1)N$ independent, extended, source signals. This approach thus has the advantage of bringing us back to the simplest type of mixtures. However, one should keep in mind that mixtures must be (over-)determined to avoid the issues that we described above for underdetermined mixtures. This may essentially be achieved by increasing the number of observed mixtures as follows: one defines an extended set of observed mixtures which does not consist of the signals of eq. (70) at a single time, but over a bounded interval of time positions [100]. This approach was applied to several linear instantaneous BSS methods defined in Section 3. For instance, a convolutive extension of FastICA was proposed in [100]. Similarly, a convolutive version of second-order diagonalization-based methods, derived from SOBI, was introduced in [101] for the case when each original source signal s_j is temporally autocorrelated.

Operating in the time domain therefore yields several attractive options for convolutive BSS methods. However, this requires one to introduce adaptive separating variables which typically consist of the impulse responses of filters which aim at, at least approximately, "matching" the filters contained in the mixing model or its inverse. This may require high-order separating filters e.g. in application domains where the non-negligible parts of the impulse responses of the mixing filters have a long duration. This may entail both reduced estimation accuracy and high computation load, then limiting the attractiveness of time-domain approaches in application fields involving these "long mixing filters". These considerations especially apply to acoustic impulse responses (see e.g. Fig. 11), and frequency-domain approaches have indeed widely been used, instead of time-domain methods, in Acoustics. On the contrary, in various electromagnetic communication applications, the effect of the propagation channel may be modelled by filters whose impulse responses are restricted to a few coefficients.

For more details about convolutive BSS methods or their acoustic/audio applications, the reader may e.g. refer to Chapters 8 and 19 of [1] or to [5, 102, 103].

6 Methods for nonlinear mixtures

Beyond the different types of linear mixtures considered above, a significantly smaller number of reported investigations concern nonlinear mixing models. The most general form of these models may be compactly expressed as in eq. (3), when interpreting this equation as follows: the value of the observed vector $x(t)$ at a single time t may depend on the *series* of values of the source vector at different times, as in the compact notations used in eq. (66) for general, i.e. convolutive, linear mixtures. In particular, the nonlinear mixing model given by eq. (3) includes *instantaneous* (or memoryless) nonlinear mixtures, which correspond to the case when $x(t)$ only depends on the value of the source vector at the same time t, i.e. $s(t)$.

Nonlinear mixtures are more complex to handle than the linear instantaneous ones that we detailed above. Difficulties appear at different stages of the standard procedure for developing BSS methods that we defined in Section 3.6. First, if trying to extend the standard separating system structure of Section 3.2 to a given class of nonlinear mixtures, i.e. to a class of MIMO functions with unknown parameter values, then for various BSS methods one again has to define a class of MIMO separating functions which implements the inverse of the considered class of MIMO mixing functions. However, unlike with linear mixtures, for many given analytical expressions of nonlinear MIMO mixing functions, the analytical expressions of the corresponding inverse functions cannot be derived (moreover, MIMO mixing functions are not necessarily invertible, i.e. bijective, unless one accepts to consider them only over a small enough bounded domain). This also means that, for nonlinear mixtures, BMI is easier to perform than BSS in situations when the analytical form of only the mixing model is known. Similarly, concerning the separation principle, a natural approach consists in trying to extend to nonlinear mixtures the principle which was the most studied for linear mixtures, that is the ICA principle. However, it has been shown (see p. 552 of [1] or [104]) that, if the mixing model is not constrained to belong to a specific class, the indeterminacies of nonlinear ICA-based methods are unacceptably high: starting from independent source signals, such methods may yield output signals which are still mixtures of the source signals, although these output signals are independent (a simple example is e.g. provided on pp. 552-553 of [1]). Related topics are also discussed in [105].

Most reported investigations for nonlinear BSS are restricted to specific classes of nonlinear mixtures, in order to reduce the above indeterminacies and/or because they are motivated by practical applications where the encountered mixing models are guaranteed to belong to known classes. Two such classes were especially studied, namely linear-quadratic instantaneous mixtures (and, to some extent, their polynomial extensions) and post-nonlinear ones. The remainder of this section is therefore focused on these two classes. For more detailed discussions about nonlinear BSS, the reader may refer to Chapter 14 of [1] or to [106, 107]. A specific type of nonlinear mixture is also discussed, in a different context, in Section 7.

6.1 Linear-quadratic mixtures and associated BSS methods

The scalar form of the linear-quadratic (instantaneous) mixing model reads

$$x_i(t) \quad = \quad \sum_{j=1}^{N} a_{ij} s_j(t) + \sum_{j=1}^{N} \sum_{k=j}^{N} b_{ijk} s_j(t) s_k(t) \qquad \forall\ i \in \{1, \ldots, P\} \tag{75}$$

where $x_i(t)$ are the values of the P observed mixed signals at time t and $s_j(t)$ are the values of the N unknown source signals which yield these observations, whereas a_{ij} and b_{ijk} are respectively the linear and quadratic mixing coefficients (with unknown values in the blind case) which define the considered source-to-observation transform. The specific version of this model which contains no second-order auto-terms (i.e. $b_{ijk} = 0$ when $k = j$) is called the bilinear mixing model. It corresponds to replacing the second sum in eq. (75) by $\sum_{j=1}^{N-1} \sum_{k=j+1}^{N}$. Similarly, the quadratic version of this model is obtained when all coefficients a_{ij} are zero.

The linear-quadratic mixing model is thus a natural extension of linear (instantaneous) mixtures, obtained by also including second-order terms (and higher-order terms for its polynomial extensions). It may thus first be seen as a generic model, to be used as an approximation (truncated polynomial series) of various, possibly unknown, models faced in practical applications. Moreover, linear-quadratic mixing has been shown to *actually* occur in some applications. It has thus mainly been used in three types of applications. The first one concerns unmixing of remote sensing data (see e.g. [108, 109]). As already discussed in Section 3.1.2, in simple situations involving single reflections, this application yields a linear instantaneous mixing model, where e.g. each source signal is the reflectance spectrum of a pure material and each associated mixing coefficient is related to a surface on Earth corresponding to this pure material. However, more complex situations with double reflections also exist, e.g. when light emitted by the sun is first reflected by a wall of a building, then reflected by the ground, and it eventually reaches the sensing device (see Fig. 12). When considering the source and observed signals as reflectance spectra (the time variable t of eq. (75) is then replaced by the wavelength index), the above situation yields observed signals in which each contribution involving double reflection is the product of the reflectances of the two pure materials (source signals) for which double reflection occurs and of a coefficient (mixing coefficient), as shown in [108]. Taking these contributions into account in addition to those associated with simple reflections, the observed data are represented according to the linear-quadratic model defined by eq. (75).

Apart from the above application, the linear-quadratic mixing model has also been considered when processing scanned images involving the show-through effect already mentioned in Section 4.3 (see e.g. [82]) and when analyzing gas sensor array data [110]. References of other investigations related to the above applications are also provided in [111].

A recent survey of the BSS methods that have been proposed for linear-quadratic (and polynomial) mixtures is also available in [111]. We hereafter summarize the main trends in that domain

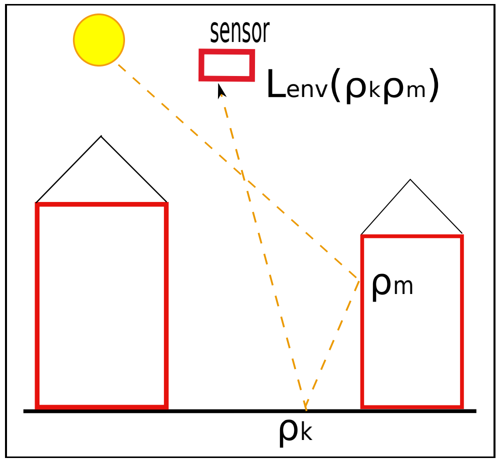

Figure 12: Double reflection of light emitted by the sun: light is first reflected by a wall of a building, then reflected by the ground, and it eventually reaches the sensing device (Reprinted from [108]).

and refer the reader to [111] for more details. These trends are consistent with the investigations previously performed for linear instantaneous mixtures, which were described in Sections 3 and 4. In particular, various reported methods for the configuration considered here may be seen as linear-quadratic extensions of ICA. To solve the above-mentioned issue concerning nonlinear separating structures (analytical form of inverse of mixing function), several of these methods use recurrent networks, which may be seen as nonlinear extensions of the original Hérault-Jutten network intended for linear mixtures (these networks may be further extended to much more general nonlinear mixtures than linear-quadratic ones only, as detailed in [112]). The parameters of these networks are adapted by using ICA-based MIMO criteria which are similar to those described in Section 3.5 for linear mixtures, namely centered output cross-moment cancellation [113], output mutual information minimization [114] and likelihood maximization (see e.g. [115]). The above methods concern i.i.d. sources. For autocorrelated sources, a few approaches based on second-order statistics and therefore related to Section 3.7 were also proposed: see e.g. [116], which is restricted to BMI and thus avoids the issue of the nonlinear separating structure often used for BSS. Still in the framework of statistical methods, Bayesian approaches have also been applied to linear-quadratic mixtures, including for autocorrelated sources (see e.g. [82]).

The other reported approaches first include a few linear-quadratic or bilinear extensions of SCA, again using either single-source zones (see e.g. the investigation in [117] or its summary in [8]) or cost functions based on L^0 pseudo-norm [118]. Besides, some linear-quadratic extensions of NMF were developed, especially for the above-mentioned remote sensing application (see especially [109] for a general configuration).

6.2 Post-nonlinear mixtures and associated BSS methods

The post-nonlinear (instantaneous) mixing model is defined by

$$x_i(t) = f_i\left(\sum_{j=1}^{N} a_{ij} s_j(t)\right) \qquad \forall\, i \in \{1, \cdots, P\} \tag{76}$$

where we use the same notations as above and we introduce single-input single-output nonlinear functions $f_i(\cdot)$. In the blind case, both a_{ij} and $f_i(\cdot)$ are unknown. However, the functions $f_i(\cdot)$ are often assumed to be strictly monotonic and, thus, invertible. This model is useful in applications where the first stage of the mixing process has a linear nature but the sensors then exhibit a nonlinear response, due to saturation or more complex nonlinear transducer phenomena. The main field of application for post-nonlinear models is the design of smart chemical sensor arrays (see e.g. [119]). These models were also applied to remote sensing data [120].

The BSS methods that have been developed for post-nonlinear mixtures are especially focused on ICA. When using this separation principle, this class of mixtures has a major attractive feature: in the determined configuration, the associated separating structure ensures ICA separability under conditions which are similar to those established for the linear case. This result was first established in [121] and later extended in other investigations. The reader is referred to the recent survey [111] for more details about these separability analyses and about practical post-nonlinear BSS methods based on ICA and Bayesian approaches.

7 Blind quantum source separation and process tomography

The above sections span various types of mixtures and source properties, but they do not include all kinds of BSS and BMI configurations, in the sense that they are restricted to "classical", i.e. non-quantum, source signals and mixing phenomena.

Independently from classical BSS and BMI, another field within the overall Information Processing (IP) domain rapidly developed during the last decades, namely Quantum Information Processing (QIP) [122, 123, 124, 125, 126]. QIP is closely related to Quantum Physics (QP). It uses abstract representations of systems whose behavior is requested to obey the laws of QP. This already made it possible to develop new and powerful IP methods, to be contrasted with classical methods such as the above-mentioned BSS and BMI approaches. These new methods manipulate the states of so-called quantum bits, or qubits. Their effective implementation then requires one to develop corresponding practical quantum systems, which is an emerging topic [122].

The gap between classical (B)SS and QIP/QP, was bridged in 2007 in [127] (followed by its extended version [128]), which introduced a new field, namely Quantum Source Separation (QSS) and especially its blind version, Blind Quantum Source Separation (BQSS). The BQSS problem consists in restoring (the information contained in) individual source qubit states, i.e. *quantum* source signals, only starting from the mixtures (in BSS terms [128]) of these source qubit states which result from their undesired coupling.

Several classes of (B)QSS methods were thus developed. In the first of them, the standard configuration of classical BSS shown in Figure 5 is modified as follows. The source signals and first part of the mixing stage here have a quantum nature. The last part of the mixing stage then converts its quantum input into data which have a classical form, but whose properties reflect their quantum origin. This conversion is performed by means of measurements. The set of resulting classical-form signals, which corresponds to the mixed vector $x(t)$ of Figure 5, is then processed by a separating system which uses classical processing. This system is the counterpart of block C of Figure 5, but it here implements a *nonlinear* function, because the mixing model (including the above quantum/classical conversion) resulting from that approach is nonlinear. This mixing model is specific to the considered BQSS problem (Heisenberg coupling + quantum/classical conversion). Various methods were therefore developed for adapting the parameters of the associated separating system. They have a relationship with classical ICA, thus leading to the introduction of Quantum-Source Independent Component Analysis (QSICA). A survey of these methods is available in [129]. Starting from the above QSICA separation principle, these methods cover various associated separation criteria, which are reminiscent of those reported in the previous sections for classical BSS, namely moment-based and cumulant-based approaches, and methods which perform output mutual information minimization or likelihood maximization (here extended to the original encountered nonlinear mixing model).

Another reported class of BQSS approaches (see e.g. [130]) is much more different from classical BSS, from the point of view of the nature and structure of the separating system, and of the considered separation principle. The separating system here directly receives mixed, i.e. coupled, *quantum* states and, after its parameters have been adapted, it only operates with quantum circuitry. In other words, the classical-BSS configuration of Figure 5 is here replaced by completely quantum data and processing means. Quantum/classical data conversion and classical processing means are only used to adapt the separating system parameters, i.e. in the part of the feedback adaptation loop which is the counterpart of the lowest part of the configuration shown in Figure 6 for adaptation in classical BSS. The proposed adaptation procedure is also specific to *quantum* BSS because, although it may eventually be shown to have some relationship with (QS)ICA, it is primarily based upon another separation principle. This principle is derived from the concept of (dis)entanglement, which appears in quantum physics and has no classical counterpart.

The above-mentioned investigations were focused on (quantum) BSS. On the contrary, a new classical-processing approach for mixed quantum sources was recently derived [131] by using an extended set of measurements, thus not only providing a BQSS method with new features, but also offering an attractive approach to achieve another goal, namely the blind estimation of the parameter values of the mixing system itself (i.e. without considering source state restoration). From the point of view of QIP, this yields a new field, namely Blind Quantum Process Tomography (BQPT), which is the blind extension of the existing field of QPT [122], that consists in *non-blindly* identifying the behavior of a quantum "system" (which is here the mixing operator). Besides, with respect to the concepts that we introduced at the beginning of this chapter, in this framework BQPT may be considered as the quantum extension of BMI.

8 Extensions and conclusion

As explained above, Blind Source Separation (BSS) methods aim at estimating a set of unknown source signals, after they were transferred through an unknown mixing function, whereas Blind Mixture Identification (BMI) methods aim at estimating that mixing function. Although these BSS and BMI problems may thus be defined in a very generic way, they give rise to many configurations, to be analyzed independently, depending on the properties of the source signals and on the mixing function faced in the considered application. In this chapter, we provided an overview of the main classes of mixing functions adressed in the literature and of the source properties exploited in BSS/BMI methods. A more detailed analysis of these fields would e.g. also address the following topics:

- Other BSS and/or BMI methods for underdetermined mixtures, beyond the comments that we provided in Section 4.1.2 about SCA-based methods: see e.g. [9, 10, 11] for some statistical methods, or Chapter 9 of [1] for an overview.

- Configurations involving complex-valued data, e.g. as in [132, 133], or discrete-valued data, e.g. as in [9, 134, 135] (besides, the measurements mentioned in Section 7 yield binary-valued signals).

- Equivariant ICA algorithms for linear instantaneous mixtures, whose performance does not depend on the mixing matrix (see e.g. [136] and the EASI algorithms presented therein ; see also [49]), and BSS methods based on natural gradient [137].

- Other BSS methods operating in a transformed domain. In this chapter, we focused on the STFT, but other time-frequency transforms exist. This especially includes quadratic time-frequency transforms, e.g. described in [60, 61]. The use of the latter transforms in the framework of BSS is detailed in Chapter 11 of [1]. The reader may also refer e.g. to [138].

- Various types of approaches based on multidimensional or joint analysis, some of which allow statistical dependence between some of the source signals. This e.g. includes taking into account that some physical objects, considered as "physical sources", are each described not by a single source signal, but by a set of such signals, thus defining a source vector and a corresponding multidimensional subspace: for instance, the data associated with the electrical activity of the heart of a pregnant woman spans a 3-dimensional subspace, whereas the electrical activity of the fetus's heart may yield a lower dimension, as discussed in [15]. Besides, the concept of Multidimensional Independent Component Analysis (MICA) was introduced in [139]. Combining MICA with the principle of invariant-feature subspaces yields Independent Subspace Analysis (ISA) [140]. In addition, Independent Vector Analysis (IVA) was especially introduced in [141] and Chapter 6 of [5], as an extension of ICA from univariate to multivariate components, which is able to take advantage of statistical dependence inside each multivariate signal, in addition to statistical independence between multivariate signals. A description of IVA methods and of their application to fMRI signals is e.g. available in [142]. Similarly, group ICA was introduced in the biomedical field in [143] for handling a group of subjects, and its application to fMRI is e.g. presented in [143] and [142].

So, well-stabilized ICA methods now exist for the most standard BSS/BMI configuration, namely determined linear instantaneous mixtures of independent source signals. Beyond that case, a variety of configurations may be defined, and the most advanced of them are still the topic of quite active research. Major developments are therefore expected to occur in the forthcoming years in the overall fields of BSS/BMI methods and related applications.

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Erratum: replace two terms $E\{r_i\}E\{q_i\}$ in (33) of [127] by $E\{r_i q_i\}$, since q_i depends on r_i.

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