Statistical intrusion detection and eavesdropping in quantum channels with coupling: Multiple-preparation and single-preparation methods

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Abstract Classical, i.e. non-quantum, communications include configurations with multiple-input multiple-output (MIMO) channels. Some associated signal processing tasks consider these channels in a symmetric way, i.e. by assigning the same role to all channel inputs, and similarly to all channel outputs. These tasks especially include channel identification/estimation and channel equalization, tightly connected with source separation. Their most challenging version is the blind one, i.e. when the receivers have (almost) no prior knowledge about the emitted signals. Other signal processing tasks consider classical communication channels in an asymmetric way. This especially includes the situation when data are sent by Emitter 1 to Receiver 1 through a main channel, and an "intruder" (including Receiver 2) interferes with that channel so as to extract information, thus performing so-called eavesdropping, while Receiver 1 may aim at detecting that intrusion, which leads to a decision problem (existence of intrusion / no intrusion). Part of the above processing tasks have been extended to quantum channels, including those that have several quantum bits (qubits) at their input and output. For such quantum

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channels, beyond previously reported work for symmetric scenarios, we here address asymmetric (blind and non-blind) ones, with emphasis on intrusion detection and additional comments about eavesdropping. To develop fundamental concepts, we first consider channels with exchange coupling as a toy model. We especially use the general quantum information processing framework that we recently developed, to derive new attractive intrusion detection methods based on a single preparation of each state. Finally, we discuss how the proposed methods might be extended, beyond the specific class of channels analyzed here.

Keywords quantum channel, exchange coupling, intrusion detection, eavesdropping, blind / unsupervised processing, single-preparation quantum information processing (SIPQIP)

1 Previous works and problem statement

The communication systems that involve classical, i.e. non-quantum, channels give rise to a variety of signal processing problems. Among them, two closely related problems are (i) channel identification, i.e. estimation, and (ii) channel equalization [31,20]. These problems may be seen as the communication version of, respectively, (i) system identification and (ii) system inversion and its multiple-signal extension to source separation, that are generic signal processing problems, which include various versions defined as follows. The basic version of system identification addresses single-input single-output (SISO) systems. It consists in estimating the unknown parameter values of such a system (i.e. of its transform) belonging to a known class, by using known values of its input (source signal s) and output (signal x). This version is stated to be non-blind (or supervised), as opposed to the more challenging, blind (or unsupervised), version of that problem, where the input values are unknown (and uncontrolled, but the input signal may be known to belong to a given class): See [1]. Both versions may then be extended to multiple-input multiple-output (MIMO) systems.

Besides, in various applications, what is needed is not the direct transform achieved by the above system, but the inverse of that transform (assuming it is invertible). For SISO non-blind and blind configurations, this is motivated by the fact that one eventually only accesses the ouput x of the above direct system, and one aims at deriving a signal y which ideally restores the original source signal s. To this end, one may use the above-mentioned system identification methods in order to first estimate the direct system, then derive its inverse and eventually transfer the output x of the direct system through the inverse system. Alternatively, one may develop methods for initially identifying the *inverse* system itself. Extended versions of this "(unknown) system inversion" task deal with MIMO configurations, where a set of original source signals s_1 to s_M are to be respectively restored by the outputs y_1 to y_M of the inverse system.

The blind MIMO version of the above system inversion problem is almost the same as blind source separation (BSS) (see e.g. [7, 22, 9, 25]): As in system inversion, BSS aims at canceling the contributions of all sources but one in each output signal of the separating system; however, in BSS, one often allows each output signal to be equal to a source signal only up to an acceptable residual transform. These transforms, called indeterminacies, cannot be avoided because only limited constraints are set on the source signals and on the direct system which combines (i.e., "mixes", in BSS terms) these signals.

Let us now consider quantum "signals" and systems, where these "signals" e.g. consist of the quantum states of quantum bits, or qubits, defined below. Then, among the above processing problems, the one which was first studied is non-blind system identification, especially[1] introduced in 1997 in [6] and often called "quantum process tomography" (QPT) by the quantum information processing (QIP) community: See e.g. [5, 27, 29, 33, 34, 2, 3, 28, 35, 36]. Besides, we introduced the field of "quantum source separation" (QSS) and especially its blind version (BQSS) in 2007: See [10]. We first mainly developed a class of BQSS methods related to the classical BSS methods based on Independent Component Analysis: See especially [11, 14]. We then proposed a second class of BQSS methods, based on output quantum state disentanglement: See especially [12, 13, 19]. Moreover, in 2015, we introduced the field of "blind quantum process tomography" (BQPT) in [15]. We then developed it especially in [18]. We also very recently [17] introduced methods for a closely related problem, namely Blind Hamiltonian Parameter Estimation (BHPE). All these QIP problems involve a quantum state transform, to be identified or inverted. Such a transform is also called a quantum process by the QIP community, or a quantum channel [29, 30], with an explicit reference to the field of communications, although the considered framework includes other quantum application fields in addition.

To solve the above QIP problems, we first proposed multiple-preparation methods, i.e. methods which require many copies of each considered quantum state value, in order to derive estimates of probabilities of associated measurement outcomes, thus using statistical approaches, as in usual QIP methods. In addition, in [16], we introduced the concept of SIngle-Preparation QIP (or SIPQIP) methods, i.e. methods that can operate with only one instance of each considered quantum state and that extract information thanks to statistical averaging over measurement outcomes associated with various states. We then especially provided a detailed report of the principles of this SIPQIP framework and of its application to BQPT in [18]. Finally, we very recently applied that framework to a variety of QIP tasks: See [17]. As shown further in this paper, this SIPQIP framework is particularly attractive for the communication applications considered here.

In multiple-source or multiple-qubit configurations, all above-defined classical or quantum processing problems are expressed symmetrically with respect to all sources/qubits. In contrast, other signal processing problems, e.g. re-

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[1] See also [29] p. 398 for the other earliest references.

lated to communications, assign a different role to different sources/qubits. First considering classical communications again, this especially involves two-source configurations, with a "main channel" from, say, Emitter 1 to Receiver 1 and with the following two possible cases. In Case 0, the above configuration, from Emitter 1, through the main channel, to Receiver 1, does not interact with its environment. In Case 1, such an interaction exists, i.e. an "intruder", often called the eavesdropper in the literature [29], interferes in some way with the above channel from Emitter 1 to Receiver 1. This yields the following two signal processing problems. On the one hand, the eavesdropper is interested in extracting information from the main channel. This information is typically derived by a Receiver 2 controlled by the eavesdropper, who may in addition control an Emitter 2, that may be considered as the jammer (this is the case in the quantum extension of this scenario analyzed further in this paper). In contrast, the eavesdropper ideally requires no prior knowledge about the data manipulated by Emitter 1 and Receiver 1. On the other hand, Receiver 1 often has a strong interest in intrusion detection, that is, in detecting that the eavesdropper is extracting, i.e. intercepting, information from the connection between Emitter 1 and Receiver 1. This receiver should perform this intrusion detection task by using only the data that he receives from the main channel (blind configuration) or by also using information that he gets from Emitter 1 (non-blind mode; some configurations are also stated to be semi-blind because Receiver 1 is provided with *very limited* prior information in addition to the received data). In all these configurations, Receiver 1 does not access information about the data possibly sent by Emitter 2 nor those obtained by Receiver 2.

How the above intrusion detection and eavesdropping capabilities extend to *quantum* channels is currently a major and still open problem. Whereas various approaches may be proposed to this end, e.g. depending on the nature of the considered quantum sources and channels, this paper aims at investigating this topic by analyzing how results from above-mentioned BQSS and related (i.e. BQPT and BHPE) investigations reported so far, hence with "symmetric" scenarios, may be exploited to derive first concepts for intrusion detection and/or eavesdropping, hence for asymmetric scenarios. We will thus only build upon the data model previously considered for BQSS and related tasks, whereas the quantum processing algorithms proposed in this paper are quite different from the above-mentioned BQSS and related algorithms, since they have very different goals. More precisely, we hereafter put the emphasis on intrusion detection and more briefly discuss eavesdropping.

The remainder of this paper is therefore organized as follows. In Section 2, we summarize the data model, i.e. the considered class of channels and its input, that was used in the above-mentioned BQSS and related methods, when addressing coupling between two qubits. We also describe measurements associated with the considered quantum states. Moving to intrusion detection in Section 3, new data models must first be derived from the above one, because the information available to Receiver 1 is not the same as in BQSS in this asymmetric scenario, with or without interaction between both qubits.

Then, two types of intrusion detection methods are proposed, respectively using the multiple-preparation and single-preparation frameworks. Finally, Section 4 contains a discussion of the features of the above methods, some considerations about the related eavesdropping problem and a conclusion.

2 Considered quantum channels: Coupling model for two qubits

The basic concepts for a single qubit, e.g. implemented as a spin 1/2, are provided in Appendix A. That description directly applies to several qubits if they are not "coupled", i.e. if they do not interact with one another. However, coupling between individual quantum states has to be considered in the QIP/QP area, in the same way as signal coupling exists in various *classical* signal processing systems. Coupling in quantum physical setups e.g. occurs when two electron spins interact through exchange. In [11], we considered a two-qubit system composed of two distinguishable [19] spins coupled according to the version of the Heisenberg model which has a cylindrical-symmetry axis, denoted Oz and collinear with the applied magnetic field. We analyzed in detail the global state of that two-qubit system resulting from that coupling and the associated measured values. As in Appendix A, the measured value of the component of each spin along axis Oz can only be $+\frac{1}{2}$ or $-\frac{1}{2}$. Therefore, when measuring the components of both spins, the obtained couple of values is equal to one of the four possible values $(+\frac{1}{2}, +\frac{1}{2})$, $(+\frac{1}{2}, -\frac{1}{2})$, $(-\frac{1}{2}, +\frac{1}{2})$ and $(-\frac{1}{2}, -\frac{1}{2})$. The probabilities of these four values are respectively denoted as p_1, p_2, p_3 and p_4 hereafter. These probabilities are related as follows to the state of the overall system composed of these two spins. This state may be expressed as a linear combination of the vectors of the four-dimensional basis $\{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$ which corresponds to the operators s_{1z} and s_{2z} respectively associated with the components of Spin 1 and Spin 2 along the symmetry axis Oz. As in Appendix A, each of the probabilities p_1 to p_4 is here equal to the squared modulus of the coefficient of the corresponding basis vector in the expression of the overall system state. In [11], we provided a detailed derivation of the expressions of these probabilities in the following configuration. The two spins are separately initialized (i.e. prepared) at time t_0, with states $|\psi_j(t_0)\rangle$ defined by

$$|\psi_j(t_0)\rangle = \alpha_j |+\rangle + \beta_j |-\rangle \tag{1}$$

where $j = 1$ for Spin 1 and $j = 2$ for Spin 2, first considering deterministic pure states. The initial state of the overall system composed of these two distinguishable spins is therefore equal to the tensor product (denoted as $\otimes$) of the states of both spins defined in (1), i.e.

$$|\psi(t_0)\rangle = |\psi_1(t_0)\rangle \otimes |\psi_2(t_0)\rangle. \tag{2}$$

That initial state $|\psi(t_0)\rangle$ is thus unentangled. The overall system state then evolves with time and the spin states thus get "mixed" (in the classical BSS

sense[2]) with one another, thus yielding an entangled state $|\psi(t)\rangle$ (except for very specific parameter values). The time evolution of the overall system state is defined by phase rotations, as in (21), and this here involves four frequencies. These frequencies depend on the Heisenberg coupling, which is especially characterized by the so-called principal value J_{xy} of the exchange tensor (see [11] for more details). We derived the expressions of the above probabilities p_1 to p_4 at an arbitrary time $t > t_0$, with respect to the polar representation of the initial qubit parameters α_j and β_j, which reads

$$\alpha_j = r_j e^{i\theta_j} \qquad \beta_j = q_j e^{i\phi_j} \qquad\qquad j \in \{1, 2\} \tag{3}$$

with $0 \leq r_j \leq 1$ and

$$q_j = \sqrt{1 - r_j^2} \tag{4}$$

due to (20). The above probabilities may then be expressed as follows:

$$p_1 = r_1^2 r_2^2 \tag{5}$$

$$p_2 = r_1^2(1 - r_2^2)(1 - v^2) + (1 - r_1^2)r_2^2 v^2$$
$$\qquad\quad -2r_1 r_2 \sqrt{1 - r_1^2}\sqrt{1 - r_2^2}\sqrt{1 - v^2}\, v \sin \Delta_I \tag{6}$$

$$p_4 = (1 - r_1^2)(1 - r_2^2) \tag{7}$$

with

$$\Delta_I = (\phi_2 - \theta_2) - (\phi_1 - \theta_1) \tag{8}$$

$$\Delta_E = -\frac{J_{xy}(t - t_0)}{\hbar} \tag{9}$$

$$v = \text{sgn}(\cos \Delta_E)\sin \Delta_E \tag{10}$$

where $\hbar$ is the reduced Planck constant. Probability p_3 is not considered, since it may be derived from the other three probabilities by means of

$$p_1 + p_2 + p_3 + p_4 = 1. \tag{11}$$

Eq. (5)-(7) yield a QSS problem because, using the classical BSS terminology, they show that some "observations" are "mixtures" of the quantities which define quantum "sources". That QSS problem is detailed in Appendix B. In most configurations, the values of the coupling parameter J_{xy} and therefore of v (see (9)-(10)) are unknown (the sign of J_{xy} is however known in some configurations) but fixed, i.e. deterministic. This corresponds to the *blind* version of this QSS problem. In this configuration, estimating the sources first requires one to estimate the unknown mixing parameter v. BQSS is thus an estimation problem [32], where one aims at deriving continuous-valued quantities. In contrast, as shown below, intrusion detection is a decision making

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[2] The terms "mixing" and "mixtures" should be considered with care when dealing with quantum data: In this paper, when speaking of random pure states, we implicitly refer to some statistical mixtures, as defined in quantum mechanics, but, except in the present note, we do not explicitly use the expression "statistical mixture".

(i.e. detection) problem [32], with a "yes/no answer" to the determination of which case, among two possible cases, is actually faced. The algorithms proposed below to answer that decision problem are therefore quite different from those introduced in our previous papers to solve the BQSS problem.

3 Intrusion detection

3.1 Data models for intrusion detection

The BQSS methods that we proposed in our previous papers to address the data model of Section 2 take advantage of all the data that are available in that model, that is, of the probabilities p_1, p_2 and p_4 that are derived (estimated in practice) from spin component measurements associated with *both* qubits. This corresponds to the above-defined symmetric scenario. In contrast, these "two-qubit probabilities", i.e. joint probabilities, are not known any more in the investigation reported in the present paper, due to the considered asymmetric scenario. More precisely, if starting from the data model of Section 2 as a toy model for quantum channels at this stage, the considered intrusion detection scenario may be defined as follows. The main channel considered in Section 1 here goes from the initial deterministic pure state $|\psi_1(t_0)\rangle$ involved in (2), provided by Emitter 1, to the results of the measurements performed by Receiver 1 at the final time t for the *first* qubit of the data model of Section 2. These measurements thus only allow Receiver 1 to access "one-qubit probabilities", i.e. marginal probabilities, associated with Qubit 1 (at the final time t).

When intrusion is actually performed, the above model also involves coupling with the *second* qubit (see Case 1 in Section 1). The probabilities of measurement results of Receiver 1 thus also depend (i) on the initial state $|\psi_2(t_0)\rangle$ involved in (2) and provided by Emitter 2, and (ii) on the qubit coupling phenomenon leading to (5)-(7) and hence on the parameter J_{xy} of that exchange coupling model. We then aim at defining and exploiting the probabilities of the measurement results of Receiver 1. This may be performed as follows, still considering the data model of Section 2. At the final time t, the measurements for each qubit with index $j \in \{1, 2\}$ define a binary random variable (RV) denoted as b_j, whose possible values are equal to $+\frac{1}{2}$ and $-\frac{1}{2}$. The two events defined by the outcomes of this RV are therefore denoted as $\{b_j = +\}$ and $\{b_j = -\}$ hereafter. The joint probabilities of the two RVs defined by the considered two qubits, namely $P(b_1 = +, b_2 = +)$, $P(b_1 = +, b_2 = -)$, $P(b_1 = -, b_2 = +)$ and $P(b_1 = -, b_2 = -)$ are nothing but the above-defined probabilities p_1, p_2, p_3 and p_4. Besides, the marginal probabilities associated with measurements performed for Qubit 1 only may be expressed as follows

$$P(b_1 = +) = P(b_1 = +, b_2 = +) + P(b_1 = +, b_2 = -)$$

$$(12)$$

$$= p_1 + p_2.$$

$$(13)$$

Using (5)-(6), this yields

$$P(b_1 = +) = r_1^2 + v^2(r_2^2 - r_1^2)$$
$$-2r_1 r_2 \sqrt{1 - r_1^2} \sqrt{1 - r_2^2} \sqrt{1 - v^2} v \sin \Delta_I.$$

$$(14)$$

Besides, $P(b_1 = -)$ provides no additional information, because

$$P(b_1 = -) = 1 - P(b_1 = +). \tag{15}$$

As stated above, this model corresponds to the case when intrusion actually occurs (Case 1), which physically corresponds to electrons being close to one another or both close to the same atom/ion, hence with exchange coupling involving $J_{xy} \neq 0$. Let us now consider the case when no intrusion is performed (Case 0), which physically corresponds to electrons being far from one another. The corresponding data model may be derived by setting $J_{xy} = 0$ in the data model defined above for Case 1. Eq. (9)-(10) then yield $v = 0$, so that (14) reduces to

$$P(b_1 = +) = r_1^2. \tag{16}$$

This result could be anticipated as follows. We here consider the case when Qubit 1 does not interact with Qubit 2, so that its state evolves according to (21) with $j = 1$. Therefore, as explained in Appendix A, at any time t, the probability of $\{b_1 = +\}$ is equal to the squared modulus of the coefficient in (21) of the vector $|+\rangle$. It is thus equal to $|\alpha_1|^2$, i.e. r_1^2 due to (3).

3.2 Multiple-preparation intrusion detection methods

The problem addressed in this paper consists of only using the measurements performed by Receiver 1 so as to determine whether intrusion occurs or not, i.e. whether the main channel is in Case 0 or Case 1. That can be seen as a hypothesis testing problem (i.e. a decision making or detection problem) [32], with hypotheses H0 and H1 respectively corresponding to the above-defined Cases 0 and 1.

To perform the above test, several methods may be proposed. Their simplest version uses a single, deterministic, value of the initial states $|\psi_j(t_0)\rangle$ with $j \in \{1, 2\}$, of the final state $|\psi(t)\rangle$ of the two-qubit system at time t and of the associated probability $P(b_1 = +)$. This method exploits the fact that, "in general", this probability does not take the same value depending whether Case 0 or Case 1 is considered, as shown by (14) and (16). By "in general", we mean that the values in (14) and (16) are different except for very specific values of the quantities that they involve, that are related to the initial quantum state (parameters r_1, r_2, Δ_I) or to the channel (parameters v and hence J_{xy} and $(t - t_0)$). We here ignore these very specific cases and we will further discuss this topic in Section 4. One should also keep in mind that, in practice, an estimate of $P(b_1 = +)$ is used and, to obtain it, one must prepare

many copies of the initial state $|\psi(t_0)\rangle$ of the two-qubit system, as explained in Appendix B.

A first intrusion detection method then consists of using a value $|\psi_1(t_0)\rangle$ of the state provided by Emitter 1, or at least a value of its parameter r_1 in (3), that Receiver 1 knows. Receiver 1 then compares his estimate of $P(b_1 = +)$ to r_1^2 and makes the following decision, based on (14) and (16): If that estimate of $P(b_1 = +)$ is "close enough" (one may aim at deriving a bound from test theory for given statistics of the considered data) to r_1^2, then Receiver 1 decides that no intrusion is being performed (Case 0); otherwise, he decides that intrusion is occurring (Case 1). Since this approach requires Receiver 1 to know a value of emitted (i.e. source) data, it can be considered to be a non-blind method. Of course, such an approach can only be used to perform detection intrusion during one or a few limited time periods, moreover jointly defined by Emitter 1 and Receiver 1 (somewhat as when using a synchronization sequence in classical communication networks) because, otherwise, Receiver 1 would have to permanently know which states are provided by Emitter 1, which would make data transmission in the main channel useless.

To reduce the above restriction about known emitted data, one may instead develop a blind variant of the above method, i.e. a variant in which Receiver 1 does not know which state is provided by Emitter 1 (and by Emitter 2) and can only use estimates of $P(b_1 = +)$. The proposed approach then consists of splitting the above-mentioned complete set of copies of the initial state $|\psi(t_0)\rangle$ in two successive subsets. An estimate of $P(b_1 = +)$ is then separately computed by Receiver 1 for each subset and these two estimates are then compared: If they are "far enough" (with the same comment as above concerning an associated bound) from one another, Receiver 1 considers that the main channel switched between Cases 0 and 1 (or between Case 1 with one value of v to Case 1 with another value) from one of the above subsets to the other.

Both variants of this method have limitations. In particular, they require Receiver 1 to estimate at least one value of the probability $P(b_1 = +)$, which requires many copies of the same state $|\psi_1(t_0)\rangle$ to be transmitted by Emitter 1 through the main channel (with a timing known by Receiver 1) and, more importantly, many copies of the same state $|\psi_2(t_0)\rangle$ to be provided by Emitter 2 (i.e. the intruder) meanwhile, which is very constraining from a practical point of view. If sticking to that multi-preparation framework that is usual in QIP, there might seem to be no solution to this problem at first glance, because estimating a probability value requires a large number of trials for the considered single experiment. However, beyond that usual QIP framework, we recently developed an original concept (see [16,18,17]), called SIngle-Preparation QIP (or SIPQIP) for its general version, and especially applied to BQSS and related tasks so far, which solves the above problem, as will now be shown. Briefly, instead of estimating a single deterministic probability from many copies of a single deterministic quantum state, SIPQIP estimates the expectation of a random probability associated with a random quantum pure state, i.e. associated with various quantum states whose coefficients (such as α_j and β_j in (1))

are randomly drawn in practice, by possibly using a single instance of each of these states.

3.3 Single-preparation intrusion detection methods

We here address the situation when Receiver 1 considers the initial state (2) of the two-qubit system, and hence the initial state (1) of each qubit, as a *random* pure state, i.e. when the parameters r_j (and hence q_j), θ_j and ϕ_j in (3) are RVs (this concept of random quantum pure state is defined in more detail in [8]). Then, the probabilities p_1, p_2 and p_4 in (5)-(7) are also RVs, and so is the probability $P(b_1 = +)$. The approach proposed here is then based on estimating the *expectation* of $P(b_1 = +)$ over random states (1). Due to (13), this expectation may be expressed with respect to the expectations of p_1 and p_2. A major property is then that all these expectations may in practice be estimated by using only one instance of each of the considered states (1). This property was theoretically justified in [16,18] and confirmed by numerical tests in [16,18,17]. Its relevance may be outlined as follows. For each expectation $E\{p_k\}$ of a random probability p_k to be estimated, in practice the expectation operator $E\{.\}$ is replaced by a sample mean, i.e. by a *sum* (of values, moreover normalized). Similarly, each probability p_k is replaced by a sample frequency, i.e. by a *sum* (of 1 and 0, depending whether the considered event occurs or not for each trial defined by a preparation of the initial quantum states (1) and by an associated measurement of the considered spin component, for each of the two spins; this summation is here again followed by a normalization, by the total number of trials). $E\{p_k\}$ is therefore estimated by a (normalized) "sum of sums", which may then be reinterpreted as a single global sum, and what primarily matters is the total number of preparations of initial quantum states (1) involved in that global sum, whereas the number of preparations for each state value (1) may be decreased, down to 1.

   In our previous papers, we applied the above analysis to the probabilities p_1, p_2 and p_4 of the data model (5)-(7), in order to achieve BQSS [16, 17], BQPT [16,18] and BHPE [17]. Here, we apply it to a new SIPQIP task, namely intrusion detection, thus introducing its single-preparation version. We therefore consider the expectations of (14) and (16). For Case 1, Eq. (14) thus yields

$$E\{P(b_1 = +)\} = E\{r_1^2\} + v^2(E\{r_2^2\} - E\{r_1^2\})$$
$$-2E\{r_1 r_2 \sqrt{1 - r_1^2} \sqrt{1 - r_2^2} \sin \Delta_I\}$$
$$\times \sqrt{1 - v^2} v. \tag{17}$$

This might be further simplified when moreover assuming that the RVs r_1, r_2 and Δ_I are statistically independent, as in our previous BQSS and related investigations (see e.g. [8] about random quantum sources and their independence). However, that additional assumption is not required for the task considered here.

Similarly, for Case 0, Eq. (16) yields

$$E\{P(b_1 = +)\} = E\{r_1^2\}. \tag{18}$$

The two variants of the method of Section 3.2 may then be transposed to the single-preparation framework considered here. We hereafter transpose only the first variant, because it is especially attractive: It yields a simple protocol while requesting Receiver 1 to have only limited prior knowledge about the emitted data, namely the value of $E\{r_1^2\}$. This method may therefore be stated to be blind (so-called blind signal processing methods are in fact not completely blind because they set some, although possibly very limited, conditions on the considered data) or semi-blind for the sake of clarity. It should be noted that, unlike its multiple-preparation version of Section 3.2, this single-preparation method does not require Receiver 1 to know any *individual* state value prepared by Emitter 1, which is very attractive. Moreover, it does not depend on the state values prepared by Emitter 2 (again provided the "specific values" are avoided).

This single-preparation method operates as follows. Receiver 1 gets a set of final two-qubit states $|\psi(t)\rangle$, without any request on the number of copies per state value, unlike in Section 3.2. After performing a single one-qubit measurement on Qubit 1 for each such state, Receiver 1 derives an estimate of $E\{P(b_1 = +)\}$, as explained above. Receiver 1 then compares this estimate to the known value $E\{r_1^2\}$ and makes the following decision, due to the expressions (17) and (18) respectively for Cases 1 and 0: If $E\{P(b_1 = +)\}$ is "close enough" (with the same comment as above) to $E\{r_1^2\}$, then Receiver 1 decides that no intrusion is being performed (Case 0); otherwise, he decides that intrusion is occurring (Case 1). This method again exploits the fact that, "in general" (in the same sense as above), $E\{P(b_1 = +)\}$ does not take the same value in Cases 0 and 1, as shown by (17) and (18).

4 Discussion and conclusion

A transform applied to a (possibly multi-qubit) quantum state is often referred to as a "quantum process", by the scientific community focused on quantum process tomography, or a "quantum channel", by the scientific community focused on quantum communications. So far in this paper, we focused on a particular class of such quantum processes/channels, namely two-qubit processes based on cylindrical-symmetry Heisenberg-type exchange coupling. This class of processes is relevant for describing coupling between two close electron spins, and that was our motivation for investigating such processes in our previous papers, focused on the field of spintronics and on associated data processing tasks, such as BQSS, BQPT and BHPE.

When moving to the detection intrusion task with quantum channels in the present paper, we still considered the above class of channels so far, in order to more easily develop first concepts for intrusion detection, by taking advantage of the information about these channels that was already available from our

previous investigations. However, it should be clear that, from the point of view of the detection intrusion task, the above class of channels is here only regarded as a toy model for first investigations: We do not claim that it will be relevant when then studying practical communication scenarios, depending on the considered hardware implementation. In particular, communications based on photons are discussed in Appendix C.

The competition between eavesdropping and intrusion detection, mentioned for the above new scenario, already clearly appeared in the intrusion detection methods proposed in this paper: these methods can detect intrusion "in general", i.e. except for specific values of the considered parameters (quantum states and channel parameters). Another extension of this paper therefore consists of analyzing these specific values in more detail, in order to determine whether they allow the eavesdropper to defeat the intrusion detector, while extracting useful information from the main channel. More generally speaking, in this paper we focused on the capabilities of the proposed approaches in terms of intrusion detection, but the associated eavesdropping capabilities should also be analyzed in our future work.

Finally, if one e.g. aims at developing intrusion detection methods that are statistical in the sense that they are based on averages of measurement outcomes (to estimate probabilities or their expectations), the following feature, specific to the most advanced methods that we proposed in this paper, should be kept in mind because it is likely to remain of high interest in future methods too. Our general SIPQIP framework for quantum processing makes it possible to use only one instance of each source state (i.e. emitted state), and this is especially attractive in communication scenarios, because (i) the receiver of the main channel (Receiver 1) should preferably not require any control on the states provided by the emitter of the main channel (Emitter 1) and (ii) anyway, that receiver surely has no control on the states provided by the intruder, i.e. jammer (Emitter 2). This ability of our SIPQIP methods to operate with one instance of each state is obtained by using *expectations* of probabilities, the latter probabilities being random-valued because we consider random quantum pure states. In contrast, a drawback of usual QIP methods is that they require many copies of a single state (or many copies per state, if considering several states) to estimate the individual probabilities associated with that state.

A Definition of a single qubit

Qubits are widely used instead of classical bits for performing computations in the field of QIP [29]. Whereas a classical bit can only take two values, usually denoted as 0 and 1, at an initial time t_0 a qubit with index j has a quantum state expressed, for a pure state, as

$$|\psi_j(t_0)\rangle = \alpha_j|+\rangle + \beta_j|-\rangle \tag{19}$$

in the basis defined by the two orthonormal vectors that we hereafter[3] denote $|+\rangle$ and $|-\rangle$, where α_j and β_j are two complex-valued coefficients constrained to meet the condition

$$|\alpha_j|^2 + |\beta_j|^2 = 1 \tag{20}$$

which expresses that the state $|\psi_j(t_0)\rangle$ is normalized. In most of the literature, α_j and β_j are deterministic, i.e. fixed, values so that $|\psi_j(t_0)\rangle$ is a *deterministic* pure state. In part of our investigations dealing with BQSS and related tasks, we also considered the case when α_j and β_j are random variables (RVs), so that $|\psi_j(t_0)\rangle$ is a *random* pure state (See e.g. [11, 14, 8, 18]).

From a Quantum Physics (QP) point of view, the above abstract mathematical model especially applies to electron spins 1/2, which are quantum (i.e. non-classical) objects. The component of such a spin, with index j, along a given arbitrary axis Oz defines a two-dimensional linear operator s_{jz}. The two eigenvalues of this operator are equal to $+\frac{1}{2}$ and $-\frac{1}{2}$ in normalized units, and the corresponding eigenvectors are therefore denoted as $|+\rangle$ and $|-\rangle$. The value obtained when measuring this spin component can only be $+\frac{1}{2}$ or $-\frac{1}{2}$. Moreover, let us assume this spin is in the state $|\psi_j(t_0)\rangle$ defined by (19) when performing such a measurement. Then, the probability that the measured value is equal to $+\frac{1}{2}$ (respectively $-\frac{1}{2}$) is equal to $|\alpha_j|^2$ (respectively $|\beta_j|^2$), i.e. to the squared modulus of the coefficient in (19) of the associated eigenvector $|+\rangle$ (respectively $|-\rangle$).

The above discussion concerns the state of the considered spin at a given initial time t_0. This state then evolves with time. The spin is here supposed to be placed in a static magnetic field and thus coupled to it. The time interval when it is considered is assumed to be short enough for the coupling between the spin and its environment to be negligible. In these conditions, the spin has a Hamiltonian [29]. Therefore, if the spin state $|\psi_j(t_0)\rangle$ at time t_0 is defined by (19), it then evolves according to Schrödinger's equation and its value at any subsequent time t is

$$|\psi_j(t)\rangle = \alpha_j e^{-i\omega_p(t-t_0)}|+\rangle + \beta_j e^{-i\omega_m(t-t_0)}|-\rangle \tag{21}$$

where the real (angular) frequencies ω_p and ω_m depend on the considered physical setup and i is the imaginary unit.

B Quantum source separation problem associated with the considered quantum channels

We here detail the quantum source separation problem associated with the "mixing model" (5)-(7). That model involves the following items. The observations are the probabilities p_1, p_2 and p_4 measured for each choice of the initial states (1) of the qubits. More precisely, these probabilities are not known exactly but estimated in practice. The procedure that we used to this end e.g. in [11, 14], and that is also widely employed in the QIP literature [3, 6], operates as follows for each choice of the initial states (1) of the qubits. We repeatedly perform two operations: i) we first initialize these qubits according to (1) and ii) after a fixed time interval when coupling occurs, we measure the two spin components along Oz associated with the system composed of these two coupled qubits. The relative frequencies of occurrence of all four possible couples of values of spin components (i.e. $(+\frac{1}{2}, +\frac{1}{2})$ to $(-\frac{1}{2}, -\frac{1}{2})$) then yield estimates of the corresponding probabilities. This approach therefore requires a large number (typically from a few thousand up to a few hundred thousand [11, 19]) of copies of the considered two-qubit state. At this stage, we ignore the resulting estimation errors and therefore consider the exact mixing model (5)-(7). Using standard BSS

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[3] These vectors $|+\rangle$ and $|-\rangle$ are often respectively denoted as $|0\rangle$ and $|1\rangle$ (see e.g. [29]), especially when considering an abstract view of qubits. When having in mind the physical implementation of qubits as electron spins, as in most of the present paper, the notations $|+\rangle$ and $|-\rangle$ are also widely used, with a reference to spin component measurements along the quantization axis, as detailed further in this paper.

notations, the observation vector is therefore $x = [x_1, x_2, x_3]^T$, where ^{T} stands for transpose and[4]

$$x_1 = p_1, \qquad x_2 = p_2, \qquad x_3 = p_4. \tag{22}$$

Eq. (5)-(7) show that the source vector to be retrieved from these observations turns out to be $s = [s_1, s_2, s_3]^T$ with $s_1 = r_1$, $s_2 = r_2$ and $s_3 = \Delta_I$. The parameters q_j are then derived from (4). The four phase parameters in (3) cannot be individually extracted from their combination Δ_I (anyway, only the phase differences $(\phi_j - \theta_j)$ have a physical meaning [18]). The transform from the sources to the observations defined by the non-linear mixing model (5)-(7) involves a single "mixing parameter", namely v. As shown by (10), this parameter always meets the condition $0 \leq v^2 \leq 1$.

C Communications based on photons

Communications based on photons deserve the following comments. First considering the classical framework, everyday communications use electromagnetic waves propagating either in free space or in a solid medium, e.g. an optical fiber [26]. Such media are non-magnetic, and their electric properties are classically described by the induction vector $\vec{D}$ (H. Lorentz), representing a local mean of the microscopic electric vector [24]. $\vec{E}$ being the applied field, in vacuum, $\vec{D} = \varepsilon_0 \vec{E}$ (SI units, ε_0: vacuum permittivity), and in a dielectric medium $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$. The polarization $\vec{P}$ may be seen as the response to the excitation $\vec{E}$, ferroelectrics, with a spontaneous polarization, being an exception. Most dielectric media are *linear,* i.e. $\vec{P}$ increases linearly with the excitation (description using a scalar or more generally a tensor not depending upon the excitation). Moreover, the appearance of the laser in 1960, i.e. of intense coherent electromagnetic sources, allowed the development of *non-linear* optics. Turning now to the quantum behavior associated with these phenomena, one should again make a distinction bewteen linear and non-linear setups. One first thinks of an electromagnetic wave propagating in vacuum space (or possibly in a linear medium): Already in 1930 Dirac [21] considered a weak electromagnetic beam and its associated photons; a device separates this beam into two partial beams, which are then made to interfere. One could then a priori think that two distinct photons possibly interfere. But, in such conditions, according to the general principles of quantum mechanics "*each photon then interferes only with itself. Interference between two different photons can never occur*". With respect to photon-based quantum communications addressed in the present paper, this entails that, if only considering communications through vacuum space or a linear medium, no entanglement is created in the transmission channel itself. This should be contrasted with the scenarios considered above, where entanglement is created by the channel itself (here with exchange coupling), whereas the original two-qubit state associated with Emitter 1 and Emitter 2 is unentangled.

The above manifestation of the superposition principle for photons, or the presence of (previously prepared) entangled states when more than one photon are implied may also be found in dielectrics, but other quantum phenomena may also be found in some optically *non-linear* dielectric materials: 1) two intense laser beams at frequencies ω_1 and ω_2 may allow fluorescence at $\omega_1 + \omega_2$; there, two photons with respective frequencies ω_1 and ω_2 generate a photon with frequency $\omega_1 + \omega_2$ [4]. 2) spontaneous emission may sometimes allow emission at the difference frequency: The material receives a laser beam with frequency ω_p (p: pump), and emits at both ω (with a material dependent value) and $\omega_p - \omega$ (so-called optical parametric fluorescence); here, a photon with frequency ω_p generates two photons,

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[4] It should be noted that the observed signals involved in this QSS problem have a specific nature, as compared with standard non-quantum BSS problems. In the latter problems, each value of an observed signal is usually the value of a measured physical quantity, such as the value of a voltage measured at a given time. In contrast, as shown by (22), each value of an observed signal is here the value of a *probability* (which is estimated in practice). The overall signal composed of all successive values of a given observation (e.g. all values of x_1) therefore consists of a set of values of probabilities (e.g. all values of p_1), which depend on the values of the coefficients used for initializing the qubit states.

with respective frequencies ω and $\omega_p - \omega$ [4]. Quantum communications take profit of the superposition principle, e.g. when involving two photons in an entangled state, and of the no-cloning theorem, both specific to quantum mechanics. Future quantum communication networks should make use of quantum teleportation - which allows transport of information, presently using entangled photon qubits - and of quantum repeaters interconnecting quantum nodes [23]. An eavesdropper, trying to access the information circulating within such a network could e.g. try and operate by interacting with a repeater. Besides, one may imagine a scenario involving transmission through quantum channels, by means of photons, be they initially entangled or not, mainly with free propagation (linear medium), but now also with a non-linear medium inserted (e.g. by an eavesdropper) in part of the overall transmission path forming what we called the "main channel" between Emitter 1 and Receiver 1 above. One might then investigate to which extent an "intruder", composed of Emitter 2 and Receiver 2, would thus be able to interact with the main channel so as to extract information from it (eavesdropping), and to which extent Receiver 1 would be able to detect this intrusion. The relevance and attractiveness of this scenario need to be further investigated.

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Conflict of interest

The authors declare that they have no conflict of interest.

Data availability statement

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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