

Blind quantum source separation: quantum-processing qubit uncoupling systems based on disentanglement

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Abstract

Blind Quantum Source Separation (BQSS) deals with multi-qubit states, called “mixed states”, obtained by applying an unknown “mixing function” (which typically corresponds to undesired coupling, e.g. between qubits implemented as close electron spins $1/2$) to unknown multi-qubit “source states”, which are product states (and pure in the simplest case, considered in this paper). Some other properties are also possibly requested from these source states and/or mixing function. Using mixed states, BQSS systems aim at restoring (the information contained in) source states, during the second phase of their operation (“inversion phase”). To this end, they estimate the unmixing function (inverse of mixing function), during the first phase of their operation (“adaptation phase”). Most previously reported BQSS systems first convert mixed states into classical-form data, that they then process with classical means. Besides, they estimate the unmixing function by using statistical methods related to classical Independent Component Analysis. On the contrary, the new BQSS systems proposed here use only quantum-form data and quantum processing in the inversion phase, and they use classical-form data during the adaptation phase only. Moreover, their unmixing function estimation methods are essentially based on using unentangled source states during that phase. They mainly consist of disentangling the output quantum state of the separating system (for a few source states). Afterwards, they can also restore entangled source states. They yield major improvements over previous systems, concerning restored source parameters, associated indeterminacies and approximations, number of source states required for adaptation, numbers of source state preparations in adaptation and inversion phases. Numerical tests confirm that they accurately restore quantum source states.

Keywords: separation and restoration of quantum states, entanglement, disentanglement, blind or unsupervised quantum processing, control of quantum adaptive system, cylindrical-symmetry Heisenberg coupling

1. Introduction

Blind (or unsupervised) Source Separation (BSS) is a generic classical (i.e. non-quantum) signal processing problem, which may be briefly defined as the estimation of a set of source signals which have unknown values but some known properties, using known values of a set of mixed signals which result from the application of an (almost) unknown mixing function to these source signals. Depending on the considered source signal properties and/or type of mixing function, various classes of BSS methods, including Independent Component Analysis (ICA), have been developed and are e.g. detailed in the books [7],[9],[14] [31],[35] or encyclopedia [23]. These publications also present quite varied applications of BSS methods. Such applications are described in Appendix A.1 and Appendix A.2.

Beyond the above diversity of classical signals which lead to BSS problems, one may anticipate that “quantum signals” will require quantum extensions of BSS methods, as the field of Quantum Information Processing [3],[26],[38],[40],[47] keeps on extending in the coming years. Quantum signals are especially defined in terms of quantum bit (i.e. qubit) states, with qubits e.g. physically implemented as electron spins. Therefore, quantum BSS methods are expected to involve “mixtures”

of qubit states, where the term “mixtures” is used in the above classical BSS sense¹, i.e. combinations of these states, e.g. due to electron spin coupling. Such coupling may occur in future quantum computers, when qubit registers involve electron spins situated close to one another.

This led us to introduce the field of Quantum Source Separation (QSS), and especially its Blind version² (BQSS), first proposed in [11] and then especially detailed in [17] and [21] (see Appendix A.3 and Appendix A.4 for applications of BQSS). The BQSS problem thus consists of restoring the information contained in individual quantum source states which have unknown values but some known properties, using known values of the mixtures of these source states which result from their *undesired*³ coupling, when the coupling operator is unknown (or only partly known). One may think of restoring the source states by transferring their available mixtures through the inverse of the coupling operator. However, this cannot be straightforwardly performed, because this operator is *unknown*. This

¹We are not considering quantum states which are statistical mixtures.

²See [17] for more details about its non-blind version, which is not considered in the present paper.

³On the contrary, a two-qubit gate using liquid NMR *takes advantage* [46] of the scalar coupling.

may suggest one to first identify (i.e. estimate the parameters of) this coupling operator but, again, this is not straightforward, because its input values are *unknown*. Besides, as detailed in [17], quantum state tomography and quantum process tomography (QPT) techniques reported so far [38], which were e.g. used in [48] for two-qubit systems, cannot achieve BQSS (QPT and its blind extension are discussed in more detail in Appendix A.4).

In almost all our previous works, we solved the BQSS problem by first converting the known mixed quantum data into classical-form signals [21] and then processing the latter signals with classical means. We thus first introduced a simple but somewhat constraining BQSS algorithm focused on first-order statistics [11],[17]. We then developed a more advanced approach derived from the maximum likelihood principle [12] (see also different considerations about this approach in [21]). Next, we proved the “separability” (defined below) in the sense of ICA, of the considered nonlinear mixing model [16], which allowed us to develop a first BQSS method based on ICA of these classical-form data [13], and then its approximate version using Edgeworth expansion [15]. Finally, we developed a method focused on second-order statistics, which combines various attractive features [21]. A survey of all these methods and of their numerical performance is provided in [21].

The above methods allow one to efficiently process the data derived from quantum/classical conversion. However, this initial conversion itself yields significant limitations, as detailed below. To reduce them, we here investigate a very different approach, which consists of using *quantum* processing, especially in the “inverting block” (defined below) of the separating system. In the short conference papers [19] and [20], we only briefly introduced the first versions of such systems combining quantum processing and blind adaptation. These preliminary versions yield limitations. In the current journal paper, we reduce them and extend this Blind Quantum-Processing Source Separation (BQPSS) framework much further.

As in classical BSS [14], [23], the development of complete BQPSS methods consists of defining the following items for each version of the proposed methods: the considered mixing model (see Section 2 below), the proposed separating system structure (Section 3), the proposed separation principle (see Section 4, where the separability properties resulting from this separation principle are also analyzed), the proposed separation criterion (Section 5) and the proposed separation algorithm (Section 5). In this paper, starting from the proposed separation principles, we derive different associated scenarios in terms of separation criterion and algorithm. We then present the numerical performance of two BQPSS methods thus obtained in Section 6. In Section 7, we summarize the features of this class of methods and compare them with those of our previous approaches. We eventually draw conclusions from this overall investigation in Section 8.

2. Mixing model

In [17], we first detailed the required concepts for a single qubit and then presented the type of coupling between two

qubits that we also consider here and that defines the “mixing model”, in (Q)SS terms, of the current investigation. We hereafter summarize the major aspects of that discussion, which are required in this paper.

A classical bit has *two* allowed values only, usually denoted 0 and 1. On the contrary, a qubit with index i considered at a given time t_0 has a quantum state and, if this state is pure, it may be equal to *any* unit-norm vector belonging to a two-dimensional space \mathcal{E}_i (see Appendix B.1). This state may then be expressed as

$$|\psi_i(t_0)\rangle = \alpha_i|+\rangle + \beta_i|-\rangle \quad (1)$$

in the basis of \mathcal{E}_i defined by the two orthonormal vectors that we hereafter⁴ denote $|+\rangle$ and $|-\rangle$, where α_i and β_i are two complex-valued coefficients.

In the BQSS configuration studied in this paper, we first consider a system composed of two distinguishable qubits, called “qubit 1” and “qubit 2” hereafter, at a given time t_0 . This system has a quantum state. If this state is pure, it belongs to the four-dimensional space \mathcal{E} defined as the tensor product (denoted \otimes) of the spaces \mathcal{E}_1 and \mathcal{E}_2 respectively associated with qubits 1 and 2, i.e. $\mathcal{E} = \mathcal{E}_1 \otimes \mathcal{E}_2$. We hereafter denote as \mathcal{B}_+ the basis of \mathcal{E} composed of the four orthonormal vectors $|++\rangle, |+-\rangle, |-+\rangle, |--\rangle$, where e.g. $|+-\rangle$ is an abbreviation for $|+\rangle \otimes |-\rangle$, with $|+\rangle$ corresponding to qubit 1 and $|-\rangle$ corresponding to qubit 2. At time t_0 , any pure state of this two-qubit system may be expressed as

$$|\psi(t_0)\rangle = c_1(t_0)|++\rangle + c_2(t_0)|+-\rangle + c_3(t_0)|-+\rangle + c_4(t_0)|--\rangle \quad (2)$$

with complex-valued coefficients $c_1(t_0)$ to $c_4(t_0)$ which are such that this state has unit norm. This state may also be represented by the corresponding vector of components in basis \mathcal{B}_+ , which reads

$$C_+(t_0) = [c_1(t_0), c_2(t_0), c_3(t_0), c_4(t_0)]^T \quad (3)$$

where T stands for transpose.

In particular, we study the case when the two qubits are independently initialized (i.e. prepared), with states defined by (1), with $i = 1$ for qubit 1 and $i = 2$ for qubit 2. We then have

$$|\psi(t_0)\rangle = |\psi_1(t_0)\rangle \otimes |\psi_2(t_0)\rangle \quad (4)$$

$$= \alpha_1\alpha_2|++\rangle + \alpha_1\beta_2|+-\rangle + \beta_1\alpha_2|-+\rangle + \beta_1\beta_2|--\rangle \quad (5)$$

and

$$C_+(t_0) = [\alpha_1\alpha_2, \alpha_1\beta_2, \beta_1\alpha_2, \beta_1\beta_2]^T. \quad (6)$$

Besides, we consider the case when the two qubits physically consist of two electron or nuclear spins 1/2, which have undesired coupling after they have been initialized according to

⁴These vectors $|+\rangle$ and $|-\rangle$ are often respectively denoted as $|0\rangle$ and $|1\rangle$ (see e.g. [38]). We had to use the notations $|+\rangle$ and $|-\rangle$ in [17], to avoid confusion, and we keep them here.

(4). The considered coupling is based on the Heisenberg model (see Appendix B.2) with a cylindrical-symmetry axis collinear to Oz , the direction common to the applied magnetic field and to our first chosen quantization axis. The time interval when these spins are studied is supposed to be short enough for their coupling with their environment to be negligible [30]. We denote as $C_+(t)$ the counterpart of (3) at an arbitrary time $t > t_0$, associated with the coupled state

$$|\psi(t)\rangle = c_1(t)|++\rangle + c_2(t)|+-\rangle + c_3(t)|-+\rangle + c_4(t)|--\rangle \quad (7)$$

of the two-qubit system at that time. The effect of the above Heisenberg coupling upon the qubit pair state, from $C_+(t_0)$ to $C_+(t)$, may then be represented as (see Appendix B.3)

$$C_+(t) = MC_+(t_0). \quad (8)$$

In basis \mathcal{B}_+ , the temporal evolution of the system's quantum state from t_0 to t is thus represented by the matrix M of (8). Then [17]:

$$M = QDQ^{-1} = QDQ \quad (9)$$

with

$$Q = Q^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

and D equal to

$$\begin{bmatrix} e^{-i\omega_{1,1}(t-t_0)} & 0 & 0 & 0 \\ 0 & e^{-i\omega_{1,0}(t-t_0)} & 0 & 0 \\ 0 & 0 & e^{-i\omega_{0,0}(t-t_0)} & 0 \\ 0 & 0 & 0 & e^{-i\omega_{1,-1}(t-t_0)} \end{bmatrix} \quad (11)$$

where i is the imaginary unit. The four real (angular) frequencies $\omega_{1,1}$ to $\omega_{1,-1}$ in (11) depend on the physical setup. Their values are unknown in practice (see Appendix C) and supposedly constant over time.

In this paper, we thus consider the following situation. The values of a set of coupled states $|\psi(t)\rangle$ are known, and it is also known that each of them is derived from a corresponding state $|\psi(t_0)\rangle$, defined by (5), through the coupling model (8), with M defined above. However, only the class of functions (defined by (9)-(11)), to which this coupling model belongs, is initially known, whereas the values of its parameters $\omega_{1,1}$ to $\omega_{1,-1}$ are initially unknown. Moreover, the values of all states $|\psi(t_0)\rangle$ are also unknown and we aim at estimating them (up to some residual transforms if such transforms cannot be avoided). We thus face a BQSS problem, as defined in Section 1. The unknown source state vector and known mixed state vector involved in this problem are, respectively, defined by $|\psi(t_0)\rangle$ and $|\psi(t)\rangle$, and the mixing model is (8), with M defined by (9)-(11). The scalar form of that mixing model is provided in Appendix D.

3. Separating system structures

3.1. Overall structures and phases of operation

As outlined in Section 1, the BQSS problem essentially consists of developing a separating system which eventually operates as follows. This system receives quantum states (or associated measurement results), that it therefore knows and that are mixtures of quantum source states (or of their parameters) that it does not know. It aims at recovering these source states (or some of their parameters), from their known mixtures only. BQSS is therefore an inverse problem, and the separating system contains an “inverting block”, which essentially receives the known mixed states and outputs the estimated source states.

Besides, we consider cases when the mixing function is only partly known: the separating system knows that the mixing function belongs to a given class of functions which depend on a set of mixing parameters, but this system initially does not know the (supposedly constant) parameter values of the encountered mixing function. The separating system then includes corresponding tunable separating parameters, whose values are initially undefined and must be selected, so as to “match” the parameter values of the mixing function. To this end, the separating system also includes an “adapting block”, and the operation of the overall separating system typically consists of two successive phases. First, during the adaptation phase, the adapting block uses a set of mixed states, in order to select the values of the separating parameters. Then, in the inversion phase, the separating parameters remain fixed to their above-defined values, and the corresponding function implemented by the inverting block is used to restore the source states (or their parameters) from the mixtures received by this block. More precisely, the source states are thus restored, but possibly only up to some residual transforms, as detailed further in this paper.

Different configurations may be developed, using inverting and adapting blocks which have classical and/or quantum forms. The “least quantum” configurations operate as follows. One first converts the above coupled quantum states $|\psi(t)\rangle$ into data which have a classical form (but whose properties reflect their quantum origin). To this end, for each considered state $|\psi(t)\rangle$, one first repeatedly prepares $|\psi(t)\rangle$ and performs measurements for that state, and one then derives the frequencies of occurrence of these measurement results, which are estimates of the probabilities of these measurement results. This “repeated write/read” (RWR) procedure is e.g. detailed in [11], [17]. One then processes these estimated probabilities with classical means only. We used this approach in almost all our previous investigations: see [11], [12], [13], [15], most of [17] and the survey in [21]. The main features of this class of classical-processing BQSS methods are summarized in Table 1 (see p. 11), when comparing these features with those introduced throughout this paper for the new class of BQSS methods proposed in this paper. The latter methods are quite different from the previous ones from several points of view. In particular, their separating system structure is based on a

quantum-processing inverting block⁵, combined with an associated adapting block, as detailed in the remainder of this Section 3. Moreover, this class of methods uses new separation principles, leading to associated separation criteria and algorithms, as explained in the subsequent sections.

3.2. Quantum-processing inverting block

As stated above, the separating system proposed in this paper uses a fully quantum inverting block. The output quantum state of that block is expressed as

$$|\Phi\rangle = c_1|++\rangle + c_2|+-\rangle + c_3|-+\rangle + c_4|--\rangle \quad (12)$$

in the standard basis associated with the output of the inverting block. This state may also be represented by the corresponding vector of complex-valued components of $|\Phi\rangle$, denoted as

$$C = [c_1, c_2, c_3, c_4]^T. \quad (13)$$

The input of this inverting block is directly (i.e. without quantum/classical conversion) the available quantum state $|\psi(t)\rangle$ defined by (7), which corresponds to $C_+(t)$. Then

$$C = UC_+(t) \quad (14)$$

where the matrix U defines the unitary quantum-processing operator applied by the inverting block to its input $C_+(t)$. It should be noted that the quantum mixing model (8) and the quantum separating sub-system defined by (14) are thus linear and therefore simpler than the classical-form *nonlinear* mixing model and the associated classical-form *nonlinear* separating system used in our previous classical-processing BQSS methods. Besides, during the inversion phase, the separating sub-system defined by (14) restores each source quantum state by using only a *single* prepared instance of the quantum state $|\psi(t)\rangle$ corresponding to this source state. On the contrary, with our above-defined classical-processing BQSS methods, the restoration of a single source vector during the inversion phase requires one to perform many (typically 10^4 : see [17]) source qubit preparations in our “repeated write/read” (RWR) procedure, and the restored source parameters are obtained with estimation errors due to this procedure.

The inverting (or unmixing) matrix U is selected as follows. Let us first consider the ideal case, i.e. the situation when the mixing matrix M defined by (9) is completely known, which means that the *values* of all four parameters $\omega_{1,1}$ to $\omega_{1,-1}$ of the class of functions (9)-(11) are known. The matrix U is then directly set to the inverse of the known matrix M , because this guarantees that the output $|\Phi\rangle$ of the inverting block exactly restores the unknown source state $|\psi(t_0)\rangle$: using (9)-(10), this corresponds to

$$U = M^{-1} \quad (15)$$

$$= QD^{-1}Q \quad (16)$$

and (8) and (14) then show that

$$C = C_+(t_0), \quad (17)$$

i.e.

$$|\Phi\rangle = |\psi(t_0)\rangle. \quad (18)$$

The structure of the ideal inverting block defined by (16) is shown in Fig. 1, here with

$$\tilde{D} = D^{-1}. \quad (19)$$

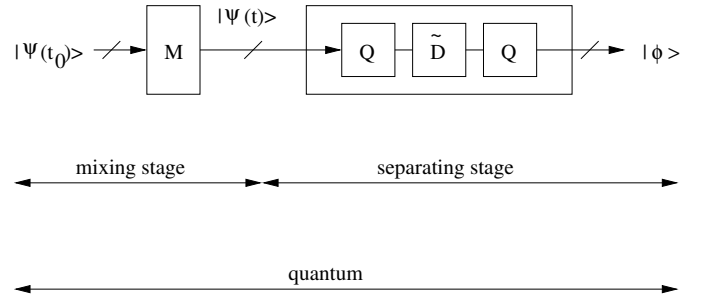


Figure 1: Mixing stage + quantum-processing inverting block of separating system.

In this ideal case, no parameters of the separating system need to be tuned, so that this system is restricted to the above inverting block. Therefore, in this case, not only the sources (and their available mixture) have a quantum form but also the *whole* processing system used to achieve QSS. This configuration therefore corresponds to quantum-source quantum-processing (only) source separation, or full-quantum source separation. We introduced that separating system, limited to the ideal case, in [17], where we also proposed a decomposition of the quantum operators Q and D^{-1} into a combination of simpler quantum gates. This quantum-processing system is thus only based on the usual approach of quantum computing, thoroughly presented in the reference book [38], which e.g. states that “a quantum computer is built from a quantum circuit containing wires and elementary quantum gates to carry around and manipulate the quantum information” (p. 17).

In the current paper, still using the above quantum processing framework, we aim at providing solutions for the much more difficult case when the values of the four mixing parameters $\omega_{1,1}$ to $\omega_{1,-1}$ involved in D are unknown. We therefore introduce an extended inverting block which contains free parameters, that are blindly tuned so that they “match” the unknown values of $\omega_{1,1}$ to $\omega_{1,-1}$, using an automatic adaptation procedure which guarantees that $|\Phi\rangle$ becomes equal to $|\psi(t_0)\rangle$, or at least that $|\Phi\rangle$ has only acceptable differences (called “indeterminacies”, as in BSS) with respect to $|\psi(t_0)\rangle$. This tunable inverting block should have enough flexibility to be able to handle any possible value of the considered mixing matrix M . Moreover, we constrain it as much as possible, i.e. so that it handles *only* the class (9)-(11) of possible matrices M , because this simplifies the structure of this inverting block and this reduces the risk

⁵In [18], we also introduced another type of BQSS methods, which uses a hybrid (i.e. quantum + classical) structure for the overall combination of the mixing stage and inverting block.

for its tunable parameters to be set to inadequate values during their tuning procedure⁶.

The above principles are applied as follows. In this investigation, the set of possible mixing matrices M is known to span the class of matrices defined by (9), where D is a diagonal matrix, and its diagonal elements have unit modulus and unknown phases, due to (11). Extending the approach presented above for the ideal case, the class of tunable inverting matrices U should here span the inverse of the above-defined class of matrices M . Therefore, instead of the single value of U defined by (16) in the ideal case, we here choose the inverting operator represented by U to belong to the functional class defined by

$$U = Q\tilde{D}Q \quad (20)$$

with

$$\tilde{D} = \begin{bmatrix} e^{i\gamma_1} & 0 & 0 & 0 \\ 0 & e^{i\gamma_2} & 0 & 0 \\ 0 & 0 & e^{i\gamma_3} & 0 \\ 0 & 0 & 0 & e^{i\gamma_4} \end{bmatrix} \quad (21)$$

where γ_1 to γ_4 are free real-valued parameters. The general structure of this class of inverting blocks defined by (20) is shown in Fig. 1. It again uses the quantum operator Q . The tunable block corresponding to (21), introduced in the current paper, may be decomposed as shown in Fig. 2, where the closed (i.e. black) and open circle notations respectively indicate conditioning on the qubit being set to one or zero, as in [38] p. 184. In the approaches defined below, the values of the parameters γ_1 to γ_4 are controlled by classical-form signals. These parameters may e.g. be increasing functions of control voltages.

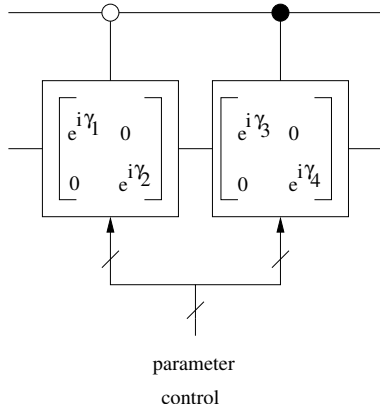


Figure 2: Implementation of quantum operator defined by matrix \tilde{D} , used in inverting block.

This inverting block, shown in Fig. 1, thus yields a feedforward data path in our separating system, i.e. it goes from the input of this system fed with the available mixture $|\psi(t)\rangle$ corresponding to $C_+(t)$, to the output of this system which yields the state $|\Phi\rangle$ corresponding to C .

⁶On the contrary, simple examples skipped here due to space constraints show that allowing the separating system to achieve *any* (linear, memoryless, unitary) transform would yield unacceptably high indeterminacies.

Setting the free parameters γ_1 to γ_4 of \tilde{D} so that (19) is met guarantees that the output state $|\Phi\rangle$ becomes equal to its desired value (18). However, condition (19) cannot here be used as a *practical* procedure for directly assigning \tilde{D} , because we consider the case when D is unknown. Instead, a procedure for adapting the parameters of \tilde{D} is therefore required. We define the corresponding adapting block of the proposed separating system in Section 3.3. Before proceeding to that topic, it is worth noting that the global, i.e. mixing + separating, operator, which defines the expression of the output state $|\Phi\rangle$ of the separating system with respect to the source state $|\psi(t_0)\rangle$, is obtained by combining (8) and (14), and is therefore defined by

$$C = GC_+(t_0) \quad (22)$$

with

$$G = UM \quad (23)$$

$$= Q\Delta Q \quad (24)$$

where

$$\Delta = \begin{bmatrix} e^{i\delta_1} & 0 & 0 & 0 \\ 0 & e^{i\delta_2} & 0 & 0 \\ 0 & 0 & e^{i\delta_3} & 0 \\ 0 & 0 & 0 & e^{i\delta_4} \end{bmatrix} \quad (25)$$

and

$$\delta_1 = \gamma_1 - \gamma_{1d} \quad (26)$$

$$\delta_2 = \gamma_2 - \gamma_{2d} \quad (27)$$

$$\delta_3 = \gamma_3 - \gamma_{3d} \quad (28)$$

$$\delta_4 = \gamma_4 - \gamma_{4d} \quad (29)$$

where we define the “desired values” of γ_1 to γ_4 as

$$\gamma_{1d} = \omega_{1,1}(t - t_0) \quad (30)$$

$$\gamma_{2d} = \omega_{1,0}(t - t_0) \quad (31)$$

$$\gamma_{3d} = \omega_{0,0}(t - t_0) \quad (32)$$

$$\gamma_{4d} = \omega_{1,-1}(t - t_0). \quad (33)$$

Without knowing the values of $\omega_{1,1}$ to $\omega_{1,-1}$, we aim at blindly tuning γ_1 to γ_4 so that they reach their unknown desired values γ_{1d} to γ_{4d} , which indirectly forces δ_1 to δ_4 to become equal to zero (all this possibly up to some acceptable indeterminacies). Thus, as shown by (24), we obtain $G = I$ and therefore $|\Phi\rangle = |\psi(t_0)\rangle$. The scalar form of the global model (22) is provided in Appendix E.

3.3. Adapting block for tuning the inverting block

Various structures may be developed for the adapting block which controls the values of the parameters γ_1 to γ_4 of the above-defined quantum-processing inverting block. We here consider the case when the input of this adapting block is connected to the output $|\Phi\rangle$ of the inverting block, whose value depends on the available mixed state $|\psi(t)\rangle$ and on the current values of γ_1 to γ_4 (similarly, when performing classical BSS with ICA methods, the input of the adapting block is usually set to the output of the inverting block). Then, as in the global

configuration, different options may be considered, depending on which parts of the adapting block respectively have classical and quantum forms. Again, the least quantum option consists of first converting the quantum input $|\Phi\rangle$ of the adapting block into classical-form signals, by means of measurements. The considered separation principle, which defines the constraint set on the control signals of the sub-block \tilde{D} of the inverting block, is then expressed with respect to the classical-form signals which result from the above quantum/classical conversion. The processing based on these signals then uses classical means. The resulting structure of the adapting block and corresponding global configuration are shown in Fig. 3 (the adapting block is in the lower part of the figure). The adapting block thus yields a feedback data path in the separating system, from the output of this system (which is also the output of the inverting block) to the adaptive sub-block \tilde{D} of that inverting block.

The states $|\Phi\rangle$ derived by the inverting block for successive values of the source state $|\psi(t_0)\rangle$ thus have two possible uses: (i) they may be used as the final outputs of the complete separating system, (ii) they may be used as the inputs of the adapting block. This deserves special care, because these $|\Phi\rangle$ are *quantum* states, which must therefore fulfill the no-cloning theorem [38], which has no equivalent in the classical framework, and which is related to the fan out operation for the output of a quantum circuit: *a single instance* of an unknown quantum state (here $|\Phi\rangle$) cannot be copied to be used as the inputs of several subsequent sub-systems. Therefore, a single instance of $|\Phi\rangle$ cannot be sent both to the output of our complete separating system (dash-dotted line in rightmost part of Fig. 3) and to its internal adapting block (dashed line in lower part of Fig. 3).

This therefore requires us to investigate whether the operation of the complete separating system structure in Fig. 3 is compatible with the above constraint. We here prove that it is, by introducing a simple two-phase procedure for using this structure. Starting from arbitrary values of the separating system parameters γ_1 to γ_4 , we first adapt them (with the methods proposed further in this paper), at this stage using the outputs $|\Phi\rangle$ of the inverting block only as the inputs of the adapting block. This corresponds to the adaptation phase of the separating system. We then freeze these parameters γ_1 to γ_4 (i.e. we freeze the signals, provided by the adapting block to sub-block \tilde{D} of the inverting block, which control γ_1 to γ_4) and do not send states $|\Phi\rangle$ anymore to the adapting block. We then start the inversion phase of the separating system. During this phase, the above fixed values of γ_1 to γ_4 thus remain such that, for any value of the source state $|\psi(t_0)\rangle$, the corresponding output $|\Phi\rangle$ of the inverting block is equal to that source state $|\psi(t_0)\rangle$ (possibly up to some indeterminacies due to the considered adaptation method, and assuming this method provided a relevant solution). During the inversion phase, we therefore use these outputs $|\Phi\rangle$ of the inverting block only as the outputs of our complete separating system, to be sent to a subsequent system.

Another structure for the adapting block is also considered at an intermediate stage, further in this paper. It is described in Appendix F.

4. Separation principles

4.1. Overall approach and disentanglement principle

The next step of the procedure that we defined in Section 1 for developing a complete BQPSS method consists of specifying its separation principle. For the innovative approach proposed in this paper, this requires a long analysis. The intermediate stages of that analysis are therefore detailed in Appendix G, whereas we here only summarize them and detail the final outcome of that analysis, to be used in the next steps of this investigation.

So, as detailed in Appendix G.1, we take advantage of the approaches which were used in *classical* BSS for developing separation principles. This leads us to develop a first original BQSS separation principle, called the “disentanglement principle” because it consists of adapting the tunable parameters of the inverting block of the separating system so that the quantum states $|\Phi\rangle$ available at the output of that block become disentangled for at least two (non-redundant) source states $|\psi(t_0)\rangle$. The investigation of that separation principle is presented in Appendix G.2. This principle is expressed with respect to quantum-form data, namely the coefficients of the states $|\Phi\rangle$: a state $|\Phi\rangle$ defined by (12) is unentangled if and only if

$$c_1c_4 = c_2c_3. \quad (34)$$

This separation principle therefore corresponds to the global architecture of Fig. F.6 but is not directly applicable to the target architecture of Fig. 3, which controls the inverting block by means of a property expressed with respect to classical-form signals.

Two modified separation principles suited to the latter architecture are then derived by taking advantage of the results previously obtained for the above disentanglement-based principle. The first of them is based on a preliminary method, which only performs measurements along the above-defined O_z axis for the states $|\Phi\rangle$, and which yields too limited separation capabilities. We therefore then extend it by also performing other measurements for (other instances of) the states $|\Phi\rangle$, along the O_x axis orthogonal to O_z . That second measurement-based separation principle is shown to be equivalent to the above disentanglement-based principle and thus to yield the required separation capabilities, in addition to being directly applicable to the separating structure of Fig. 3. Therefore, respectively in Sections 4.2 and 4.3 below, we detail the definition of these preliminary and final versions of the measurement-based separation principle eventually selected in this paper. Their properties, including their separation capabilities, are addressed in more detail in Appendix G.3 and Appendix G.4.

4.2. Method based on measurements along O_z axis

As announced above, we here consider the separating system structure shown in Fig. 3, and we now aim at completing the definition of its adapting block. This first consists of defining the type of measurements performed by the first sub-block of that adapting block. To this end, we especially take into account our previous classical-processing QSS methods,

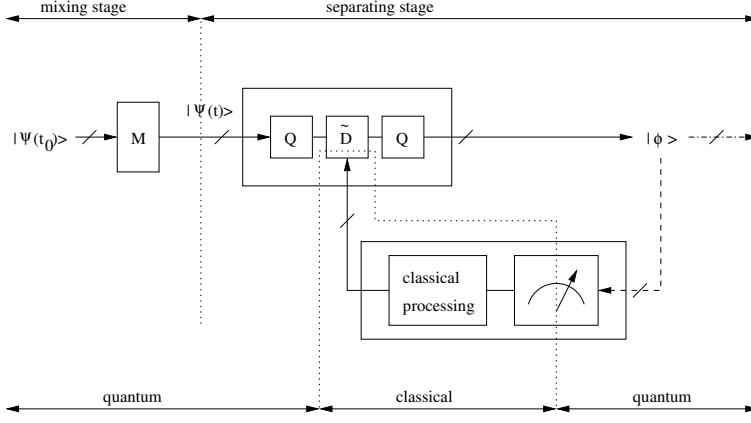


Figure 3: Global (i.e. mixing + separating) configuration including a quantum-processing inverting block and a classical-processing adapting block. Each quantum state $|\Phi\rangle$ is used only once (no cloning): see p. 6.

which first perform repeated (preparations, and) measurements along the Oz axis for the coupled qubits (see e.g. [11], [17]). We here adapt these methods by again performing repeated (preparations, and) measurements along the Oz axis, but now for *the two output qubits of our separating system*. Applying the “repeated write/read” (RWR) procedure of [11], [17], summarized in Section 3.1, to these two qubits shows that each such couple of measurements here has four possible results, namely $(+\frac{1}{2}, +\frac{1}{2})$, $(+\frac{1}{2}, -\frac{1}{2})$, $(-\frac{1}{2}, +\frac{1}{2})$ and $(-\frac{1}{2}, -\frac{1}{2})$ in normalized units. Their probabilities are respectively denoted as P_{1z} , P_{2z} , P_{3z} and P_{4z} . As explained e.g. in [11], [17], these probabilities are equal to the squared moduli of the associated coefficients in the expression (12) of the overall output state $|\Phi\rangle$ of the separating system corresponding to an initialization of the considered source state $|\psi(t_0)\rangle$, that is

$$P_{1z} = |c_1|^2, P_{2z} = |c_2|^2, P_{3z} = |c_3|^2, P_{4z} = |c_4|^2. \quad (35)$$

Considering one or several values of the source state $|\psi(t_0)\rangle$, we thus obtain (estimates of) one or several sets of the resulting four probability values defined by (35). We eventually have to define a method for using these probabilities in the classical-processing sub-block of the adapting block of Fig. 3, in order to adapt the tunable parameters γ_1 to γ_4 of the inverting block. Ideally, we would like to enforce the above-defined disentanglement condition (34), which may be seen as composed of modulus-based and phase-based sub-conditions that are detailed in the appendices of this paper: see (G.2) and (G.15). However, (35) shows that P_{1z} to P_{4z} only give access to the moduli of the complex-valued coefficients c_1 to c_4 involved in conditions (34), (G.2) and (G.15). Therefore, the best use we can make of P_{1z} to P_{4z} consists of using them to tune γ_1 to γ_4 so that condition

$$P_{1z}P_{4z} = P_{2z}P_{3z} \quad (36)$$

is met: we select this condition because (35) shows that it is equivalent to the modulus-based sub-condition (G.2) involved in the above-defined single-state disentanglement principle.

This approach yields a potential separation principle, defined by (36). Appendix G.3 details its properties and limitations, which lead us to extend it as follows.

4.3. Method based on measurements along Oz and Ox axes

We here still consider the separating system structure of Fig. 3, which performs measurements for the output of that system. In our first papers, we used measurements along the Oz axis only (for other qubits than the above separating system output). Here, we are naturally led to also consider measurements along other axes, as will now be explained. The disentanglement constraint (34) that we would like to impose concerns the *complex-valued* quantities c_1 to c_4 . Moreover, in (35) we saw that measurements along the Oz axis only give access to the moduli of these quantities. However, we may hope that measurements along other directions give access to other combinations of these quantities, and thus to combinations involving the phases of c_1 to c_4 . In our first scenario, we therefore also perform measurements along the Ox axis⁷ (orthogonal to Oz) for other preparations⁸ of the same (two or more) output states $|\Phi\rangle$ as those used for measurements along the Oz axis. These measurements along the Ox axis are again used in our “repeated write/read” (RWR) procedure. Each such couple of measurements again has four possible results, namely $(+\frac{1}{2}, +\frac{1}{2})$, $(+\frac{1}{2}, -\frac{1}{2})$, $(-\frac{1}{2}, +\frac{1}{2})$ and $(-\frac{1}{2}, -\frac{1}{2})$ in normalized units, but now with probabilities which are respectively denoted as P_{1x} , P_{2x} , P_{3x} , P_{4x} , and whose expressions are provided in (I.1)-(I.4).

We then use the probabilities associated with the Oz and Ox axes by enforcing the output states $|\Phi\rangle$ to meet not only constraint (36), but also its counterpart for the Ox axis, that is:

$$P_{1x}P_{4x} = P_{2x}P_{3x}. \quad (37)$$

For the considered two states (at least), the joint condition (36) and (37) yields more than a partial constraint on the phases

⁷We hereafter consider the general case of a possibly non-isotropic exchange tensor, i.e. possibly $J_z \neq J_{xy}$, using the notations of Appendix C. On the contrary, when it is known a priori that the exchange tensor is isotropic, i.e. $J_z = J_{xy}$, it may be shown that measurements along the Ox axis can be avoided for the BQSS method described here.

⁸The measurements along the Ox axis are performed for *other instances* of the same states $|\Phi\rangle$ than the instances used in measurements along the Oz axis, since s_z and s_x do not commute and hence cannot be both measured for the same arbitrary state instance.

of c_1 to c_4 : it is completely equivalent to the disentanglement constraint (34), as shown in Appendix I (for states meeting its conditions). The method based on (36) and (37), obtained here, is moreover applicable to the separating system structure of Fig. 3 and is therefore the method used in the remainder of this paper. Appendix G.4 discusses its features in more detail and describes a second, slightly different, scenario for applying it.

5. Separation criteria and algorithms

Various separation criteria and practical blind adaptation procedures may then be derived from the above separation principles. We here focus on a simple version of such procedures. The first step of this procedure uses a set of source states indexed by n with $n \in \{1, \dots, N_z\}$ and $N_z \geq 2$. The corresponding probabilities (35) are denoted as $P_{1z}(n)$ to $P_{4z}(n)$ and estimates of them are used in practice. This step of the procedure aims at ensuring (36) for all considered source states and thus (G.9). Due to (27)-(28), this is achieved by adapting one of the parameters γ_2 and γ_3 , while the other one, as well as γ_1 and γ_4 , are set to constant, arbitrary, values⁹. The separation criterion used in this adaptation consists of looking for one of the values of γ_2 (or γ_3) which correspond to the global minimum of the cost function

$$F_z = \sum_{n=1}^{N_z} |f_z(n)|^p \quad (38)$$

with

$$f_z(n) = P_{1z}(n)P_{4z}(n) - P_{2z}(n)P_{3z}(n) \quad (39)$$

and e.g. $p = 1$ or 2 . Using $p = 1$ is well suited to the sweep-based algorithm described hereafter. On the contrary, $p = 2$ is e.g. better suited when the cost function F_z is minimized by means of gradient-based algorithms, because F_z is thus differentiable everywhere. The global minimum of F_z is equal to zero (when ignoring estimation errors for $P_{1z}(n)$ to $P_{4z}(n)$). It is reached for the first solution of (36), and therefore of (G.2) and (G.3), but not for their spurious solution (again under the above-mentioned condition on source states).

A simple algorithm for reaching the *global* minimum value of F_z is a sweep-based approach: this consists of increasing γ_2 (or γ_3) with a small discrete step over a wide enough bounded interval, in computing the values (estimates in practice) of F_z which correspond to each tested value of γ_2 , and in keeping the value of γ_2 which minimizes F_z . One then freezes γ_2 (and γ_3).

Similarly, the second step of our basic procedure uses a set of source states indexed by n with $n \in \{1, \dots, N_x\}$ and $N_x \geq 2$. The corresponding probabilities involved in (37) are denoted as $P_{1x}(n)$ to $P_{4x}(n)$. This step of the procedure aims at ensuring (in

addition to the constraint set by the first step) that (37) is met for all considered source states and thus that (G.20) is met. Due to (26) and (29), this is achieved by adapting one of the parameters γ_1 and γ_4 , while the other one is set to a constant arbitrary value, and γ_2 and γ_3 remain fixed to their values obtained in the first step of this procedure, as explained above. The separation criterion used in this adaptation consists of looking for one of the values of γ_1 (or γ_4) which correspond to the global minimum of the cost function

$$F_x = \sum_{n=1}^{N_x} |f_x(n)|^p \quad (40)$$

with

$$f_x(n) = P_{1x}(n)P_{4x}(n) - P_{2x}(n)P_{3x}(n). \quad (41)$$

Here again, a simple algorithm for reaching the *global* minimum value of F_x consists of a sweep on γ_1 (or γ_4). Two approaches may be employed to define the set of source states used in this second step of our procedure. The first approach is based on the first scenario of Section 4.3 and consists of using the same set of states as in the first step of our procedure, therefore with $N_x = N_z$. The second approach is based on the second scenario of Appendix G.4 and consists of using (at least) two arbitrary states (so that $N_x = 2$ at least). The latter approach is less constraining than the former.

This algorithm may then be further extended so as to remove the remaining indeterminacies, using the approach defined in Appendix C. Besides, some of its features are discussed in more detail in Appendix J.

6. Numerical results

We hereafter present the performance of two BQPSS algorithms, namely (i) the basic algorithm defined in the previous section, with $p = 1$ in (38) and (40), and (ii) its extension derived from Appendix C. These two algorithms are hereafter respectively referred to as our “first algorithm” and “second algorithm”¹⁰. The physical implementation of qubits is only an emerging topic, which is beyond the scope of this paper. Therefore, we here assess the performance of the above BQPSS algorithms by means of tests performed with data derived from our software simulation of the behavior of coupled qubits. The version of this software intended for classical-processing QSS methods was described in [17]. Its fundamental principles are here extended to quantum-processing QSS methods, so as to perform each elementary test as follows.

We here use the polar representation of the source qubit parameters α_i and β_i in (1), which reads

$$\alpha_i = r_i e^{i\theta_i} \quad \beta_i = q_i e^{i\phi_i} \quad i \in \{1, 2\} \quad (42)$$

with $0 \leq r_i \leq 1$ and

$$q_i = \sqrt{1 - r_i^2} \quad (43)$$

⁹More precisely, (26)-(29) show that condition (G.9) meets the following two properties: (i) it does not depend on γ_1 and γ_4 , (ii) it does not depend on γ_2 and γ_3 *separately* but only on their difference $\gamma_3 - \gamma_2$. Besides, these two properties are met not only at the point of interest defined by condition (G.9), but for any value of the function (38) to be optimized. This may be seen by combining (38), (39), (35), (E.1)-(E.4) and (26)-(29).

¹⁰These algorithms may be called BQPSS-D1 and BQPSS-D2, where the letter “D” refers to the fact that both versions are essentially based on the Disentanglement principle.

because $|\psi_i(t_0)\rangle$ has unit norm. In the adaptation and inversion phases of each elementary test, our software creates a set of source states $|\psi(t_0)\rangle$ defined by (5) and (42), with the following parameter values: r_1 and r_2 are randomly selected with a uniform distribution over $]0, 1[$, q_1 and q_2 are derived from (43), and θ_1 , θ_2 , ϕ_1 and ϕ_2 are randomly selected with a uniform distribution over $[0, 2\pi[$. This software then derives a set of coupled states $|\psi(t)\rangle$ corresponding to the above source states $|\psi(t_0)\rangle$, coupled according to (D.1)-(D.4) with given values for their four parameters $\omega_{k,i}(t - t_0)$. The latter parameters are defined by (C.1)-(C.4). We therefore fix their values by setting $g\mu_e B(t - t_0)/\hbar = 175.8$, $J_{xy}(t - t_0)/\hbar = 39.26$ and $J_z(t - t_0)/\hbar = 130.9$ in the adaptation phase (in the inversion phase, the above values are kept for our first algorithm but multiplied by 2 for our second algorithm, as shown by the above descriptions of these algorithms). The motivations for using these values are provided in Appendix K.

During the first and second steps of the adaptation procedures, this software uses N_z and N_x above-defined coupled states $|\psi(t)\rangle$, respectively, to blindly adapt the separating parameters γ_1 to γ_4 . More precisely, a first type of tests is performed with $N_z = N_x = 2$. This value is selected because it is the minimum required number of states. However, it may yield limited performance, due to possible redundancy of the considered states (see Appendix G.2.1.2 and Appendix I), since these states are here created randomly, without selecting them so as to guarantee that the above redundancy is avoided, in order to be coherent with the targetted *blind* configuration. Therefore, another type of tests is also performed, with $N_z = N_x = 10$, since this reduces the probability to get such redundant states (see Appendix G.2.1.2). Still in the adaptation phase, in each sweep on γ_2 (with $\gamma_1 = \gamma_3 = \gamma_4 = 0$) or on γ_1 (with $\gamma_4 = 0$ for our first algorithm and γ_4 varied according to (C.8) for our second algorithm), we use 10^3 values of the γ_j parameter(s) varied in this sweep. During that adaptation phase, for each choice of the repeatedly prepared source state and γ_j parameters, a set of K measurements is performed at the output of the separating system, for each considered direction (Oz or Ox), in order to derive probability estimates from the relative frequencies of occurrence of measured values. This number K of measurements is varied in the tests reported below.

During the inversion phase, the above software is then used to successively process 10^4 other coupled states $|\psi(t)\rangle$ with the above estimates of the separating parameters γ_j (multiplied by 2 for our second algorithm). It thus derives estimates $|\Phi\rangle$ of the actual source states $|\psi(t_0)\rangle$ from which these coupled states $|\psi(t)\rangle$ were computed.

For each considered set of conditions and each BQPSS algorithm, we perform 100 above-defined elementary tests with different sets of source states (so that we perform 100 estimations of the same set of mixing parameter values). Analyzing the estimated source states $|\Phi\rangle$ and comparing them to the actual source states $|\psi(t_0)\rangle$ then makes it possible to check that the proposed algorithms succeed in separating these sources, and to determine the accuracy of this separation, i.e. essentially the magnitude of the deviation of the estimated source states

$|\Phi\rangle$ with respect to the actual source states¹¹ $|\psi(t_0)\rangle$. Based on this principle, the performance of the tested BQPSS algorithms is assessed by two criteria, which are detailed in Appendix L. Briefly, they are related to the Root Mean Square Error (RMSE) between (i) the coefficients $c_1(t_0)$ to $c_4(t_0)$ of the actual source states $|\psi(t_0)\rangle$ defined by (2) and (5), and (ii) their respective estimates c_1 to c_4 which are the coefficients of the output state $|\Phi\rangle$ of the separating system, defined by (12). Moreover, they are built so as to be insensitive to the intrinsic indeterminacies of the considered BQPSS algorithms, i.e.

- The first criterion is insensitive to the permutation and component-wise phase indeterminacies of our first algorithm. It thus only measures the remaining error for the *moduli* of the coefficients of the above quantum states. It is therefore denoted as $RMS E_m$.
- The second criterion completely takes into account the complex values of the above *coefficients*, except that it is insensitive to the global (i.e. on overall quantum states) phase factor obtained with our second algorithm. It is therefore denoted as $RMS E_c$.

The test results thus obtained are shown in Fig. 4 and 5. They confirm the above theoretical analyses from the following points of view. Our first algorithm achieves a low $RMS E_m$ (down to around 10^{-3}) but a high $RMS E_c$ (between 10^{-1} and 1), due to its indeterminacies. On the contrary, in the considered conditions, both $RMS E_m$ and $RMS E_c$ can be reduced down to roughly 10^{-3} for our second algorithm. This confirms that this algorithm is especially attractive, because it removes all the indeterminacies which have a physical influence. Besides, much lower errors are obtained when using $N_z = N_x = 10$ source states in adaptation than when using $N_z = N_x = 2$. This can be explained as follows: analyzing the distribution (not shown in the figures) of the 100 error values obtained over the considered 100 elementary tests shows that a few of these values are especially high for $N_z = N_x = 2$, due to the rare cases when the considered two source states are close to redundancy, whereas these errors are more uniformly distributed over lower values when using $N_z = N_x = 10$, as expected from Appendix G.2.1.2.

Besides, concerning our second algorithm, it may be noted that $RMS E_c$ is higher than $RMS E_m$. This is normal, since $RMS E_m$ only takes into account errors related to the moduli of the quantum state coefficients, whereas $RMS E_c$ takes into account errors related to their phases in addition. So, as requested, the performance criterion $RMS E_c$ takes into account all types of errors which are relevant in the considered application, and it thus yields higher error values than the performance criterion $RMS E_m$, because the latter criterion only measures one aspect of the above errors. Finally, Fig. 4 and 5 show that $RMS E_c$

¹¹This performance assessment procedure can only be used when *developing* and testing the considered BQPSS algorithms, with actual source values $|\psi(t_0)\rangle$ which are *known* (but not used in the BQPSS algorithms themselves). On the contrary, in the actual setup which is to be eventually used, the actual source states are *unknown*, and one precisely aims at estimating them ! They cannot therefore be compared to their estimated values.

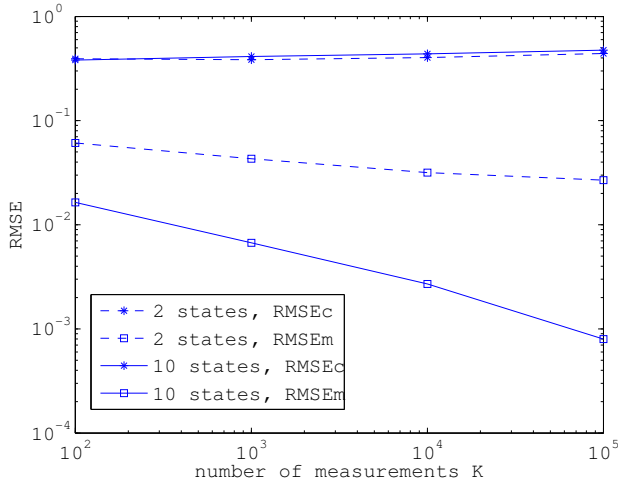


Figure 4: Root Mean Square Errors $RMSE_c$ and $RMSE_m$ of our first algorithm vs. number K of measurements performed for output states, in each direction, during adaptation. Number of source states used in adaptation: $N_z = N_x = 2$ or $N_z = N_x = 10$, depending on considered plot (see box in figure).

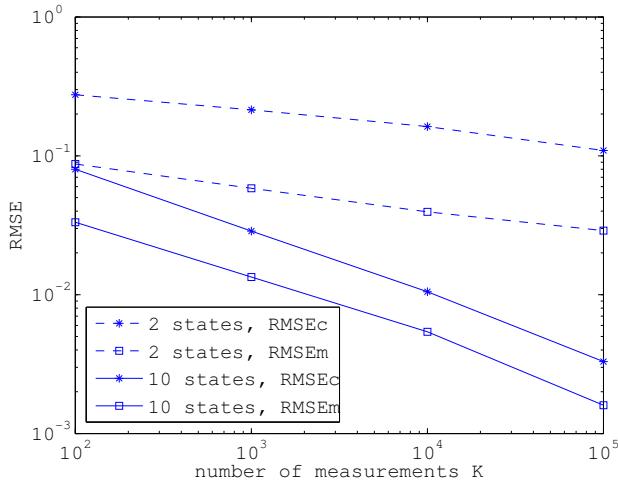


Figure 5: Same as Fig. 4 for our second algorithm.

and $RMSE_m$ decrease when the number K of measurements is increased. This is expected too, because a higher K yields more accurate estimates of the required probabilities. In the considered conditions, setting $K \approx 10^5$ allows one to achieve $RMSE_c$ and $RMSE_m$ around 10^{-3} , which corresponds to quite accurate quantum state estimation. Modifying K and/or some other above-defined test parameters makes it possible to even further reduce estimation errors, if required.

7. Discussion

Having detailed the features of the proposed BQSS systems in the previous sections, we hereafter summarize them and compare them with those of our previous approaches.

Most previously reported BQSS systems first convert the available mixed quantum states into classical-form data, by per-

forming measurements along one direction. They then process the latter data with classical means only, especially estimating the unmixing function by using statistical methods related to classical Independent Component Analysis. The main features of these BQSS methods are summarized in Table 1.

Table 1 also contains the main features of the new class of BQSS systems proposed in this paper. These systems use a quantum-processing inverting block, that therefore restores the source states in quantum form, which is well suited to connecting the output of that block to a quantum-processing computer. They estimate the unmixing function thanks to a separation principle based on output disentanglement, which uses measurements along two directions. They are therefore primarily based on what could be called a (multi-state) “Unentangled Component Analysis”. As compared with our previous BQSS systems, the new systems proposed in this paper yield major improvements in terms of restored source parameters and associated indeterminacies and approximations, number of source states required to perform adaptation and numbers of source state preparations in the adaptation and inversion phases (see numerical values in Table 1).

Moreover, in Appendix G.2.2.2 we show that, once the unmixing function has been estimated, the proposed systems can restore *any* two-qubit pure source state, i.e. entangled or not. We thus introduced an extended field, that may be called “Blind Quantum State Restoration” or BQSR (i.e. methods for restoring any overall multi-qubit source state, not only product states), which includes the originally considered BQSS field (i.e. restoration of separate single-qubit states, whose tensor product is the overall considered source state¹²).

8. Conclusion

Blind (or unsupervised) Quantum Source Separation (BQSS) systems address the situation when available multi-qubit “mixed states” result from the application of an unknown “mixing function” to unknown multi-qubit “source states”, where each source state is the tensor product of *separate* single-qubit components (and each of these components is a pure state in the simplest case). Some other properties are also possibly requested from these source states and/or mixing function. Using these available mixed states, BQSS systems aim at restoring (the information contained in) the source states, during the second phase of their operation, called the “inversion phase”. To this end, they typically start by estimating the “unmixing function”, i.e. the inverse of the mixing function, during the first phase of their operation, called the “adaptation phase”. They are therefore quantum adaptive systems, i.e. quantum systems which adapt their behavior to the data that they process, as classical adaptive systems do [32].

In this paper, we proposed a new class of BQSS systems, whose features are summarized in Section 7. Their performance

¹²In other words, BQSS always requests (at least) the following known property from the multi-qubit source states: they are tensor products of single-qubit states.

Feature	properties of BQSS methods and references in this paper		
	previous methods	new methods	page number
nature of processing means of inverting block	classical	quantum	4
type of mixing function and inverting block	nonlinear	linear	4
restored source parameters (defined in (42))	3 + 2 = 5 real parameters: r_1, r_2 and $\Delta_I = [(\phi_2 - \theta_2) - (\phi_1 - \theta_1)]$; q_i are then derived from (43)	4 complex (\Rightarrow 6 meaningful real) parameters: $\alpha_1, \alpha_2, \beta_1, \beta_2$	21 (see (G.26)), 22
indeterminacies	none e.g. only if: $r_1 < \frac{1}{2} < r_2, 0 \leq \Delta_I \leq \frac{\pi}{2}$	none for final method	16, 21, 22
separation principle used in adaptation phase	statistical, related to Independent Component Analysis	disentanglement or associated probabilities	18 21, 21
axes of measurements of spin components in adaptation phase	O_z	O_z, O_x	21
number of states $ \psi(t_0)\rangle$ required in adaptation phase	$\approx 10^3$	2 (minimum)	(19 and) 20, 7, 22
total number of preparations of all states $ \psi(t_0)\rangle$ in adaptation phase	$\approx 10^3 \times 10^4 = 10^7$	$\approx 10^7$ (basic algo.) or $\approx 10^4$ (improved algorithm)	23 23
accuracy of restored source parameters in inversion phase	approximation: frequencies of occurrence in repeated measurements	no approximation for ideal value of U	4 (see (18))
number of preparations of each state $ \psi(t_0)\rangle$ in inversion phase	$\approx 10^4$	1	4

Table 1: (i) Summary of the general features (defined in Column 1 of this table) of the previous BQSS methods summarized in Section 3.1 (Column 2) and of the new BQSS methods proposed in this paper (Column 3). (ii) For these new methods, references to the main pages of this paper where the considered features are presented (Column 4).

was validated by means of numerical tests. Validation with real coupled qubits will become possible when the corresponding physical devices are available. For the time being, we plan to investigate several aspects of the multiple extensions of BQSS suggested by this paper. In particular, up to now we only considered two-qubit Heisenberg coupling, both because this is a concrete and relevant mixing model, and because this first step already required very detailed investigations for introducing all BQSS concepts and methods of interest. But the approach thus proposed is expected to have much wider applicability, and we especially plan to extend it to other mixing models.

Appendix A. Applications of blind classical and quantum source separation and system identification

As shown in Section 1, blind classical and quantum source separation may first be addressed as generic, hence somewhat abstract, data processing problems. Moreover, both of them lead to practical applications, that we discuss in this appendix,

where we also show that some of these applications are linked to system identification.

Appendix A.1. Applications of blind classical source separation

Different aspects may be distinguished in the applications of classical BSS (see e.g. the overviews in [8], [21], [23], [31]). The first one consists of directly providing the human end-user of the considered application with the output signals of the separating system, i.e. with the estimated source signals. This e.g. concerns applications in the audio field: when using a set of microphones to record signals which are mixtures of a source signal of interest, that is useful speech, and of interfering source signals (other speech signals and/or “noise”), one may process all recorded signals with BSS methods, so as to especially extract the useful speech signal and to provide it to the end-user’s ears. Similarly, in the biomedical field, a wide range of signals, such as electrocardiograms (ECG), electroencephalograms (EEG) or magnetoencephalograms (MEG) contain undesired components in addition to the useful ones. By

recording a given type of such signals with multichannel data acquisition means and processing these recordings with BSS, one may extract or at least enhance the components of interest. These extracted components may then be provided to the medical human expert’s eyes, thus helping him analyzing these signals. For instance, this approach was used in [10] to process multichannel ECG recordings which are mixtures of large-magnitude mother’s heartbeats, low-magnitude fetus’s heartbeats and noise components. This made it possible to extract fetus’s heartbeats, which are hardly visible in the original recordings.

A modified version of the above configurations leads one to the second aspect of applications of classical BSS. In this second framework, BSS is used as the *pre-processing* block of a more complete automated processing system: BSS then aims at “denoising” measured signals, i.e. at extracting/enhancing components of interest, which are then sent to a subsequent block of the system, which typically analyzes them. For instance, in the audio field, after BSS extracted a speech signal of interest from “noise” as explained above, this extracted signal may be sent to an automatic speech recognition system, which converts speech signal waveforms into the associated sentences [27], that may then be used to control some actuators (see e.g. voice control of some car functions performed by a car driver). Similarly, once the above-mentioned ECG (or EEG, MEG ...) components of interest have been separately extracted, they may be sent to an automatic medical diagnosis system. Related applications also exist in other fields, such as radiocommunications, as e.g. discussed in [21], [23].

Appendix A.2. Connection with blind classical system identification

Moreover, the field of classical BSS is closely linked to the field of blind system identification [1], and more specifically to blind mixture identification (BMI). This is e.g. discussed in [23] and may be summarized as follows. Various BSS methods estimate the inverse of the mixing function applied to the source signals. They thus allow one to estimate this mixing function itself. The applications related to BSS thus include various applications of BMI, as a spin-off.

These BSS and BMI tasks are so closely connected that some applications need both of them, i.e. they require one to estimate both the source signals and the mixing transform. For instance, in the field of Earth observation (often referred to as “remote sensing”), a hyperspectral imaging sensor e.g. situated on a satellite provides reflectance spectra. More precisely, the data typically derived for each pixel of such an image sensor consist of a reflectance spectrum corresponding to the region of Earth’s surface seen by this pixel. These data for one pixel thus define a scalar function which depends on wavelength. Each value of this function is equal to the fraction of light power reflected by the considered region on Earth and sent to the considered sensor pixel, with respect to the light power received from the sun by the considered region on Earth, over the considered narrow wavelength band. If the considered region is covered with various pure materials, the overall spectrum recorded for that region is, in the simplest case, a linear combination of the reflectance

spectra of the pure materials present in this region (which are the source signals), with each coefficient of this combination (mixing coefficient) equal to the fraction of that region covered with the considered pure material (see details e.g. in [4], [23]). To analyze the considered scene, one should process the recorded mixed spectra so as to extract from them both the spectra of the pure materials (BSS task), which define the nature of the materials present in this region, and the mixing coefficients (BMI task), which define the abundances of these pure materials in this region.

Appendix A.3. Applications of blind quantum source separation

Although the above classes of applications only concern blind *classical* source separation and system identification, they are quite enlightening when aiming at determining the applications of the very general blind *quantum* source separation and system identification problems that are already of interest today, or that are likely to develop in the future. Splitting these applications in the same way as above, let us first consider those which are focused on estimating the source “signals” (as opposed to the mixing function), i.e. here the source quantum states, after they were altered by undesired coupling (i.e. “mixing” in the BSS sense). These restored states are then used as the inputs of subsequent “quantum computing means” in a broad sense, i.e. processing means which manipulate these states in quantum form or which convert them into classical-form data at some stage, by using measurements (the result of which may then be used by a human operator or stored in some device). BQSS is thus essentially used as a pre-processing block in a “quantum computer”, so as to restore non-modified data before the target task of that computer is carried out with the latter data.

More specifically, one may anticipate situations where data will be stored in a register of qubits of a quantum computer, for subsequent use. Due to non-idealities of the physical implementation of that register, the qubits which form it may be coupled¹³. As time goes on, the register state will therefore evolve in a complicated way due to qubit coupling, thus making the final value of that register state not directly usable in the target quantum application. BQSS may then be used to restore the initially stored register state, before providing it to the target application.

Appendix A.4. Connection with blind quantum process tomography

Moreover, the above discussion of the relationship between blind source separation and system (or mixture) identification

¹³In this appendix, we only aim at outlining general approaches for applying BQSS methods so as to handle states resulting from a certain type of undesired coupling. Of course, depending on the considered physical setup and application, the type of coupling to be handled may be different from the specific Heisenberg coupling analyzed in detail in the present paper (it may e.g. involve more than two qubits), and the proposed general approaches for applying BQSS should then be refined accordingly.

in the classical framework suggests the following major spin-off of blind source separation performed in the quantum domain.

Independently from BQSS, the QIP community already addressed the quantum version of system identification and called it Quantum Process Tomography (QPT) [6], [37], [38], [41], [43], [48]. That problem was only studied in its *non-blind* version, i.e. identifying the considered quantum process by using not only its outputs but also the associated inputs. Transposing the above comments about the link between classical BSS and BMI to the quantum world recently led us to introduce the following idea in [22]¹⁴: by estimating the inverse of the mixing function, BQSS is also essentially able to estimate this function itself, i.e. the parameters of the considered coupling operator (possibly up to some residual transforms, called indeterminacies as in classical BSS and BMI [9], [23]). Thus, generally speaking, BQSS opens the way to introducing the blind version of QPT (called BQPT), i.e. performing QPT essentially *without knowing the values of the input quantum states* of the considered process (but e.g. requesting them to be unentangled). Again transposing to the quantum world our above comments about classical BSS and BMI, the applications related to BQSS thus include applications of BQPT, as a spin-off. Such applications may be defined as follows.

Non-blind QPT is widely recognized to be a major tool for QIP: for instance, see [6], [37], [38], [41], [43], [48], which e.g. state that QPT “is considered the gold standard for fully characterising quantum systems, and in particular for characterising the quantum logic gates that form the basic elements of a quantum computer” [43] and that “accurate characterization of two-qubit gates will be critical for any realization of quantum computation” [48].

The BQPT methods considered when extending the above standard QPT tool to its blind version therefore especially have two potential applications. The most natural one is when the input states of the considered process indeed cannot be known. BQPT methods could then be of interest for characterizing quantum gates while they are operating and when only their results (output states) are available to the user who is to characterize them. This on-line characterization may be useful e.g. if the transform performed by a quantum gate slowly evolves over time (e.g. due to aging) and must be monitored, by characterizing it from time to time. Besides, BQPT may be of even higher interest in more standard configurations, where the process input states may be prepared and known: BQPT then avoids the complexity of *accurately preparing the specific states* which are required by standard QPT methods, because BQPT can use *any* input states (unentangled in the version of BQPT associated with the BQSS methods proposed in the present paper).

A more detailed analysis of BQPT is beyond the scope of the present paper: this paper already required detailed derivations for introducing the proposed class of BQSS methods itself, and resulting spin-offs for BQPT will be addressed in future papers.

¹⁴More precisely, the new field of BQPT defined hereafter was implicitly present since the first publications dealing with BQSS: see e.g. the estimation of the mixing parameter in the early paper [11]. But [22] was the first paper where the emphasis was explicitly put on that BQPT concept and where the associated terminology was introduced.

Appendix B. About quantum concepts

This appendix aims at bringing both additional information and suggestions on possible readings to the reader unfamiliar with quantum physics.

Appendix B.1. The concept of quantum pure state

Surely, as stressed by Peres [39], “quantum phenomena do not occur in a Hilbert space, they occur in a Laboratory”. However, the importance of abstract concepts and tools of Quantum Physics (QP) should not be underestimated, as Dirac has warned us a long time ago [25], writing that e.g. a book on QP “if not purely descriptive of experimental work, must be essentially mathematical”.

In order to describe the physical state of a quantum system, QP uses the concepts of pure state and statistical mixture. This paper and consequently this appendix are restricted to pure states. A quantum pure state is the result of a preparation, a concept introduced by Margenau [36]. The mathematical description of the (pure) states of a quantum system is made thanks to the introduction of a complex Hilbert space, called the state space, with the corresponding properties of its elements (vectors, or kets) and the existence of a scalar product. The statistical interpretation of QP imposes us to consider only normed vectors, and two vectors differing by a phase factor $e^{i\phi}$, ϕ being any angle, should be considered as describing the same physical state. The space state of a spin 1/2 is a two-dimensional complex Hilbert space, and that of a pair of distinguishable spins is the tensor product of the two corresponding state spaces. If $|1, m_1\rangle$ and $|2, m_2\rangle$ (eigenkets of s_{1z} and s_{2z} respectively, with corresponding eigenvalues $m_1 = \pm 1/2$ and $m_2 = \pm 1/2$) are basis kets for spins 1 and 2 respectively, abbreviated as $|1, \pm\rangle$ and $|2, \pm\rangle$, a basis of the corresponding state space of the pair consists of the four kets $|+, +\rangle$, $|+, -\rangle$, $|-, +\rangle$ and $|-, -\rangle$, where e.g. $|-, +\rangle$ is an abbreviation (also denoted as $| - + \rangle$) for the tensor product ket $|1, -1/2\rangle \otimes |2, +1/2\rangle$.

If a quantum system Σ is first prepared, and can then be considered as being isolated until it interacts with an apparatus M chosen to inform us upon one property (to which is formally attached a so-called observable) of Σ , in an act called a measurement, the measurement generally alters the physical state of Σ . If it is possible to repeat the preparation in the same pure state, then the result of each measurement of that observable has a random behaviour (except in specific situations where the result is systematically the same one), but a high number of such measurements allows us to calculate a mean value. The experimenter is interested in this result. M obeys the laws of quantum physics (while the classical approximation should be valid for its own behaviour when isolated). The description of the behaviour of the coupled system consisting of Σ and M , which obey the laws of QP, is another chapter of QP [39].

Appendix B.2. Origin of the Heisenberg coupling

In its simplest form, the Heisenberg coupling between two distinguishable neighboring electron spins \vec{s}_1 and \vec{s}_2 takes the form $-2J\vec{s}_1\vec{s}_2$ (Heisenberg isotropic model). This section aims

at giving a qualitative idea of the physical origin of this coupling. It originates in the exchange interaction, a purely quantum phenomenon, which appears each time there exists a coupling between two identical particles with a half-integer spin, as e.g. occurs with the two electrons in the atom of Helium 4 (or in the Hydrogen molecule), each with a spin $1/2$, and coupled by the Coulomb repulsion. While classical mechanics can distinguish two identical particles, e.g. two electrons, through their trajectories, in the quantum point of view e.g. the two electrons in the Helium 4 atom are indistinguishable, and this is always true for two identical particles [25] (this book includes Dirac's own contribution to the subject, explaining why one also speaks of the "Heisenberg-Dirac model"). Moreover, from this fact and the general principles of quantum theory, any physical quantum state of a pair of identical particles is either symmetrical in (i.e. unaffected by) the exchange of the particles, or antisymmetrical in this exchange (the pair state just changes its sign). Experience has taught us that when a particle has a half-integer spin, any physical state of a pair of these particles is antisymmetrical in the exchange, and these particles are called fermions. Otherwise (no spin, or integer spin), that state is symmetrical in that exchange and the particles are called bosons. Any antisymmetrical eigenstate of the Hamiltonian of a pair of uncoupled electrons is a product of a symmetrical (resp. antisymmetrical) spatial state with an antisymmetrical (resp. symmetrical) spin state. If that state involves two distinct one-electron spatial wave-functions ψ and φ (e.g. in the Helium atom, the orbital $1s$ and $2s$ states), one gets another eigenstate with the same energy just by exchanging the electrons: this degeneracy is called *exchange degeneracy*. Considering the Helium 4 atom, one may accept a rough treatment through perturbation theory, considering now that these electrons are not independent, but coupled by the Coulomb repulsion. This repulsion partially lifts the exchange degeneracy and generates a singlet level and a triplet level (three linearly independent stationary states). Each of these four states is still antisymmetric. In the singlet level, the state is a product of an antisymmetrical spin state and a symmetrical spatial (so-called orbital) state, and the states of the triplet level are the product of one and the same antisymmetrical orbital state with three different symmetrical spin states. Two integrals, K and J , respectively called the Coulomb and the exchange integrals, appear in the calculation. K may receive a classical interpretation, describing the Coulomb repulsion between two charges with respective densities $e |\psi(\vec{r}_1)|^2$ and $e |\varphi(\vec{r}_2)|^2$. J then describes the Coulomb repulsion between two charges with respective densities $e\varphi^*(\vec{r}_1)\psi(\vec{r}_1)$ and $e\varphi(\vec{r}_2)\psi^*(\vec{r}_2)$, a formal interpretation, through the concept of so-called exchange charges, without classical analog. One may then show that the results are formally identical with that which would exist if two (now distinguishable) electron spins were coupled by a Hamiltonian $-2J\vec{s}_1\vec{s}_2$ [5], [42]. This Hamiltonian therefore does not describe a true coupling between the electrons, but is an effective Hamiltonian resulting from the indistinguishability of the electrons, the fact that they are fermions, and the existence of their Coulomb repulsion. The exchange integral J , related to the overlap of the involved one-electron orbital wave-functions, equals zero when these wave-functions do not overlap, and that

coupling is significant only for nearby electrons. Blokhinsev's book [5] thoroughly treats the behaviour of identical particles, the exchange degeneracy and its lifting by the Coulomb repulsion, and the Helium 4 atom case. Historically, the exchange coupling was first identified in Helium 4, through its optical spectra, by Heisenberg. Shortly afterwards, Heisenberg identified the exchange coupling as the origin of the previously mysterious "molecular field" introduced by Weiss in 1906 in order to explain the existence of ferromagnetic solids. Stevens [42] gives a short presentation of the exchange interaction in magnetically ordered solids, including the existence of solids with *anisotropic* exchange coupling. A detailed treatment, including more complex exchange interactions than the so-called direct exchange between two neighboring atoms, and the corresponding models, may be found in [28]. In that context, the Heisenberg model with cylindrical symmetry is a rough description of the effect of magnetic anisotropy upon exchange.

Appendix B.3. Time evolution of an isolated pair of spins $1/2$ with a cylindrical Heisenberg coupling

In this paper and in our previous ones, the two qubits were separately prepared at time t_0 . We are therefore interested in the time-behavior of a pair of spins $1/2$, after its preparation, at t_0 , in a state $|\psi(t_0)\rangle$ equal to

$$(\alpha_1 | + \rangle + \beta_1 | - \rangle) \otimes (\alpha_2 | + \rangle + \beta_2 | - \rangle). \quad (\text{B.1})$$

Therefore:

$$|\psi(t_0)\rangle = \alpha_1\alpha_2 | ++ \rangle + \alpha_1\beta_2 | +- \rangle + \beta_1\alpha_2 | -+ \rangle + \beta_1\beta_2 | -- \rangle \quad (\text{B.2})$$

where the kets $| ++ \rangle$, $| +- \rangle$, $| -+ \rangle$ and $| -- \rangle$ were defined in Appendix B.1. We denoted this basis as \mathcal{B}_+ and we also write these kets as $\{| i \rangle$. The state of the pair at time t may then be written:

$$|\psi(t)\rangle = \sum_{i=1}^4 c_i(t) | i \rangle. \quad (\text{B.3})$$

The time evolution of an isolated system initially prepared in some pure state is driven by its Hamiltonian, and predictions upon this time-behavior necessitate the knowledge of its energy levels and eigenstates. We first briefly consider a pair of spins $1/2$ coupled by an isotropic Heisenberg Hamiltonian $-2J\vec{s}_1\vec{s}_2$. Introducing the total angular momentum $\vec{S} = \vec{s}_1 + \vec{s}_2$, one first writes

$$2\vec{s}_1\vec{s}_2 = \vec{S}^2 - \vec{s}_1^2 - \vec{s}_2^2. \quad (\text{B.4})$$

Then, from elementary quantum mechanics, one gets the following energies for this isotropic Hamiltonian:

$$E(S) = -2J[S(S+1) - s_1(s_1+1) - s_2(s_2+1)] \quad (\text{B.5})$$

and presently $s_1 = s_2 = 1/2$ and therefore $S = 0$ or 1 . A basis of eigenkets common to \vec{S}^2 , S_z , \vec{s}_1^2 and \vec{s}_2^2 are the kets denoted as $| S, m_S \rangle$, and the allowed values for m_S are $1, 0$ and -1 if $S = 1$, and only $m_S = 0$ if $S = 0$. These eigenkets are

therefore $|1, 1\rangle, |1, 0\rangle, |1, -1\rangle$, corresponding to a triplet with energy $E(1)$, and $|0, 0\rangle$ corresponding to a singlet with energy $E(0)$. The $|S, m_S\rangle$ states are related to the $|i\rangle$ states as follows:

$$|1, 1\rangle = |++\rangle, \quad |1, 0\rangle = \frac{|+-\rangle + |-+\rangle}{\sqrt{2}}, \quad (\text{B.6})$$

$$|0, 0\rangle = \frac{|+-\rangle - |-+\rangle}{\sqrt{2}}, \quad |1, -1\rangle = |--\rangle. \quad (\text{B.7})$$

If this pair of spins $1/2$ possesses the following internal Heisenberg coupling with cylindrical symmetry:

$$H_{HC} = -2[J_{xy}(s_{1x}s_{2x} + s_{1y}s_{2y}) + J_z s_{1z}s_{2z}], \quad (\text{B.8})$$

the $|S, m_S\rangle$ kets are still eigenstates of H_{HC} (an accident occurring for spins $1/2$) [18], and this keeps true if a Zeeman term $H_Z = g\mu_e(s_{z1} + s_{z2})B$ is added. When $B \neq 0$ (static field along Oz) the triplet is split, and the energies of the states $|1, 1\rangle, |1, 0\rangle, |0, 0\rangle$ and $|1, -1\rangle$ (abbreviated as $|j\rangle$, with $j = 1$ to 4) are respectively:

$$\hbar\omega_{1,1} = g\mu_e B - \frac{J_z}{2}, \quad \hbar\omega_{1,0} = \frac{J_z}{2} - J_{xy}, \quad (\text{B.9})$$

$$\hbar\omega_{0,0} = \frac{J_z}{2} + J_{xy}, \quad \hbar\omega_{1,-1} = -g\mu_e B - \frac{J_z}{2}. \quad (\text{B.10})$$

Denoting the total Hamiltonian of the pair as $H = H_{HC} + H_Z$, the evolution of the system from a state $|\psi(t_0)\rangle$ to a state $|\psi(t)\rangle$ obeys the Schrödinger equation:

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle. \quad (\text{B.11})$$

When $|\psi(t)\rangle$ is developed over the $|j\rangle$ kets, i.e. the $|S, m_S\rangle$ orthonormal basis eigenkets of H :

$$|\psi(t)\rangle = \sum_{j=1}^4 b_j(t) |j\rangle, \quad (\text{B.12})$$

then Eq. (B.11) leads to a system of four uncoupled linear differential equations:

$$i\hbar \frac{d}{dt} b_j(t) = \hbar\omega_j b_j(t) \quad (\text{B.13})$$

where ω_1 to ω_4 is a short writing for $\omega_{1,1}$, $\omega_{1,0}$, $\omega_{0,0}$ and $\omega_{1,-1}$ respectively, with the solutions $b_j(t) = e^{-i\omega_j(t-t_0)} b_j(t_0)$, and therefore:

$$|\psi(t)\rangle = \sum_{j=1}^4 e^{-i\omega_j(t-t_0)} b_j(t_0) |j\rangle. \quad (\text{B.14})$$

Since $|1, 1\rangle = |++\rangle$ and $|1, -1\rangle = |--\rangle$, $b_1(t_0) = \alpha_1\alpha_2$ and $b_4(t_0) = \beta_1\beta_2$. The detailed expression of $|\psi(t)\rangle$ and a matrix formulation are given in [17], [18]. It should be clear that the possibility of a matrix formulation has nothing to do with the nature of the coupling between the qubits. It results from two general properties of QP: the existence of the principle of superposition and of the linear character of the Schrödinger equation. The cylindrical symmetry of the Heisenberg coupling and the fact that the spins are $1/2$ are reflected in the form of the expression for $|\psi(t)\rangle$ and the numerical values of J_z , J_{xy} and B are reflected in the numerical values of the ω_j .

Appendix C. Removing indeterminacies with additional constraints

In Appendix G.2.2.2 we show that, for the considered mixing model and separation principle (multi-state disentanglement), the class of separating systems defined by (20)-(21) and (10) yields limited indeterminacies only, namely a permutation of the two output qubit states and a phase factor between both components of each output qubit state. We hereafter show how to remove both indeterminacies, by further constraining the considered configuration.

It should first be noted that we did not exploit the whole available knowledge about the considered mixing model yet: we already stated that the frequencies $\omega_{1,1}$ to $\omega_{1,-1}$ in (11) have unknown values, but we now take advantage of their properties in addition. By deriving the expressions of these frequencies from (B.9)-(B.10) and using (30)-(33), we get

$$\gamma_{1d} = \omega_{1,1}(t - t_0) = \frac{1}{\hbar} \left[g\mu_e B - \frac{J_z}{2} \right] (t - t_0) \quad (\text{C.1})$$

$$\gamma_{2d} = \omega_{1,0}(t - t_0) = \frac{1}{\hbar} \left[-J_{xy} + \frac{J_z}{2} \right] (t - t_0) \quad (\text{C.2})$$

$$\gamma_{3d} = \omega_{0,0}(t - t_0) = \frac{1}{\hbar} \left[J_{xy} + \frac{J_z}{2} \right] (t - t_0) \quad (\text{C.3})$$

$$\gamma_{4d} = \omega_{1,-1}(t - t_0) = \frac{1}{\hbar} \left[-g\mu_e B - \frac{J_z}{2} \right] (t - t_0) \quad (\text{C.4})$$

where

- g is the principal value of the considered isotropic \bar{g} tensor and μ_e is the Bohr magneton, i.e. $\mu_e = e\hbar/2m_e = 0.927 \times 10^{-23} JT^{-1}$,
- B is the magnitude of the applied magnetic field (which should be high enough for the considered coupling model to be relevant),
- J_{xy} and J_z are the principal values of the exchange tensor.

Eq. (C.1)-(C.4) first entail

$$\gamma_{1d} + \gamma_{2d} + \gamma_{3d} + \gamma_{4d} = 0. \quad (\text{C.5})$$

Since we aim at tuning γ_1 to γ_4 until they reach values which meet (C.5), we may constrain them to meet condition

$$\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 = 0 \quad (\text{C.6})$$

throughout this adaptation procedure. Our calculations show that imposing this condition in addition to the constraints set in Appendix G.2.2.2 does not reduce the indeterminacies obtained in Appendix G.2.2.2 enough. Therefore, we do not consider condition (C.6) hereafter (note that it could be used instead to reduce the number of degrees of freedom of the separating system). Still, the above discussion suggests a useful extension, which consists of looking for other relationships between γ_{1d} to γ_{4d} . The relationships of interest are those which do not involve the parameters, among those of (C.1)-(C.4), whose values are quite hard to determine experimentally and therefore considered as unknown, that is J_{xy} and J_z . On the contrary, the time interval $(t - t_0)$ and the magnitude B of the magnetic field may

be measured and the principal value g may be experimentally determined. Eq. (C.1) and (C.4) then yield

$$\gamma_{1d} - \gamma_{4d} = \frac{2}{\hbar} g \mu_e B (t - t_0) \quad (\text{C.7})$$

and the value of the right-hand term of that equation is known. Using the same idea as above, we therefore constrain the parameters γ_1 and γ_4 to be jointly tuned, so as to always meet

$$\gamma_1 - \gamma_4 = \frac{2}{\hbar} g \mu_e B (t - t_0). \quad (\text{C.8})$$

Eq. (C.7), (C.8), (26), (29) then guarantee that we always have

$$\delta_4 = \delta_1. \quad (\text{C.9})$$

In particular, condition (C.9) is met for all solutions (i.e. values of δ_1 to δ_4 and associated states $|\Phi\rangle$) of the separation principle studied in Appendix G.2.2.2. Eq. (G.21) then reduces to

$$\delta_5 = -k\pi, \quad (\text{C.10})$$

so that the phase indeterminacies due to the factors $e^{i\delta_5}$ in the output state expressions (G.22), (G.23) reduce to sign indeterminacies between both components of each qubit, since $e^{i\delta_5} = \pm 1$.

The only remaining indeterminacies at this stage are therefore the above signs and the permutation between qubits. Both of them may be removed as follows. The above adaptation phase uses a given value of the time interval, denoted $(t - t_0)$, between the preparation of the source qubit state $|\psi(t_0)\rangle$ and the use of the resulting mixed state $|\psi(t)\rangle$. This adaptation phase yields a set of values for the separating system parameters γ_1 to γ_4 . In the approach considered up to now, the same values of $(t - t_0)$ and of γ_1 to γ_4 are then used in the inversion phase. We here propose a modified approach where, in the inversion phase, the time interval between source qubit state preparation and use of resulting mixed state is twice larger¹⁵ than the interval $(t - t_0)$ used in the adaptation phase, and the values of γ_1

to γ_4 obtained at the end of the adaptation phase are also multiplied by two before being used during the inversion phase. Eq. (26)-(33) show that the values of δ_1 to δ_4 used in the inversion phase are thus equal to the double of the corresponding values obtained in the adaptation phase. This factor of two between the values used in these two phases therefore also exists for any linear combination of δ_1 to δ_4 with fixed coefficients. In particular, let us consider the two combinations which appear in the left-hand terms of (G.9) and (G.20). In the adaptation phase, they are respectively constrained to become equal to $m\pi$ and $2k\pi$ by the multi-state disentanglement principle, as shown in Appendix G.2. Therefore, the corresponding combinations of the values of δ_1 to δ_4 used in the inversion phase are respectively equal to $m'\pi$ and $2k'\pi$, with $m' = 2m$ and $k' = 2k$. In other words, during the inversion phase, when still using an unentangled state as the source state and the above-defined conditions for the mixing and separating stages, the corresponding separating system output again meets (G.9) and (G.20), but it should be understood that all quantities in this use of (G.9) and (G.20) refer to their version during the inversion phase, i.e. their version “with a prime sign in all notations”. Therefore, the integers m and k used in the inversion phase are equal to the double of those obtained in the adaptation phase and are thus *guaranteed to be even*, which was our goal.

In this inversion phase, since (G.9) and (G.20) are met, one then derives in the same way as in Appendix G.2.2 that (G.21), (G.22), (G.23) are met. Using in addition the constraint (C.8) in this inversion phase (therefore implicitly “with a prime sign in all notations”), condition (G.21) again reduces to (C.10). Finally, one should take into account that, in this inversion phase, m and k (“with a prime sign”) are *guaranteed to be even*. Therefore (i) among the two solutions (G.22) and (G.23), it is guaranteed that the obtained solution is (G.22) and (ii) the phase parameter δ_5 is a multiple of 2π and therefore the phase factor $e^{i\delta_5}$ in (G.22) is equal to one. As an overall result, the output state (G.22) of the separating system reduces to (G.26). Adding the constraints proposed in this appendix to the approach of Appendix G.2.2.2 therefore guarantees that the separating system restores the unentangled source states without any indeterminacies.

Appendix D. Scalar form of mixing model

The scalar form of the mixing model defined in Section 2 may be derived from the equations of that section and was provided in [17]. It reads

$$c_1(t) = \alpha_1 \alpha_2 e^{-i\omega_{1,1}(t-t_0)} \quad (\text{D.1})$$

$$c_2(t) = \frac{1}{2} \left[(\alpha_1 \beta_2 + \beta_1 \alpha_2) e^{-i\omega_{1,0}(t-t_0)} + (\alpha_1 \beta_2 - \beta_1 \alpha_2) e^{-i\omega_{0,0}(t-t_0)} \right] \quad (\text{D.2})$$

$$c_3(t) = \frac{1}{2} \left[(\alpha_1 \beta_2 + \beta_1 \alpha_2) e^{-i\omega_{1,0}(t-t_0)} - (\alpha_1 \beta_2 - \beta_1 \alpha_2) e^{-i\omega_{0,0}(t-t_0)} \right] \quad (\text{D.3})$$

$$c_4(t) = \beta_1 \beta_2 e^{-i\omega_{1,-1}(t-t_0)}. \quad (\text{D.4})$$

¹⁵In this paper, it is assumed that the time interval $(t-t_0)$ cannot be decreased freely, otherwise one would choose to set it to zero, which would avoid any coupling phenomenon and the associated QSS problem. In other words, we are interested in situations when the existence of a minimum time interval between the preparation of the source qubit states and the use of their coupled version is an undesired but unavoidable phenomenon, and this interval is above a fixed minimum value. In such systems, we can then reasonably assume that one can increase this interval if required by the considered approach. We here use this idea in the inversion phase (possibly at the expense of reducing the speed of operation of the overall quantum processing system during the inversion phase). This approach also requires some cooperation between the mixing and separating stages: both stages should simultaneously switch their parameter values (width of time interval and values of γ_1 to γ_4), in order to simultaneously switch from the adaptation phase to the inversion phase. One might argue that, when adding this principle of operation, the proposed QSS approach becomes “less blind”. However (i) it is still quite blind, in the sense that the separating system is not requested to know the values of the source qubit states corresponding to the mixed states $|\psi(t)\rangle$ that it uses, (ii) this approach as well as all classical BSS methods cannot be claimed to be fully blind anyway, because they set some conditions on source properties and on the mixing model (which is the reason why some authors call classical BSS methods “myopic” or “semi-blind” rather than “blind”, with a degree of blindness which depends on the considered method).

Appendix E. Scalar form of global model

The scalar form of the global model (22) may be derived either directly from the equations of Sections 2 to 3.2, or more easily as follows. Comparing (24) to (9) shows that the global model has the same expression as the mixing model, except that the exponentials in (11) are replaced by those in (25). Applying the same substitution to the scalar form (D.1)-(D.4) of the mixing model therefore yields the scalar form of the global model, which may eventually be expressed as

$$c_1 = \alpha_1 \alpha_2 e^{i\delta_1} \quad (\text{E.1})$$

$$c_2 = \frac{1}{2} e^{i\delta_2} [(\alpha_1 \beta_2 + \beta_1 \alpha_2) + (\alpha_1 \beta_2 - \beta_1 \alpha_2) e^{i(\delta_3 - \delta_2)}] \quad (\text{E.2})$$

$$c_3 = \frac{1}{2} e^{i\delta_2} [(\alpha_1 \beta_2 + \beta_1 \alpha_2) - (\alpha_1 \beta_2 - \beta_1 \alpha_2) e^{i(\delta_3 - \delta_2)}] \quad (\text{E.3})$$

$$c_4 = \beta_1 \beta_2 e^{i\delta_4}. \quad (\text{E.4})$$

Appendix F. Adapting block using a quantum-form property

Beyond the adapting block detailed in Section 3.3, the very next step towards increasingly quantum approaches consists of considering a separation principle, i.e. a property here again imposed on the separating system output, which is directly defined with respect to the quantum form $|\Phi\rangle$ of that output, as shown in Fig. F.6. In the framework of Section 3.3, i.e. when addressing the second step of the procedure that we defined in Section 1 for developing a complete BQPSS method, using such a quantum-form property is still a conceptual idea (this property is therefore shown inside an elliptic box in Fig. F.6), as opposed to practical processing blocks (represented in square boxes in this and previous figures). This still leaves open how this property is used to close the feedback loop of Fig. F.6 (as shown by the “?” in the adapting block of that figure). One may convert the quantum data into classical-form signals at some stage of this feedback loop. One might instead wonder whether all this feedback processing could be performed in quantum form but, again, the quantum nature of this feedback structure deserves some care: [38] p. 23 e.g. states that “There are a few features allowed in classical circuits that are not usually present in quantum circuits. First of all, we don’t allow ‘loops’, that is, feedback from one part of the quantum circuit to another”. In Section 4 and in the associated appendix, i.e. when addressing the third step of the procedure that we defined in Section 1 for developing a complete BQPSS method, we describe the separation principle that we propose for the approach of Fig. F.6, and we complete the description of its feedback loop, using quantum/classical conversion.

Appendix G. Developing the separation principles

Appendix G.1. From classical to quantum BSS separation principles

As explained in Section 4, we here detail the third step of the procedure that we defined in Section 1 for developing a complete BQPSS method. This step consists of specifying the considered separation principle. To this end, we may take advantage of the following two results, available in classical BSS.

First, a *specific approach* to classical BSS is defined for random sources and is based on the independence principle used in ICA methods, which consists of forcing the statistical mutual independence of the separating system outputs. This approach may be justified as follows. In ICA methods, the unknown source signals are requested to be stochastic and mutually statistically independent. The resulting known mixtures of these source signals are not independent however, due to the mixing phenomenon. The mixing operator therefore destroys the independence property of the original source signals. This suggests one to tune the parameters of the associated class of separating operators so as to restore in the separating system outputs the property met by the source signals, i.e. to tune them so that the separating system outputs become independent. One then hopes that this constraint is sufficient to force the separating system outputs to become equal to the source signals, or at least equal to them up to residual transforms (called indeterminacies) which correspond to limited, and therefore acceptable, modifications of the source signals. If this behavior is indeed obtained for the considered mixing operator, this mixing¹⁶ model is stated to be ICA separable. Therefore, for a given mixing model, once a separation principle has been proposed, such as the independence principle, one must then analyze whether that model is separable from the point of view of that principle. This has been performed for various mixing models in classical BSS, thus showing that only part of them are ICA separable (see e.g. [8], [9], [33], [44], [45]).

Still in the classical framework, one may build other separation principles by just transposing the interpretation that we provided above for the independence principle to other source properties. A *general approach* thus consists of exhibiting a property met by the source signals in the considered configuration but lost in their available mixtures, and in tuning the parameters of the separating system so that the outputs of that system meet the above property. This allows one to *propose* the separation principle based on the selected property. One must then *analyze* whether that property guarantees separability (i.e. yields acceptable indeterminacies) for the considered mixing model. Some classical BSS methods are based on this general approach, although this is most often not stated explicitly in their description.

Taking into account the above state of the art for classical BSS suggests several approaches for building separation prin-

¹⁶More precisely, ICA separability is a property of the combination of (i) the considered mixing model and (ii) the separating model which is selected accordingly. It is therefore a property of the considered global (i.e. mixing + separating) model.

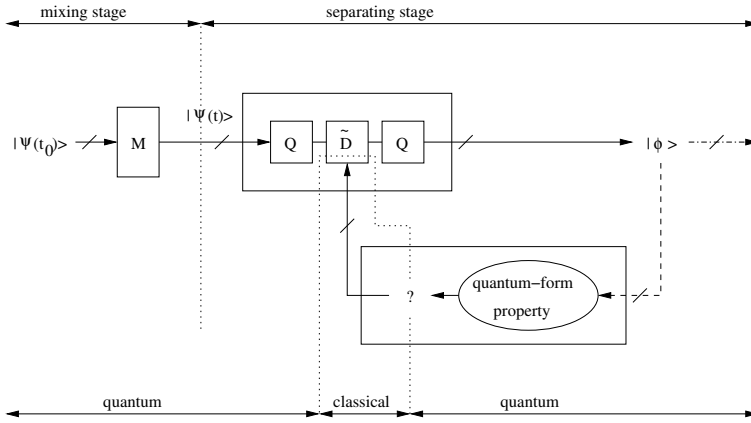


Figure F.6: Global configuration using a quantum-processing inverting block, and a quantum-form property in the separation principle of the adapting block (this separation principle controls sub-block \tilde{D}). Each quantum state $|\Phi\rangle$ is used only once (no cloning): see p. 6.

principles in the framework of BQPSS. The most conservative approach consists of sticking as much as possible to the commonly used “specific approach” described above, namely classical ICA. One then uses classical-form outputs of the separating system, as in Fig. 3 (the type(s) of measurements performed to derive them should then be defined), and adapts the parameters of the separating system so that the above classical-form signals become mutually statistically independent. We will not initially base our developments on this approach, because there is no a priori guarantee that this separation principle remains relevant when moved in an *ad hoc* way from the classical to the quantum framework. Nevertheless, we will come back to this approach, as a by-product of the other solutions considered hereafter.

The BQSS separation principles that we propose below are based on the “general approach” that we defined above for classical BSS. They rely on the observation that, in all variants of the BQSS problem studied in this paper, the source state (4) meets a property which is lost in the corresponding available mixed state defined by (7) and (D.1)-(D.4): this source state is not entangled. The first separation principle that we propose therefore consists of adapting the tunable parameters of the sub-block \tilde{D} of our separating system so that the quantum output $|\Phi\rangle$ of this system becomes disentangled. This separation principle is therefore called the “*disentanglement principle*” hereafter. It is based on a property of quantum states and therefore corresponds to the separating structure shown in Fig. F.6. It is initially more justified than the above BQSS approach based on sticking to classical ICA, because it is derived from the fundamental property of the quantum sources faced here, namely their unentanglement. This approach then requires us to analyze whether that disentanglement principle guarantees separability for the considered mixing model. That principle (applied to one and then several states) and its separability properties are studied in Appendix G.2.

Before moving to the analysis of the above disentanglement principle, we here complete the definition of all separation principles studied in this paper. Modified versions of the above disentanglement-based approach are derived by replacing the

above quantum property (that is: output disentanglement) by other properties which have a direct relationship with it, but which concern associated classical-form signals (this corresponds to Fig. 3). This yields different versions, depending on which types of measurements are performed to derive these classical-form signals and how these signals are exploited. Appendix G.3 and Appendix G.4 describe two such versions and make the connection with the above disentanglement-based approach.

Appendix G.2. Method based on disentanglement principle

In this Appendix G.2, we consider the adaptation phase (except at the end of Appendix G.2.2.2, starting from the paragraph including (G.25), where we also make the connection with the inversion phase). We here analyze the properties of the first method proposed above for adapting the parameters γ_1 to γ_4 of sub-block \tilde{D} of the separating system during the adaptation phase. In other words, these parameters are here tuned so that the output state $|\Phi\rangle$ is forced to be unentangled, considering a single state $|\Phi\rangle$ as a first step (our investigation then leads us to consider *two* such states). Starting from the very general expression (12) of that state, it may be shown¹⁷ that this unentanglement condition reads

$$c_1 c_4 = c_2 c_3. \quad (\text{G.1})$$

We now investigate whether this constraint guarantees separability, i.e. whether it forces the output state $|\Phi\rangle$ to become equal to the source state $|\psi(t_0)\rangle$ up to acceptable indeterminacies. Condition (G.1) involves complex-valued parameters. It may equivalently be expressed in terms of two sub-conditions, respectively concerning the moduli and phases of these parameters. We first study the modulus-based sub-condition and then combine it with the phase-based sub-condition, in order to eventually analyze the complete unentanglement condition.

¹⁷We introduced condition (G.1) in [19], [20] and we then justified it in more detail in [24].

Appendix G.2.1. Modulus-based sub-condition of disentanglement principle

Appendix G.2.1.1 Single-state modulus-based sub-condition.

We start by considering a single output state $|\Phi\rangle$ and analyzing the first of the above-defined sub-conditions, which reads

$$|c_1 c_4| = |c_2 c_3|. \quad (\text{G.2})$$

Appendix H shows that¹⁸ (G.2) is equivalent to

$$\sin(\delta_3 - \delta_2) [B_1 \sin(\delta_3 - \delta_2) + B_2 \cos(\delta_3 - \delta_2)] = 0 \quad (\text{G.3})$$

with δ_2 and δ_3 defined in (27) and (28), and

$$B_1 = (A_1^2 - A_2^2)^2 - 4 \sin^2(\xi_1 - \xi_2) A_1^2 A_2^2 \quad (\text{G.4})$$

$$B_2 = 4 A_1 A_2 (A_1^2 - A_2^2) \sin(\xi_1 - \xi_2) \quad (\text{G.5})$$

where we use the polar representations

$$\alpha_1 \beta_2 = A_1 e^{i\xi_1} \quad (\text{G.6})$$

$$\beta_1 \alpha_2 = A_2 e^{i\xi_2}. \quad (\text{G.7})$$

Condition (G.3) has two solutions, respectively corresponding to

$$\sin(\delta_3 - \delta_2) = 0 \quad (\text{G.8})$$

and to the cancellation of the second factor of (G.3). The first solution, corresponding to (G.8), is

$$\delta_3 - \delta_2 = m\pi \quad (\text{G.9})$$

where m is an integer. The corresponding output of our separating system is defined by (E.1)-(E.4). It may easily be shown that, when (G.9) is met, (E.2)-(E.3) become

$$c_2 = \alpha_1 \beta_2 e^{i\delta_2} \quad \text{if } m \text{ even} \quad (\text{G.10})$$

$$c_2 = \beta_1 \alpha_2 e^{i\delta_2} \quad \text{if } m \text{ odd} \quad (\text{G.11})$$

$$c_3 = \beta_1 \alpha_2 e^{i\delta_2} \quad \text{if } m \text{ even} \quad (\text{G.12})$$

$$c_3 = \alpha_1 \beta_2 e^{i\delta_2} \quad \text{if } m \text{ odd}. \quad (\text{G.13})$$

Comparing these expressions, Eq. (E.1), (E.4), and the associated state (12) to (5) shows that the output state $|\Phi\rangle$ of our separating system then partly succeeds in restoring the source state $|\psi(t_0)\rangle$: c_1 to c_4 are equal to the corresponding coefficients in (5) up to two phenomena which correspond to usual indeterminacies in classical BSS, namely a possible permutation of c_2 and c_3 , depending whether m is odd or not, and some phase factors. Due to these phase factors, $|\Phi\rangle$ may still be entangled. This means that only setting this single-state modulus-based constraint on the separating system output would yield excessive indeterminacies (on the contrary, a global phase on $|\Phi\rangle$ is not an issue, since it has no physical consequence).

The second solution of (G.3), corresponding to the cancellation of its second factor, reads

$$\delta_3 - \delta_2 = -\text{Arctan} \frac{B_2}{B_1} + m\pi \quad (\text{G.14})$$

where m is an integer. It therefore generally has a more complex expression than (G.9), so that the corresponding output state $|\Phi\rangle$ of our separating system does not seem to restore the source state $|\psi(t_0)\rangle$ up to acceptable indeterminacies. This second solution of (G.3) therefore appears to be a spurious one from the point of view of BQSS. If the parameters γ_1 to γ_4 of the separating system were controlled only so as to ensure that condition (G.2) is met, the existence of this second solution of (G.3) would be an issue, because the considered control procedure would not guarantee which of the two solutions of (G.3) is reached by the values taken by γ_1 to γ_4 , and this procedure could therefore provide the above spurious solution. We now show how to avoid this issue.

Appendix G.2.1.2 Multi-state modulus-based sub-condition.

The above spurious solution is avoided by introducing an extended version of our BQSS method, which uses *two* (or more) source states $|\psi(t_0)\rangle$, and which adapts the *single* set of parameters γ_1 to γ_4 so as to ensure that (G.2) is met for each of these two source states, instead of the single state considered up to this point. This extended method removes the spurious solution of (G.2) and (G.3) as follows.

The second solution of (G.3) depends on the values of the parameters B_1 and B_2 and therefore on the considered source state. Therefore, a single set of parameters γ_1 to γ_4 cannot simultaneously correspond to the second solution of (G.3) for two source states, provided these states yield parameter values B_1 and B_2 resulting in different values for this second solution of (G.3). These states are then stated to be non-redundant. This non-redundancy assumption is hereafter assumed to be met for the considered data. Besides, more than two source states may be jointly used, to increase the probability that they do not all yield the same second solution of (G.3), i.e. that they are non-redundant.

On the contrary, the first solution of (G.3) is defined by (G.9) and therefore applies to any source state. So, by tuning the adaptive parameters γ_1 to γ_4 of our separating system in such a way that (G.2) is guaranteed to be met for both (or all) non-redundant source states, we guarantee that these parameters γ_1 to γ_4 become such that the *first* solution of (G.3) is reached.

Appendix G.2.2. Complete condition of disentanglement principle

Appendix G.2.2.1 Single-state complete condition.

We here first consider the phase-based sub-condition involved in (G.1), for a single output state $|\Phi\rangle$. This sub-condition reads¹⁹

$$\arg(c_1 c_4) = \arg(c_2 c_3) + 2k\pi \quad (\text{G.15})$$

or equivalently

$$\arg(c_1 c_4 c_2^* c_3^*) = 2k\pi \quad (\text{G.16})$$

¹⁸Some calculations in this paper require all four coefficients c_1 to c_4 in (G.2) to be non-zero. This is not mentioned everywhere.

¹⁹As stated above, some calculations in this paper assume all coefficients c_1 to c_4 to be non-zero. This is especially required for the phase of the complex numbers involved in (G.15) to be defined.

where $*$ stands for conjugation and k is an integer. Moreover, for any output state $|\Phi\rangle$ of our separating system, Eq. (H.1)-(H.2) yield

$$c_1 c_4 c_2^* c_3^* = c_5 e^{i(\delta_1 + \delta_4 - 2\delta_2)} \quad (\text{G.17})$$

with

$$c_5 = \alpha_1 \alpha_2 \beta_1 \beta_2 \left[\alpha_1 \alpha_2 \beta_1 \beta_2 + \frac{1}{4} (\alpha_1 \beta_2 - \beta_1 \alpha_2)^2 \times (1 - e^{i2(\delta_3 - \delta_2)}) \right]^* \quad (\text{G.18})$$

We now analyze the case when both the phase condition (G.16) (with (G.17)-(G.18)) and the modulus condition (G.2) are met, for the considered output state $|\Phi\rangle$. As shown in Appendix G.2.1.1, due to the constraint set by the modulus condition (G.2), $|\Phi\rangle$ can only correspond to one of the two solutions of (G.3). We therefore successively consider each of these two solutions of (G.3), and we combine its modulus condition with the phase condition (G.16) in order to obtain the overall disentanglement condition for that state. We start with the first solution of (G.3), which is defined by (G.9). Combining that condition with (G.18), the latter equation reduces to

$$c_5 = |\alpha_1 \alpha_2 \beta_1 \beta_2|^2 \quad (\text{G.19})$$

which is a real positive number. Due to (G.17), condition (G.16) then reads

$$\delta_1 + \delta_4 - 2\delta_2 = 2k\pi. \quad (\text{G.20})$$

The complete disentanglement condition for the first solution of (G.3) therefore consists of (G.9) and (G.20).

Inserting (G.20) into (G.10)-(G.13) and using (E.1) and (E.4) then makes it possible to express the components of the resulting output state of our separating system with respect to δ_1 and δ_4 only. This state is defined by (12). Denoting

$$\delta_5 = \frac{\delta_4 - \delta_1}{2} - k\pi, \quad (\text{G.21})$$

additional manipulations then show that this state may be expressed as follows if m is even:

$$|\Phi\rangle = e^{i\delta_1} (\alpha_1|+\rangle + \beta_1 e^{i\delta_5}|-\rangle) \otimes (\alpha_2|+\rangle + \beta_2 e^{i\delta_5}|-\rangle) \quad (\text{G.22})$$

whereas, if m is odd

$$|\Phi\rangle = e^{i\delta_1} (\alpha_2|+\rangle + \beta_2 e^{i\delta_5}|-\rangle) \otimes (\alpha_1|+\rangle + \beta_1 e^{i\delta_5}|-\rangle). \quad (\text{G.23})$$

For this first solution of (G.3), the disentanglement principle is therefore such that, for the resulting values of γ_1 to γ_4 , the output state $|\Phi\rangle$ obtained for any unentangled source state (4) is also unentangled. In addition, (G.22) and (G.23) show that this output meets another property: the factors of this tensor product (G.22) or (G.23) are respectively equal to each of the source qubit states (1), possibly up to two transforms, defined as follows. The first transform is a permutation: for instance, the first output qubit corresponds to the first factor in the tensor products (G.22) and (G.23) so that, for the solution defined by (G.23), it depends on α_2 and β_2 , i.e. it restores information corresponding to the *second* source qubit. The second transform is

a phase $e^{i\delta_5}$ for the second qubit state component with respect to the first one, separately for each output qubit (the phase $e^{i\delta_1}$ should be ignored, since it applies to all qubit state components and therefore has no physical consequence). Output permutations and constant factors (here reduced to phases) are also very common in classical BSS, where they are called ‘‘indeterminacies’’. We will show how to remove them, for the proposed BQSS method, in Appendix G.2.2.2.

Before that, we now investigate the output state obtained, with the single-state complete disentanglement condition, for the second solution of (G.3). That second solution corresponds to the cancellation of the second factor of (G.3). This fixes the value of $\tan(\delta_3 - \delta_2)$, and therefore the value of $(\delta_3 - \delta_2)$ up to a multiple of π . That value depends on the considered source state. Eq. (G.18) then shows that this fixes c_5 , to a value which depends on the considered source state. Let us denote as ϕ_5 the corresponding phase of c_5 . Using (G.17), Eq. (G.16) becomes

$$\phi_5 + \delta_1 + \delta_4 - 2\delta_2 = 2k\pi. \quad (\text{G.24})$$

The second, spurious, solution of (G.3), obtained in Appendix G.2.1.1 for the single-state modulus-based sub-condition, therefore here does yield an associated solution (which depends on the considered source state) for the single-state complete disentanglement condition. However, as in Appendix G.2.1.1, the solution obtained here has a complex expression, so that the corresponding output state $|\Phi\rangle$ of our separating system does not seem to restore the source state $|\psi(t_0)\rangle$ up to acceptable indeterminacies. The second solution obtained here therefore appears to be a spurious one from the point of view of BQSS. In other words, the *single-state* disentanglement principle is not sufficient for solving the considered BQSS problem in a satisfactory way. As in Appendix G.2.1, we will now show how to solve this issue by considering more than one source state.

Appendix G.2.2.2 Multi-state complete condition.

In the first multi-state scenario proposed hereafter, we use the unentanglement condition (G.1) for output states $|\Phi\rangle$ corresponding to two (or more) source states. To this end, we start by taking into account that the modulus-based sub-condition (G.2) is met for these states $|\Phi\rangle$. Appendix G.2.1.2 then shows that (G.9) is met, i.e. we thus avoid the above-defined spurious solution, which is our motivation for considering more than one source state. We then combine the constraint (G.9), i.e. the first solution of the modulus-based sub-condition considered in Appendix G.2.2.1, with the phase-based sub-condition defined in that appendix, i.e. (G.16). We thus obtain the complete disentanglement condition for these two (or more) states $|\Phi\rangle$. The calculations required for the above-mentioned combination were already performed in Appendix G.2.2.1: we here consider two states instead of one, but since their modulus-based sub-conditions here yield (G.9), Appendix G.2.2.1 here shows that also taking into account the phase-based sub-condition for both states yields (G.20)²⁰. Therefore, the resulting output state of our separating system is here again defined by (G.22) or (G.23).

²⁰One may wonder whether imposing the above disentanglement condition for *more than* two (non-redundant) source states entails more constraining con-

In a second scenario, we again first use (at least) two states $|\Phi\rangle$ in order to impose the constraints (G.2) and thus (G.9), but we then use a *single and arbitrary* state $|\Phi\rangle$, on which we impose the constraint (G.16) in addition to (G.9). The same analysis as above shows that the overall set of constraints thus imposed on the parameters of the separating system again consists of (G.9) and (G.20), which then again leads to (G.22) or (G.23). This second scenario is less constraining than the first one because, in the second part of its operation, it uses one state instead of two, and this state may be arbitrary instead of using the same states as in the first part of its operation.

In both scenarios, the output state of our separating system has the permutation and phase indeterminacies which appear in (G.22) and (G.23) and which were discussed in Appendix G.2.2.1. As explained in Appendix C, these indeterminacies may here be removed, by adding two constraints to the considered configuration. Their effects during the inversion phase may be summarized as follows. The first constraint guarantees that

$$\delta_4 = \delta_1, \quad (\text{G.25})$$

which simplifies (G.21). To put it briefly, the second constraint entails that, during the inversion phase, the above integers k and m are even. All this guarantees that only solution (G.22), as opposed to (G.23), is reached, and that it reduces to

$$|\Phi\rangle = e^{i\delta_1} (\alpha_1|+\rangle + \beta_1|-\rangle) \otimes (\alpha_2|+\rangle + \beta_2|-\rangle). \quad (\text{G.26})$$

This restores the state (4) up to a global phase which has no physical consequence. It should be noted that the BQSS method proposed at this stage only requires a few quantum source states in the adaptation phase, whereas the classical-processing methods mentioned in Section 3.1 typically need 10^3 source states [17]. We discuss the properties of the resulting practical algorithms in Appendix J.

We stress that the above BQSS method only requires the source states to be unentangled *during the adaptation phase*, not during the inversion phase. More precisely, as shown above, by using unentangled states during the adaptation phase, this method fixes the parameters γ_1 to γ_4 of the separating system so that, during the subsequent inversion phase, conditions (G.9), (G.20) and (G.25) are met, and k and m are even. Combining all these conditions, Eq. (24), (25) and (10) show that G becomes equal to the identity matrix (up to the phase factor $e^{i\delta_1}$, which should be ignored, as explained above). Therefore, during the inversion phase, for any two-qubit source state $|\psi(t_0)\rangle$, entangled or not, (22) shows that the corresponding output state $|\Phi\rangle$ of our separating system is equal to $|\psi(t_0)\rangle$ (up to the phase factor $e^{i\delta_1}$).

The above theoretical investigation leaves one question open: we are considering the global architecture of Fig. F.6 and we

ditions for the separating system parameters than conditions (G.9) and (G.20) obtained here for only two states. The answer is no: in [24], we showed that imposing the above disentanglement condition for *all possible pure and unentangled source states* yields conditions (G.9) and (G.20) only. However, using more than two source states may here be of interest for the same reason as in Appendix G.2.1.2, i.e. when aiming at ensuring the above-mentioned non-redundancy of these source states.

still have to define how the quantum-form property considered here, namely the disentanglement condition (G.1) applied to two (or more) states in the above first scenario, may be used in practice to complete the feedback loop of Fig. F.6. This issue is addressed in Appendix G.3 and Appendix G.4, where we first consider other separation principles, which then allows us to connect them to the above one.

Appendix G.3. Using measurements along Oz axis in disentanglement-related principle

Back to the separating system structures introduced in Section 3, we here start from the version shown in Fig. 3, and we now aim at completing the definition of its adapting block. The first method that we propose to this end is presented in Section 4.2. This approach yields a potential separation principle, defined by (36), which is initially justified by disentanglement-based considerations. In addition, it has relationships with classical ICA, as shown in [19]. The BQSS methods based on this principle may thus be considered as Quantum-Source Independent Component Analysis (QSICA) methods. The other methods presented in this paper are also related to QSICA, but this topic is not discussed in more detail here (see also [20]), to limit the length of this paper.

Once we have selected the approach based on Condition (36), we must analyze whether it yields a relevant separation principle, i.e. whether it guarantees separability. This requires no additional calculations here because, as mentioned in Section 4.2, that separation principle is equivalent to the modulus-based sub-condition (G.2) involved in the above-defined single-state disentanglement principle, and we already analyzed the separability properties of sub-condition (G.2) in Appendix G.2.1.1, and even of its multi-state extension in Appendix G.2.1.2. This shows that, even when applying the constraint (36) to several source states so as to avoid the spurious solution defined in Appendix G.2.1.1, this only guarantees that the output state of the separating system meets (E.1), (E.4), (G.10)-(G.13). As already noted in Appendix G.2.1.1, this state succeeds only partly in restoring the source state and may still be entangled. To avoid these restrictions, we hereafter extend the approach based on (36).

Appendix G.4. Using measurements along Oz and Ox axes, link with disentanglement principle

The first scenario that we propose for a BQSS method based on measurements performed along the Oz and Ox axes is described in Section 4.3. It compares as follows to the method of the first scenario of Appendix G.2.2.2, which concerns the initial disentanglement principle: as stated in Section 4.3, although they started from different points of view, these methods lead to the same solution. Besides, they are complementary rather than redundant, as will now be explained. The approach of Appendix G.2.2.2 was first needed, to introduce the most natural separation principle, that is the disentanglement constraint (G.1) (eventually applied to several output states). However, it did not yield a practical procedure for tuning the parameters of the separating system so as to ensure this property

(G.1), which involves quantum-form data. The approach then used in Appendix G.3 (including Section 4.2) and extended in the current appendix (including Section 4.3) relies on the results of Appendix G.2.2.2, while bringing an additional feature: the constraints (36) and (37) that it uses concern classical-form data, namely probabilities, which may be accessed (estimated) in practice, and therefore used to tune the parameters of the separating system. The approach introduced in this appendix thus opens the way to a practical implementation (here again, the remaining indeterminacies are eventually removed by using the method defined in Appendix C). This implementation is described in Section 5. Since this approach is equivalent to the disentanglement-based approach of Appendix G.2.2.2 we here also made the final required step for the latter approach, that is, we also defined a way to implement it.

This equivalence of these two approaches also implies that the probability-based approach proposed here yields the same expression as in the first scenario of Appendix G.2.2.2 for the separating system output, that is (G.22)-(G.23) or its refined version (G.26) when using the principle of Appendix C.

The probabilities involved in (36) and (37) may also be used in a second scenario, based on a two-stage approach. In the first stage of that approach, we again impose the constraint (36) on two states $|\Phi\rangle$. Because of (35), Eq. (36) is equivalent to (G.2). The analysis of Appendix G.2.1 then implies that (G.9) is met. Then, in the second stage of that approach, we require (G.9) to still be met and we consider two (or more) states $|\Phi\rangle$, which are not necessarily the same as in the first stage and on which we impose the constraint (37) in addition. Due to (G.9), these states also meet (36). Here again, Appendix I then shows (for states meeting its conditions) that (G.20) is met, in addition to (G.9). The separating system output is thus here again defined by (G.22)-(G.23), or its refined version (G.26) when using the principle of Appendix C, for any output state resulting from an unentangled source state.

Appendix H. Transforming the single-state modulus-based sub-condition

We here show how to derive (G.3). Starting from (E.1)-(E.4) lengthy calculations yield

$$c_1 c_4 = \alpha_1 \alpha_2 \beta_1 \beta_2 e^{i(\delta_1 + \delta_4)} \quad (\text{H.1})$$

$$c_2 c_3 = e^{i2\delta_2} \left[\alpha_1 \alpha_2 \beta_1 \beta_2 + \frac{1}{4} (\alpha_1 \beta_2 - \beta_1 \alpha_2)^2 \times (1 - e^{i2(\delta_3 - \delta_2)}) \right]. \quad (\text{H.2})$$

Condition (G.2) then becomes

$$|\alpha_1 \alpha_2 \beta_1 \beta_2| = \left| \alpha_1 \alpha_2 \beta_1 \beta_2 + \frac{1}{4} (\alpha_1 \beta_2 - \beta_1 \alpha_2)^2 (1 - e^{i2(\delta_3 - \delta_2)}) \right|. \quad (\text{H.3})$$

Using (G.6)-(G.7), Eq. (H.3) may be rewritten as

$$|A_1 A_2| = \left| A_1 A_2 + \frac{1}{4} (A_1 e^{i(\xi_1 - \xi_2)/2} - A_2 e^{i(\xi_2 - \xi_1)/2})^2 \times (1 - e^{i2(\delta_3 - \delta_2)}) \right|. \quad (\text{H.4})$$

This equation may be solved by considering its connection with the generic equation

$$|a_1| = |a_1 + (a_2 + ib_2)| \quad (\text{H.5})$$

where a_1 , a_2 and b_2 are real numbers. The latter equation is equivalent to

$$a_2^2 + 2a_1 a_2 + b_2^2 = 0. \quad (\text{H.6})$$

In the specific case when this generic equation is applied to (H.4), some calculations yield

$$a_1 = A_1 A_2 \quad (\text{H.7})$$

$$a_2 = \frac{1}{4} \left\{ \left[(A_1^2 + A_2^2) \cos(\xi_1 - \xi_2) - 2A_1 A_2 \right] \times (1 - \cos[2(\delta_3 - \delta_2)]) + (A_1^2 - A_2^2) \sin(\xi_1 - \xi_2) \sin[2(\delta_3 - \delta_2)] \right\} \quad (\text{H.8})$$

$$b_2 = \frac{1}{4} \left\{ (A_1^2 - A_2^2) \sin(\xi_1 - \xi_2) (1 - \cos[2(\delta_3 - \delta_2)]) - \left[(A_1^2 + A_2^2) \cos(\xi_1 - \xi_2) - 2A_1 A_2 \right] \times \sin[2(\delta_3 - \delta_2)] \right\}. \quad (\text{H.9})$$

Inserting these expressions in (H.6), simple but lengthy calculations show that (H.6) is equivalent to (G.3).

Appendix I. Links between probability-based and disentanglement-based separation conditions

We here consider quantum states $|\Phi\rangle$ defined by (12) and moreover constrained to be created by starting from unentangled states (5) and successively applying to them the cylindrical-symmetry Heisenberg mixing and unmixing operations (8) and (14), with (20). Appendix G.3 showed that, if condition (36) is met for (at least) two (non-redundant) source states, then (E.1), (E.4), (G.9) and (G.10)-(G.13) are satisfied.

In this appendix, we derive the additional conditions which are met when using the approach proposed in the first scenario of Appendix G.4, i.e. when also constraining the above quantum states $|\Phi\rangle$ to meet condition (37). Standard quantum calculations show that the probabilities involved in that condition (37) read

$$P_{1x} = \frac{1}{4} |c_1 + c_2 + c_3 + c_4|^2 \quad (\text{I.1})$$

$$P_{2x} = \frac{1}{4} |c_1 - c_2 + c_3 - c_4|^2 \quad (\text{I.2})$$

$$P_{3x} = \frac{1}{4} |c_1 + c_2 - c_3 - c_4|^2 \quad (\text{I.3})$$

$$P_{4x} = \frac{1}{4} |c_1 - c_2 - c_3 + c_4|^2. \quad (\text{I.4})$$

By combining (37) and (I.1)-(I.4), one can express condition (37) as

$$\Re[(c_1^2 + c_4^2 - c_2^2 - c_3^2)(c_1 c_4 - c_2 c_3)^*] = 0 \quad (\text{I.5})$$

where $\Re[.]$ stands for real part. This condition may then be transformed in the case when, in addition, condition (36) is met

for the above-mentioned states $|\Phi\rangle$. To this end, we insert (E.1), (E.4) and (G.10)-(G.13) into (I.5), which thus becomes, whatever m

$$\Re[(\alpha_1^2 \alpha_2^2 e^{i2\delta_1} + \beta_1^2 \beta_2^2 e^{i2\delta_4} - \alpha_1^2 \beta_2^2 e^{i2\delta_2} - \beta_1^2 \alpha_2^2 e^{i2\delta_2}) \times (\alpha_1 \alpha_2 \beta_1 \beta_2)^* (e^{i(\delta_1 + \delta_4)} - e^{i2\delta_2})^*] = 0. \quad (\text{I.6})$$

We constrain our separating system to meet this condition for several source states, indexed by n with $n \in \{1, \dots, N_x\}$ and $N_x \geq 2$. These states are defined by the values of the corresponding parameters $\alpha_1(n)$, $\beta_1(n)$, $\alpha_2(n)$, $\beta_2(n)$. Inserting these parameters into (I.6) yields a set of N_x equations. The solutions of these equations are determined by introducing the following property, which may be proved by considering the polar representation of the considered complex numbers.

Property 1. *Let d be a non-zero complex number. Let $d(n)$, with $n \in \{1, \dots, N\}$ and $N \geq 2$, be a set of non-zero complex numbers such that*

$$\Re[d(n)d^*] = 0 \quad \forall n \in \{1, \dots, N\}. \quad (\text{I.7})$$

Then the phases of the complex numbers $d(n)$ are all equal, up to multiples of π .

We apply this property to (I.6) by selecting

$$d(n) = [\alpha_1^2(n)\alpha_2^2(n)e^{i2\delta_1} + \beta_1^2(n)\beta_2^2(n)e^{i2\delta_4} - \alpha_1^2(n)\beta_2^2(n)e^{i2\delta_2} - \beta_1^2(n)\alpha_2^2(n)e^{i2\delta_2}][\alpha_1(n)\alpha_2(n)\beta_1(n)\beta_2(n)]^* \quad (\text{I.8})$$

$$d = e^{i(\delta_1 + \delta_4)} - e^{i2\delta_2} \quad (\text{I.9})$$

and by considering the case when (I.7) is met with $N = N_x \geq 2$, the complex numbers $d(n)$ are non-zero and do *not* all have the same phase up to multiples of π . Then, Property 1 guarantees that $d = 0$, i.e.

$$\delta_1 + \delta_4 = 2\delta_2 + 2k\pi \quad (\text{I.10})$$

where k is an integer.

As an overall result, constraining the considered states $|\Phi\rangle$ to meet the probability-based conditions (36) and (37) imposes conditions (G.9) and (I.10) on the separating system parameters. The latter two conditions turn out to be exactly the same as conditions (G.9) and (G.20) obtained when constraining the separating system according to our multi-state disentanglement principle. The latter principle and the probability-based principle analyzed here are thus shown to be equivalent in the cylindrical-symmetry Heisenberg case.

Appendix J. Features of separation algorithms

The overall separation algorithm defined in Section 5 deserves the following comments.

First, the sweeps on parameters γ_j described in Section 5 define the principle of the algorithm, but its practical operation also contains the following aspect: to tune each γ_j of the quantum circuit which implements the sub-block \tilde{D} , what is controlled in practice is not γ_j itself but the value of a physical quantity, hereafter denoted V_j , which may e.g. be a voltage.

Each resulting parameter γ_j is thus a function $g_j(\cdot)$ of the control quantity V_j :

$$\gamma_j = g_j(V_j) \quad j \in \{1, \dots, 4\}. \quad (\text{J.1})$$

These functions $g_j(\cdot)$ are preferably invertible (e.g. increasing). An attractive feature of the algorithm considered here is that it does not require these functions to be known: in practice, this algorithm just performs sweeps over physical quantities V_j and keeps their values which minimize the cost functions F_z and F_x .

Besides, even when neglecting the estimation errors for the probabilities involved in (39) and (41), the accuracy of this algorithm is intrinsically limited by its sweep-based principle: this algorithm does not find the values of V_j (and thus γ_j) which exactly cancel F_z and F_x ; instead, it only keeps the values of V_j which are the closest to this minimum, among the discrete set of tested values of V_j . The accuracy of the solution thus obtained depends on the step-size used in the sweeps on V_j . This approach therefore yields a trade-off between accuracy and computational complexity: using many values of V_j (i.e. a small step-size over the fixed bounded domain of V_j to be tested) yields better accuracy but requires many state preparations and measurements.

The typical complexity of this algorithm is thus non-negligible. For each source state, the estimation of the probabilities involved in (39) typically requires 10^4 qubit preparations [17]. If this is performed 10^3 times, in order to compute the values of F_z for 10^3 values of a parameter V_j , this yields a total amount of 10^7 required qubit preparations (and the same holds for F_x). The overall typical required number of qubit preparations obtained here (10^7 , possibly multiplied by a few source states and two cost functions) turns out to be almost the same as with the classical-processing BQSS methods summarized in Section 3.1. However, it should be clear that the approach obtained here thus keeps different advantages, highlighted throughout the whole present paper, with respect to classical-processing methods. In particular, unlike our previous methods, this new approach requires repeated qubit preparations during the adaptation phase only, whereas it then completely avoids them during the inversion phase, as explained in Section 3.2. Moreover, although both types of methods require almost the same overall number of qubit preparations in the adaptation phase, the new approach needs much fewer different source states (therefore each prepared a larger number of times), that is, a few states instead of typically 10^3 . It thus requires “less information” about the sources, which may e.g. be attractive in applications where waiting for this information to be available is an issue. In other words, the limitation of this method is not due to its separation principle (which only requires a few source states), but to its algorithm considered at this stage (which requires these states and the resulting mixed states to be prepared many times). Indeed, still using the same separation principle, other more advanced algorithms may be developed in order to use a much lower number of qubit state preparations. Such an algorithm will be detailed elsewhere, to limit the length of the present paper. It uses around 10^4 qubit preparations to perform adaptation.

Appendix K. Parameter values used in tests

Conventional *ESR* generally operates at *X* or *Q* Bands (around 10 and 35 *GHz* respectively). For electron spins with $g = 2$, at 35 *GHz*, the resonance field is near 1.25 *T*. In the simulations, we used the values $g = 2$, $B = 1T$, $(t - t_0) = 10^{-9}$ s, whence $g\mu_e B(t - t_0)/\hbar = 175.8$.

Concerning the exchange coupling, we chose $J_z/k_B = 1K$, and $J_{xy}/k_B = 0.3K$ (cf. Appendix E of [17] and [29]), which lead to $J_z(t - t_0)/\hbar = 130.9$ and $J_{xy}(t - t_0)/\hbar = 39.26$.

Appendix L. Performance criteria

We here show how to derive the performance criteria used in Section 6. As explained above, our first BQPSS algorithm yields permutation and phase indeterminacies, so that each output state $|\Phi\rangle$ obtained with this algorithm during the inversion phase is defined (apart from errors due to the estimation of γ_1 to γ_4) by (G.22) or (G.23), which may also be expressed as follows. If m is even

$$|\Phi\rangle = e^{i\delta_1}\alpha_1\alpha_2|++\rangle + e^{i(\delta_1+\delta_5)}\alpha_1\beta_2|+-\rangle + e^{i(\delta_1+\delta_5)}\beta_1\alpha_2|-+\rangle + e^{i(\delta_1+2\delta_5)}\beta_1\beta_2|--\rangle \quad (\text{L.1})$$

whereas, if m is odd

$$|\Phi\rangle = e^{i\delta_1}\alpha_1\alpha_2|++\rangle + e^{i(\delta_1+\delta_5)}\beta_1\alpha_2|+-\rangle + e^{i(\delta_1+\delta_5)}\alpha_1\beta_2|-+\rangle + e^{i(\delta_1+2\delta_5)}\beta_1\beta_2|--\rangle. \quad (\text{L.2})$$

These two possible values of $|\Phi\rangle$ should be compared to the considered source state, defined by (5). Let us first consider the case when the output state (L.1) is obtained, up to estimation errors. Each estimate of a coefficient c_j in (L.1) is equal to the coefficient $c_j(t_0)$ in (5), up to estimation errors and up to a specific phase factor which should here not be considered as an error, but as an intrinsic indeterminacy of our first algorithm. These phase differences are therefore not taken into account in our first performance criterion. Hence, only the *moduli* of $c_j(t_0)$ and c_j are compared in that criterion. For (an estimate of) the state $|\Phi\rangle$ defined by (L.1), this criterion is therefore based on the Mean Square Error (MSE) for these moduli, which reads

$$\frac{1}{4} \sum_{j=1}^4 [|c_j(t_0)| - |c_j|]^2. \quad (\text{L.3})$$

The case when the output state (L.2) is obtained leads to the same considerations, except that the coefficients corresponding to the states $|+-\rangle$ and $|-+\rangle$ are permuted between (L.2) and (5). The corresponding MSE for coefficient moduli is then defined as

$$\frac{1}{4} \left\{ [|c_1(t_0)| - |c_1|]^2 + [|c_2(t_0)| - |c_3|]^2 + [|c_3(t_0)| - |c_2|]^2 + [|c_4(t_0)| - |c_4|]^2 \right\}. \quad (\text{L.4})$$

For a *single source state* $|\psi(t_0)\rangle$ and an obtained output state $|\Phi\rangle$, since the latter state may correspond either to (L.1) or to (L.2), we define the overall MSE between the moduli of the coefficients of $|\psi(t_0)\rangle$ and $|\Phi\rangle$ as the minimum between the quantities

(L.3) and (L.4). Then, for *all the source states* $|\psi(t_0)\rangle$ used in one elementary test, since the above permutation occurs for no states or all of them (depending on the value of m that they share), the overall MSE for that test is obtained by first separately computing the two mean values of (L.3) and (L.4) over all states and then deriving the minimum between these two mean values. Then computing the mean of the latter minimum over all considered elementary tests yields the overall MSE for all elementary tests. Finally taking the square root of the latter MSE yields the Root Mean Square Error (RMSE) for these moduli over all considered elementary tests. This RMSE for moduli is denoted as $RMS E_m$ in this paper.

The above criterion is also applicable to our second BQPSS algorithm (moreover, the state (L.2) need not be considered then, since our second algorithm only yields the state (L.1)). However, that criterion only characterizes the quality of the *moduli* of the coefficients c_j provided by any BQPSS algorithm. Since our second algorithm is also supposed to achieve phase properties for these coefficients, we derived a second performance criterion for also characterizing the quality of the *phases* of the coefficients c_j . If our second algorithm was supposed to make all complex-valued coefficients c_j respectively equal to $c_j(t_0)$ up to estimation errors, its modulus and phase performance would be assessed by just computing the usual RMSE for these complex-valued quantities. But the situation is slightly different here: for our second algorithm, each output state $|\Phi\rangle$ is theoretically equal to the corresponding source state $|\psi(t_0)\rangle$ up to a *global phase factor*, as shown by (G.26). This phase factor should not be considered as an error of our second algorithm, because quantum states are only defined up to a phase factor. Therefore, we define the actual MSE of a state $|\Phi\rangle$ obtained in practice, with respect to a given state $|\psi(t_0)\rangle$, as

$$MSE(|\psi(t_0)\rangle, |\Phi\rangle) = \frac{1}{4} \|C_+(t_0) - e^{i\varphi_{opt}} C\|^2, \quad (\text{L.5})$$

which is thus expressed with respect to the coefficient vectors (3) and (13) associated with the above states, and where φ_{opt} is the value of the phase parameter φ which minimizes the quantity $\frac{1}{4} \|C_+(t_0) - e^{i\varphi} C\|^2$. In other words, we measure the distance between the above states considered up to a phase factor by taking into account the class of states $e^{i\varphi}|\Phi\rangle$ corresponding to all possible values of φ and selecting the state in this class which yields the lowest difference or “best fit” with respect to $|\psi(t_0)\rangle$. This approach is similar to the more usual method used to find the best fit between signals up to a *scale* factor (see e.g. [14] p. 30). When deriving the analytical expression of φ_{opt} , one is led to introduce the phase

$$\varphi_{ext} = -\arctan \left[\frac{\sum_{j=1}^4 |c_j(t_0)c_j| \times \sin(\arg(c_j) - \arg(c_j(t_0)))}{\sum_{j=1}^4 |c_j(t_0)c_j| \times \cos(\arg(c_j) - \arg(c_j(t_0)))} \right]. \quad (\text{L.6})$$

It may then be shown that

$$MSE(|\psi(t_0)\rangle, |\Phi\rangle) = \frac{1}{4} \min \left(\|C_+(t_0) - e^{i\varphi_{ext}} C\|^2, \|C_+(t_0) + e^{i\varphi_{ext}} C\|^2 \right). \quad (\text{L.7})$$

The above definition (L.7) of this type of MSE is then straightforwardly extended to (i) all states used in one elementary test and then (ii) all elementary tests. Eventually taking the square root of the value thus obtained yields the RMSE for complex state coefficients, denoted as $RMS E_c$ in our paper.

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