From Blind Quantum Source Separation to Blind Quantum Process Tomography

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Abstract. We here extend the field of blind (i.e. unsupervised) quantum computation into two directions. On the one hand, we introduce a new class of blind quantum source separation (BQSS) methods, which perform quantum/classical data conversion by means of spin component measurements, followed by classical processing. They differ from our previous class of classical-processing BQSS methods by using extended types of measurements (three directions, possibly different for the considered two spins), which yield a more complete nonlinear mixing model. This allows us (i) to develop a new disentanglement-based separation procedure, which requires a much lower number of source values for adaptation and (ii) to restore a larger set of sources. On the other hand, these extended measurements motivate us to introduce a new research field, namely *Blind* Quantum Process Tomography, which may be seen both as the blind extension of its existing non-blind version and as the quantum extension of classical blind identification of mixing systems.

Keywords: Blind quantum system identification and inversion · Nonlinear mixture · Disentanglement-based separation principle · Unsupervised unmixing · Multidirectional measurements of spin components

1 Prior Work and Problem Statement

Source Separation (SS), also called signal separation, is a generic Information Processing (IP) problem, where the inverting block of a separating system eventually receives signals, which are mixtures of source signals that it does not know, and aims at recovering these source signals only from their known mixtures [2]. In the ideal case, the separating system initially completely knows the mixing function. On the contrary, in many applications, this system initially knows which class the mixing function belongs to, but does not know its parameter values. This system therefore contains an adapting block which is initially used to tune the parameter values of the inverting block so that the latter block achieves the inverse of the mixing function (possiby up to some indeterminacies). This adapting block thus typically aims at estimating the parameter values of

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the mixing function. The SS problem is thus closely linked to the Mixture Identification (MI) problem (see e.g. [1], and [2] pp. 65–66), which may be seen as a multiple-input multiple-output system identification problem.

The above initial adaptation/identification may be performed in two modes. In the less demanding, i.e. non-blind, mode, the adapting block receives both a set of known source values and the associated values of the mixed signals. The more challenging mode corresponds to the *Blind* (or unsupervised) Source Separation (BSS) [2] and associated Blind Mixture Identification (BMI) problems. In this blind mode, the adapting block only receives values of the mixed signals. The sources have unknown values, but they are requested to possess some known properties (e.g. they are mutually statistically independent for ICA methods).

Until recently, all (B)SS and BMI investigations were performed in a "classical", i.e. non-quantum, framework. Independently from them, another growing field within the overall IP domain is Quantum Information Processing (QIP) [7]. QIP is closely related to Quantum Physics (QP). It uses abstract representations of systems whose behavior is requested to obey the laws of QP. This already made it possible to develop new and powerful IP methods, which manipulate the states of so-called quantum bits, or qubits.

In 2007, we bridged the gap between classical (B)SS and QIP/QP, by introducing a new field, namely Quantum Source (or Signal) Separation (QSS) [3]. The QSS problem consists in restoring the information contained in individual *quantum* source signals, eventually only using the mixtures (in SS terms [4]) of states of these qubits which result from their undesired coupling. We especially developed two main classes of Blind (i.e. unsupervised) QSS (BQSS) methods for qubits implemented as spins 1/2. Briefly, in the first class (see e.g. [3–5]), we first perform a quantum/classical conversion by using monodirectional spin component measurements and then process the resulting data with classical means. In the second class (see e.g. [6]), we only use quantum processing means in the inverting block, whereas the adapting block preferably performs bidirectional spin component measurements and then classical processing.

In this paper, we first introduce a new mixing model (Sect. 2), defined by our already used spin coupling model and an *extended* set of spin component measurements (along three directions, that may moreover be different for the two spins). We then present major extensions of the above quantum-source processing methods, into two directions. On the one hand, we introduce a new class of BQSS methods (Sects. 3 and 4), which use classical processing after the quantum/classical conversion performed by the above measurements, as in our first class of methods, but which take advantage of this new set of measurements to achieve additional capabilities. On the other hand, these capabilities motivate us to explicitly introduce a new research field in Sect. 5, namely *Blind* (or unsupervised) Quantum Process Tomography (BQPT). This field may be seen both as the quantum counterpart of classical BMI when applied to separately initialized qubits as in this paper, and as the blind extension of the field of (non-blind) QPT, previously developed in the framework of QIP [7]. Conclusions are drawn from this investigation in Sect. 6.

2 New Mixing Model

2.1 Heisenberg Quantum Coupling

As stated above, qubits are used instead of classical bits for performing computations in the field of QIP [7]. In our previous papers (e.g. [3,4]), we first detailed the required concepts for a single qubit and then presented the type of coupling between two qubits that is involved in the "mixing model", in (B)SS terms, of our investigation. We hereafter summarize the major aspects of that discussion, which are required in the current paper.

A qubit with index i considered at a given time t_0 has a quantum state. If this state is pure, it belongs to a two-dimensional space $\mathcal{E}_i$ and may be expressed as

$$|\psi_i(t_0)\rangle = \alpha_i|+\rangle + \beta_i|-\rangle \tag{1}$$

in the basis of $\mathcal{E}_i$ defined by the two orthonormal vectors that we hereafter denote $|+\rangle$ and $|-\rangle$, whereas α_i and β_i are two complex-valued coefficients constrained to meet the condition

$$|\alpha_i|^2 + |\beta_i|^2 = 1 \tag{2}$$

which expresses that the state $|\psi_i(t_0)\rangle$ is normalized.

In the QSS configuration studied in this paper, we first consider a system composed of two qubits, called "qubit 1" and "qubit 2" hereafter, at a given time t_0. This system has a quantum state. If this state is pure, it belongs to the four-dimensional space $\mathcal{E}$ defined as the tensor product (denoted $\otimes$) of the spaces $\mathcal{E}_1$ and $\mathcal{E}_2$ respectively associated with qubits 1 and 2, i.e. $\mathcal{E} = \mathcal{E}_1 \otimes \mathcal{E}_2$. The considered basis of $\mathcal{E}$ is composed of the four orthonormal vectors $|++\rangle, |+-\rangle, |-+\rangle, |--\rangle$, where e.g. $|+-\rangle$ is an abbreviation for $|+\rangle \otimes |-\rangle$, with $|+\rangle$ corresponding to qubit 1 and $|-\rangle$ corresponding to qubit 2. Any pure state of the above two-qubit system may then be expressed as

$$|\psi(t_0)\rangle = c_1(t_0)|++\rangle + c_2(t_0)|+-\rangle + c_3(t_0)|-+\rangle + c_4(t_0)|--\rangle \tag{3}$$

and has unit norm. In particular, we study the case when the two qubits are separately initialized, with states defined by (1) respectively with $i = 1$ and $i = 2$. Then

$$|\psi(t_0)\rangle = |\psi_1(t_0)\rangle \otimes |\psi_2(t_0)\rangle \tag{4}$$

$$= \alpha_1\alpha_2|++\rangle + \alpha_1\beta_2|+-\rangle + \beta_1\alpha_2|-+\rangle + \beta_1\beta_2|--\rangle. \tag{5}$$

Moreover, we consider the case when the two qubits correspond to two electron or nuclear spins 1/2, called "spin 1" and "spin 2", which have undesired coupling after they have been initialized according to (4). The considered coupling is based on the Heisenberg model with a cylindrical-symmetry axis collinear to Oz, the direction common to the applied magnetic field and to our first chosen quantization axis [4]. Due to that coupling, and for negligible coupling with the environment, the state of the above system at any time $t > t_0$ reads [4]

$$|\psi(t)\rangle = c_1(t)|++\rangle + c_2(t)|+-\rangle + c_3(t)|-+\rangle + c_4(t)|--\rangle, \tag{6}$$

with

$$c_1(t) = \alpha_1\alpha_2 e^{-\mathrm{i}\omega_{1,1}(t-t_0)} \tag{7}$$

$$c_2(t) = \frac{1}{2}\left[(\alpha_1\beta_2 + \beta_1\alpha_2)e^{-\mathrm{i}\omega_{1,0}(t-t_0)} + (\alpha_1\beta_2 - \beta_1\alpha_2)e^{-\mathrm{i}\omega_{0,0}(t-t_0)}\right] \tag{8}$$

$$c_3(t) = \frac{1}{2}\left[(\alpha_1\beta_2 + \beta_1\alpha_2)e^{-\mathrm{i}\omega_{1,0}(t-t_0)} - (\alpha_1\beta_2 - \beta_1\alpha_2)e^{-\mathrm{i}\omega_{0,0}(t-t_0)}\right] \tag{9}$$

$$c_4(t) = \beta_1\beta_2 e^{-\mathrm{i}\omega_{1,-1}(t-t_0)}. \tag{10}$$

where all four (angular) frequencies $\omega_{k,l}$ are unknown in practical, i.e. non-ideal, configurations (see also Sect. 5 for more details about this physical model).

2.2 Extended Quantum/Classical Conversion

Classical-form data may be derived from the above coupled state $|\psi(t)\rangle$ by measuring the components of the considered two spins along given directions. In our first class of QSS methods [3–5], we only used measurements along Oz for both spins. We explained that this couple of measured spin components has four possible values only, namely $(+\frac{1}{2}, +\frac{1}{2})$, $(+\frac{1}{2}, -\frac{1}{2})$, $(-\frac{1}{2}, +\frac{1}{2})$ and $(-\frac{1}{2}, -\frac{1}{2})$ (in normalized units), with respective probabilities

$$p_{1zz} = |c_1(t)|^2, \quad p_{2zz} = |c_2(t)|^2, \quad p_{3zz} = |c_3(t)|^2, \quad p_{4zz} = |c_4(t)|^2. \tag{11}$$

These probabilities may be estimated by the sample frequencies of the associated measured values, using the Repeated Write Read (RWR) procedure that we proposed. This allowed us to derive a *nonlinear* mixing model, where the mixed signals are three of these (estimated) probabilities and the sources are the two moduli $|\alpha_1|$ and $|\alpha_2|$ and a single combination of the four phases of α_i and β_i.

In this paper, we extend this nonlinear mixing model by also performing other types of measurements for $|\psi(t)\rangle$ (for additional initializations of the qubits). More precisely, we first consider the case when one again measures the Oz component of spin 1, but now the Ox component of spin 2. QP then tells us that such measurements again yield the same four possible values as above and that, in particular, the probabilities of $(+\frac{1}{2}, +\frac{1}{2})$, and $(-\frac{1}{2}, +\frac{1}{2})$ respectively read

$$p_{1zx} = \frac{1}{2}|c_1(t) + c_2(t)|^2 \quad \text{and} \quad p_{3zx} = \frac{1}{2}|c_3(t) + c_4(t)|^2. \tag{12}$$

Similarly, the probabilities of the above two values when measuring the Oz and Oy components respectively of spins 1 and 2 read

$$p_{1zy} = \frac{1}{2}|c_1(t) - \mathrm{i}c_2(t)|^2 \quad \text{and} \quad p_{3zy} = \frac{1}{2}|c_3(t) - \mathrm{i}c_4(t)|^2, \tag{13}$$

and the probabilities for getting the value $(+\frac{1}{2}, +\frac{1}{2})$, for the two couples of directions (Ox, Oz) and (Oy, Oz) for spins 1 and 2, are respectively

$$p_{1xz} = \frac{1}{2}|c_1(t) + c_3(t)|^2 \quad \text{and} \quad p_{1yz} = \frac{1}{2}|c_1(t) - \mathrm{i}c_3(t)|^2. \tag{14}$$

The output of the mixing stage of the considered QSS configuration consists of all probabilities in (11)–(14) (more precisely of their *estimates* derived from our RWR procedure). It is sent to the input of the inverting block of the separating system defined in the next section.

3 Inverting Block of Separating System

We hereafter present the five steps of the operation of the classical-processing inverting block of the proposed separating system, respectively calling "Case 1" and "Case 2" the ideal and blind (Q)SS configurations defined in Sect. 1.

In both Cases, **Step 1** consists in restoring (estimates of: this is not stated everywhere below) the coefficients $c_j(t)$, with $j \in \{1, \ldots, 4\}$, from the (estimates of) probabilities derived in Sect. 2.2. To this end, we use the polar representation $c_j(t) = \rho_j e^{\mathrm{i}\xi_j}$ of these coefficients. All their moduli ρ_j are directly derived from (11). Using (11), (12) and $c_j(t) = \rho_j e^{\mathrm{i}\xi_j}$, it may then be shown that

$$\cos(\xi_1 - \xi_2) = \frac{2p_{1zx} - p_{1zz} - p_{2zz}}{2\sqrt{p_{1zz}p_{2zz}}}, \qquad \cos(\xi_3 - \xi_4) = \frac{2p_{3zx} - p_{3zz} - p_{4zz}}{2\sqrt{p_{3zz}p_{4zz}}}. \quad (15)$$

The sines of the above phase differences may then similarly be derived by using (13) instead of (12). Finally, using (14) instead, one obtains the cosine and sine of $(\xi_1 - \xi_3)$. All differences between the four phases ξ_j are thus known (modulo 2π). Moreover, a quantum state (here (6)) is only defined up to a phase factor. One may therefore arbitrarily fix one of the above phases ξ_j (e.g. to 0). As an overall result, we thus know all phases ξ_j and moduli ρ_j, i.e. all coefficients $c_j(t)$.

Keeping in mind that these restored versions of $c_1(t)$ to $c_4(t)$ here meet (7)–(10), we then process them so as to derive successive transformed versions of this set of four coefficients, which progressively bring us back to the original, or source, data defined by (1) and (4). The four transformed coefficients obtained at the output of each processing step n with $n = 2$ to 4 are denoted as c_{jn}, with $j \in \{1, \ldots, 4\}$. **Step 2** then consists in reducing its input coefficients $c_j(t)$ to expressions which only depend on a single frequency $\omega_{k,l}$. To this end, both in Cases 1 and 2, we keep $c_{12} = c_1(t)$ and $c_{42} = c_4(t)$, while respectively setting c_{22} and c_{32} to the sum and difference of $c_2(t)$ and $c_3(t)$, moreover rescaled so that the coefficients c_{j2} form a unit-norm vector, as in (3) and (6). This yields

$$c_{22} = \frac{1}{\sqrt{2}}[c_2(t) + c_3(t)] = \frac{1}{\sqrt{2}}(\alpha_1\beta_2 + \beta_1\alpha_2)e^{-\mathrm{i}\omega_{1,0}(t-t_0)} \quad (16)$$

$$c_{32} = \frac{1}{\sqrt{2}}[c_2(t) - c_3(t)] = \frac{1}{\sqrt{2}}(\alpha_1\beta_2 - \beta_1\alpha_2)e^{-\mathrm{i}\omega_{0,0}(t-t_0)}. \quad (17)$$

Step 3 then aims at compensating for the phase factors $e^{-\mathrm{i}\omega_{k,l}(t-t_0)}$ in the above c_{j2}. This is achieved by setting $c_{j3} = c_{j2} \times e^{\mathrm{i}\gamma_j}$, with $j \in \{1, \ldots, 4\}$, which yields

$$c_{13} = \alpha_1\alpha_2 e^{\mathrm{i}\delta_1}, \qquad c_{23} = \frac{1}{\sqrt{2}}(\alpha_1\beta_2 + \beta_1\alpha_2)e^{\mathrm{i}\delta_2} \quad (18)$$

$$c_{33} = \frac{1}{\sqrt{2}}(\alpha_1\beta_2 - \beta_1\alpha_2)e^{\mathrm{i}\delta_3}, \qquad c_{43} = \beta_1\beta_2 e^{\mathrm{i}\delta_4} \quad (19)$$

with

$$\delta_j = \gamma_j - \omega_{k,l}(t - t_0). \tag{20}$$

In Case 2, all parameters γ_j are adapted as explained in Sect. 4, because all $\omega_{k,l}$ are unknown. In Case 1, all parameters γ_j are set to the known values $\omega_{k,l}(t-t_0)$. All phase factors $e^{\mathrm{i}\delta_j}$ thus disappear in (18)–(19). In **Step 4**, we reduce the above coefficients to a single product of α_i and/or β_j parameters in Case 1. To this end, we use the same approach as in Step 2, i.e. in both Cases we keep $c_{14} = c_{13}$, $c_{44} = c_{43}$ and we set

$$c_{24} = \frac{1}{\sqrt{2}}[c_{23} + c_{33}], \qquad c_{34} = \frac{1}{\sqrt{2}}[c_{23} - c_{33}]. \tag{21}$$

In Case 1, this yields

$$c_{14} = \alpha_1\alpha_2, \qquad c_{24} = \alpha_1\beta_2, \qquad c_{34} = \beta_1\alpha_2, \qquad c_{44} = \beta_1\beta_2. \tag{22}$$

Step 5 then aims at deriving all source parameters α_i and β_i (i.e. a larger set of sources than in our previous classical-processing BQSS methods) from all coefficients c_{j4}. This is relevant only if these coefficients correspond to a non-entangled quantum state, i.e. a tensor product such as (4), e.g. as in Case 1. In the latter case, one computes the moduli of the outputs of this step e.g. as $\sqrt{|c_{14}|^2 + |c_{24}|^2}$ because (22) and (2) show that this yields $|\alpha_1|$ in Case 1. Moreover, one of the four phases of the parameters α_i and β_i, say $\arg(\alpha_1)$, may be arbitrarily selected. Then combining (22) with the polar expressions of c_{j4}, α_i and β_i e.g. yields $\arg(\beta_1) = \arg(\alpha_1) + \arg(c_{34}) - \arg(c_{14})$ (modulo 2π). The calculations for the other source parameters are similar and therefore skipped.

4 Interpretation and Adaptation of the Separating system

The inverting block of the separating system that we developed in our QSS method proposed in [6] only uses quantum states and quantum processing means. It is thus quite different from the inverting block depicted in Sect. 3, which receives classical-form data (which have quantum properties, however) and processes them with classical means. Yet, it may be shown that (i) the classical-form coefficients $c_j(t)$ here restored in Step 1 are those of the quantum state at the input of the inverting block of [6], again up to estimation errors, and (ii) the quantum processing achieved in that block of [6] is governed by the same equations as in Steps 2 to 4 above, although they are expressed in a quite different way in [6]. Processing Steps 2 to 4 of the block of Sect. 3 may therefore be considered as a new classical-processing counterpart of the quantum-processing block of [6] (but they here receive an approximate version of coefficients $c_j(t)$).

To blindly adapt the parameters γ_j of the inverting block of Sect. 3, we then propose a procedure which is partly related to the one introduced in [6]. We here take advantage of the availability of the complex-valued coefficients c_{j4} in classical form. On the contrary, in [6] their counterpart is only available in

quantum form, which required us to develop an adaptation criterion based on related real-valued probabilities. The new adaptation procedure proposed here consists in tuning all γ_j so as to enforce the quantum disentanglement condition

$$c_{14}c_{44} = c_{24}c_{34} \tag{23}$$

for (at least) two (non-redundant [6]) source states (5). QP calculations skipped here show that condition (23) above implies the probability-based separation conditions (17) and (24) of [6]. As shown in [6], the latter conditions themselves entail

$$\delta_3 - \delta_2 = m\pi, \qquad \delta_1 + \delta_4 = 2\delta_2 + 2k\pi, \tag{24}$$

where m and k are arbitrary integers, and these conditions ensure separability, so that they here force the coefficients c_{j4} to become equal to those in (5), up to some permutation and phase indeterminacies. This approach thus yields a new classical-processing BQSS method, which only requires a very limited number of source states for adaptation, by using the disentanglement condition (23). On the contrary, our previous, statistical, methods related to ICA [5] need hundreds to thousands of (also repeatedly prepared) source states.

5 Blind Quantum Process Tomography

The considered cylindrical-symmetry Heisenberg quantum coupling model was initially defined by the corresponding Hamiltonian (see e.g. [4]). We showed that this yields the coupled state expression in (6)–(10), moreover with

$$\omega_{1,1} = \frac{1}{\hbar}\left[GB - \frac{J_z}{2}\right], \qquad \omega_{1,0} = \frac{1}{\hbar}\left[-J_{xy} + \frac{J_z}{2}\right] \tag{25}$$

$$\omega_{0,0} = \frac{1}{\hbar}\left[J_{xy} + \frac{J_z}{2}\right], \qquad \omega_{1,-1} = \frac{1}{\hbar}\left[-GB - \frac{J_z}{2}\right]. \tag{26}$$

In these expressions, $\hbar$ is the reduced Planck constant and $G = g\mu_e$, where g is the principal value of the considered isotropic $\overline{\overline{g}}$ tensor and the constant μ_e is the Bohr magneton [4]. The value of g may be experimentally determined. B is the magnitude of the applied magnetic field, which can be known thanks to measurements. J_{xy} and J_z are the principal values of the exchange tensor, which are unknown in practice. The frequencies $\omega_{k,l}$ are thus unknown.

QPT, mentioned in Sect. 1, is a generic, therefore complex, procedure for identifying the behavior of a quantum system by applying known input states (thus in the non-blind mode) to this system and measuring its corresponding outputs. We here aim at developing an extension of QPT tailored to the Heisenberg Hamiltonian and operating in the *blind* mode, i.e. with unknown input states. To this end, we analyze the BMI capabilities of the adaptation procedure proposed in Sect. 4. Eqs. (24), (20) and (25)–(26) then yield

$$J_{xy} = \frac{\hbar}{2(t-t_0)}(\gamma_3 - \gamma_2 - m\pi), \qquad J_z = \frac{\hbar}{2(t-t_0)}(\gamma_2 + \gamma_3 - \gamma_1 - \gamma_4 + 2k\pi - m\pi). \tag{27}$$

The only unknown values of the considered Hamiltonian, namely J_{xy} and J_z, may therefore be derived from the values γ_j provided by the proposed adaptation procedure for given but unknown source states (5). For BQPT, k and m only yield sign indeterminacies in the exponentials of the process (6)–(10). Moreover, they can be set to zero if all other terms of (27) are known to be small enough.

This yields our first reported method for performing complete BQPT (with the above indeterminacies) with classical processing means. Related BQPT capabilities could however be derived from our previous BQSS methods. First, our method in [6] here also yields (27), but that BQSS method requires quantum processing means and it is much more difficult to implement them than classical ones. Second, our previous classical-processing BQSS methods only estimate the single parameter of their mixing model, which is different from here because they only use measurements along one direction. This parameter only yields J_{xy}. These BQSS methods then achieve complete BQPT for the isotropic Heisenberg model ($J_{xy} = J_z$) of [3], but only partial BQPT for the general cylindrical-symmetry Heisenberg model (arbitrary J_{xy} and J_z) used in [4,5].

6 Conclusion

Our contributions in this paper are twofold. We first proposed a new class of BQSS methods, by introducing an extended set of spin component measurements, which allowed us to restore an estimate of the entangled state $|\psi(t)\rangle$ and to develop corresponding classical-processing inverting and adapting blocks of the separating system. We then explicitly introduced a new research field, namely *Blind* Quantum Process Tomography (BQPT), as the extension of its existing non-blind version. Although not detailed in our previous papers, BQPT could be obtained as a spin-off of our corresponding BQSS methods, but with limitations (only partial identification or need for quantum processing means), which are here avoided. We plan to further develop this new class of BQSS and BQPT methods and to test their performance with simulated data.

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