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The Blind Version of Quantum Process Tomography: Operating with Unknown Input Values

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Abstract: System identification and inversion are two closely related problems, which lead to various configurations. In the quantum framework, system identification was only studied in the non-blind (or supervised) mode, i.e. with known input states (in addition to known output states). It is then called quantum process tomography. We here develop its blind (or unsupervised) version, operating with unknown (unentangled) input states (which are here not controlled by an automated loop). Our approach takes advantage of the blind quantum source separation (i.e. blind multi-qubit system inversion) methods based on disentanglement that we recently introduced. It is detailed for Heisenberg spin coupling and could be extended to other classes of processes. It estimates the parameter values of the Heisenberg Hamiltonian.

Keywords: blind quantum process tomography, blind Hamiltonian estimation, Heisenberg coupling, entanglement, blind quantum source separation, qubit uncoupling, disentanglement, measurement-based quantum control, blind quantum system identification and inversion, unsupervised adaptive quantum processing

1. INTRODUCTION

System identification and system inversion are two closely related problems. First considering classical, i.e. non-quantum, signals and systems, the basic version of system identification concerns single-input single-output (SISO) systems. It consists in estimating the unknown parameter values of such a system (i.e. of its transform) belonging to a known class, by using known values of its input (source signal s) and output (signal x). This version is stated to be non-blind (or supervised), as opposed to the more challenging, blind (or unsupervised), version of that problem, where the input values are unknown (and uncontrolled ; but the input signal may be known to belong to a given class): see Abed-Meraim et al. (1997). Both versions may then be extended to multiple-input multiple-output (MIMO) systems.

Besides, in various applications, what is needed is not the direct transform achieved by the above system, but the inverse of that transform (assuming it is invertible). For SISO non-blind and blind configurations, this is motivated by the fact that one eventually only accesses the ouput x of the above direct system, and one aims at deriving a signal y which ideally restores the original source signal s. To this end, one may use the above-mentioned system identification methods in order to first estimate the direct system, then derive its inverse and eventually transfer the output x of the direct system through the inverse system. Alternatively, one may develop methods for initially identifying the *inverse* system itself. Extended versions of this "(unknown) system inversion" task concern MIMO configurations, where a set of original source signals s_1 to s_M are to be respectively restored on the outputs y_1 to y_M of the inverse system.

The blind MIMO version of the above system inversion problem is almost the same as blind source separation (BSS) (see Comon et al. (2010)): as in system inversion, BSS aims at canceling the contributions of all sources but one in each output signal of the separating system ; however, in BSS, one often allows each output signal to be equal to a source signal only up to an acceptable residual transform. These transforms, called indeterminacies, cannot be avoided because only limited constraints are set on the source signals and on the direct system which combines (i.e., "mixes", in BSS terms) these signals.

Let us now consider quantum "signals" (i.e. states) and systems. Then, among the above problems, the one which was first studied is non-blind system identification, called "quantum process tomography" by the quantum information processing community: see Branderhorst et al. (2009), Merkel et al. (2013), Nielsen et al. (2000), Shukla et al. (2014), Takahashi et al. (2013). Besides, we introduced the field of "quantum source separation" (QSS) and especially its blind version (BQSS) in 2007: see Deville (2007). We first mainly developed a class of BQSS methods related to classical independent component analysis: see especially Deville (2012), Deville (2014a). We then proposed a second class of BQSS methods, based on output quantum state disentanglement: see Deville (2014b) for its *original* com-

plete version and Deville (2015), Deville (2016a), Deville (2016b) for *modified* methods also belonging to that class. Finally, we very recently introduced the field of "blind quantum process tomography" (BQPT) in Deville (2015). We only very briefly outlined the operation of BQPT methods derived as spin-offs of the above-mentioned modified disentanglement-based BQSS methods (see Deville (2015), Deville (2016a), Deville (2016b)).

The present paper therefore has complementary features as compared with the above ones: (i) to our knowledge, it is the first paper ever fully devoted to BQPT, (ii) we hereafter first introduce a BQPT method based on the *original* BQSS method of Deville (2014b), as opposed to the above *modified* BQSS methods, and (iii) we propose an extension of that BQPT method, that aims at removing identification indeterminacies, which was not considered in the BQPT methods outlined in our previous papers.

The remainder of this paper is organized as follows. The considered quantum process and our associated goals are defined in Section 2. Section 3 contains a summary of the principles of the BQSS method used hereafter. Then, the two proposed BQPT methods are respectively described in Sections 4 and 5. Section 6 contains test results. Some general features of BQPT and a conclusion are eventually provided in Section 7.

2. CONSIDERED QUANTUM PROCESS AND GOALS

The process to be identified may be defined as follows. We consider a system composed of two distinguishable spins. These spins are assumed to be coupled according to the version of the Heisenberg model which has a cylindrical-symmetry axis, denoted Oz (this includes the isotropic Heisenberg model as a specific case). These spins are supposed to be placed in a magnetic field (also oriented along Oz and with a magnitude B) and thus coupled to it. Moreover, we assume an isotropic $\overline{\overline{g}}$ tensor, with principal value g. The time interval when these spins are considered is supposed to be short enough for their coupling with their environment to be negligible. In these conditions, the temporal evolution of the system composed of these two spins is governed by the following Hamiltonian:

$$H = Gs_{1z}B + Gs_{2z}B - 2J_{xy}(s_{1x}s_{2x} + s_{1y}s_{2y}) - 2J_z s_{1z}s_{2z} \quad (1)$$

where:

- $G = g\mu_e$, where μ_e is the Bohr magneton, i.e. $\mu_e = e\hbar/2m_e = 0.927 \times 10^{-23} JT^{-1}$ and $\hbar$ is the reduced Planck constant,
- s_{ix}, s_{iy}, s_{iz}, with $i \in \{1, 2\}$, are the three components of the vector operator $\overrightarrow{s_i}$ associated with spin i in a cartesian frame,
- J_{xy} and J_z are the principal values of the exchange tensor.

Among the above parameters, the value of g may be experimentally determined, and B can be measured. The values of J_{xy} and J_z are here assumed to be unknown.

The Heisenberg coupling model allows us to define a configuration which is relevant with respect to the expected development of spintronics. But we stress that, beyond that model and associated applications, this paper also aims at defining the general concept of BQPT, which could also be used in other applications by transposing the procedures described below to other quantum processes.

We here assume that each spin i, with $i \in \{1, 2\}$, is initialized, at a given time t_0, with the pure state

$$|\psi_i(t_0)\rangle = \alpha_i|+\rangle + \beta_i|-\rangle \quad (2)$$

where $|+\rangle$ and $|-\rangle$ are eigenkets of s_{iz}, for the eigenvalues $1/2$ and $-1/2$ respectively. These spins are then coupled according to the above-defined model for $t \geq t_0$.

Hereafter, we consider the state of the overall system composed of these two distinguishable spins. At time t_0, this state is equal to the tensor product of the states of both spins defined in (2). It therefore reads

$$|\psi(t_0)\rangle = |\psi_1(t_0)\rangle \otimes |\psi_2(t_0)\rangle \quad (3)$$

$$= \alpha_1\alpha_2|++\rangle + \alpha_1\beta_2|+-\rangle + \beta_1\alpha_2|-+\rangle + \beta_1\beta_2|--\rangle \quad (4)$$

in the four-dimensional basis $\mathcal{B}_+ = \{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$.

The state of this two-spin system then evolves with time. Its value $|\psi(t)\rangle$ at any subsequent time t may be derived from its above-defined Hamiltonian. In Deville (2012), we showed that it is defined by

$$C_+(t) = MC_+(t_0) \quad (5)$$

where $C_+(t_0)$ and $C_+(t)$ are the column vectors of components of $|\psi(t_0)\rangle$ and $|\psi(t)\rangle$, respectively, in basis $\mathcal{B}_+$. For instance, as shown by (4),

$$C_+(t_0) = [\alpha_1\alpha_2, \alpha_1\beta_2, \beta_1\alpha_2, \beta_1\beta_2]^T \quad (6)$$

where ^{T} stands for transpose. Moreover, the matrix M of (5), which defines the transform applied to $|\psi(t_0)\rangle$, reads

$$M = QDQ^{-1} = QDQ \quad (7)$$

with

$$Q = Q^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

and D equal to

$$\begin{bmatrix} e^{-\mathrm{i}\omega_{1,1}(t-t_0)} & 0 & 0 & 0 \\ 0 & e^{-\mathrm{i}\omega_{1,0}(t-t_0)} & 0 & 0 \\ 0 & 0 & e^{-\mathrm{i}\omega_{0,0}(t-t_0)} & 0 \\ 0 & 0 & 0 & e^{-\mathrm{i}\omega_{1,-1}(t-t_0)} \end{bmatrix} \quad (9)$$

where i is the imaginary unit. The four real (angular) frequencies $\omega_{1,1}$ to $\omega_{1,-1}$ in (9) depend on the physical setup. In Deville (2012), we showed that they read

$$\omega_{1,1} = \frac{1}{\hbar}\left[GB - \frac{J_z}{2}\right],\ \omega_{1,0} = \frac{1}{\hbar}\left[-J_{xy} + \frac{J_z}{2}\right], \quad (10)$$

$$\omega_{0,0} = \frac{1}{\hbar}\left[J_{xy} + \frac{J_z}{2}\right],\ \omega_{1,-1} = \frac{1}{\hbar}\left[-GB - \frac{J_z}{2}\right]. \quad (11)$$

Since the values of the parameters J_{xy} and J_z of the Hamiltonian of (1) are presently unknown, the values

of the parameters $\omega_{1,1}$ to $\omega_{1,-1}$ of the quantum process involved in (5) are also unknown. The problem addressed here may thus be considered from two points of view:

- We mainly aim at achieving (blind) QPT, i.e. at estimating the matrix M involved in (5).
- This problem is closely related to the (blind) estimation of the Hamiltonian of (1), that is, to the estimation of its parameters J_{xy} and J_z.

However, some differences may exist between these two problems, due to the potential indeterminacies of these two estimation tasks that may result from the considered blind approach. In particular, from the above definition of these two tasks, one can already anticipate at this stage that the following phenomenon may occur. The Hamiltonian estimation procedure to be proposed may suffer from indeterminacies, in the sense that it may yield estimates of J_{xy} and J_z which are equal to the actual values of J_{xy} and J_z only up to additive constants. Let us assume that these constants are such that the estimates of $\omega_{1,1}(t-t_0)$ to $\omega_{1,-1}(t-t_0)$ derived from those of J_{xy} and J_z by means of (10)-(11) are equal to their actual values only up to constants *which are multiples of* 2π. Then, these additive constants obtained when estimating J_{xy} and J_z eventually have no influence on the exponentials involved in the estimate of the matrix of (9) and therefore on the estimate of the process defined by the matrix M involved in (5). Unlike the considered Hamiltonian estimation task, the associated (B)QPT task to be eventually considered therefore has no indeterminacies in such a situation. These considerations lead us to analyze the properties of the proposed methods both in terms of (B)QPT and (blind) Hamiltonian estimation in the remainder of this paper.

3. PREVIOUS SOURCE SEPARATION METHOD

As stated above, in this paper we take advantage of the BQSS method that we proposed in Deville (2014b). Therefore, in this section, we summarize the principles of that BQSS method which are of importance in the framework of this paper.

For each available state $|\psi(t)\rangle$, derived from an unknown source state $|\psi(t_0)\rangle$ by means of (5), that BQSS method creates a two-qubit output quantum state $|\Phi\rangle$, which aims at being equal to $|\psi(t_0)\rangle$ ideally, or equal to $|\psi(t_0)\rangle$ up to some indeterminacies if such indeterminacies cannot be avoided. To this end, we proposed to transfer $|\psi(t)\rangle$ through a so-called inverting block, which uses quantum processing means only and whose operation may be defined as follows. The output quantum state $|\Phi\rangle$ of that block and therefore of our overall separating system reads

$$|\Phi\rangle = c_1|++\rangle + c_2|+-\rangle + c_3|-+\rangle + c_4|--\rangle. \quad (12)$$

It may also be represented by the corresponding vector of components of $|\Phi\rangle$ in the basis $\mathcal{B}_+$ associated with the output of the inverting block. That vector is denoted as

$$C = [c_1, c_2, c_3, c_4]^T. \quad (13)$$

We then have

$$C = UC_+(t) \quad (14)$$

where U defines the unitary quantum-processing operator applied by our separating system to its input $C_+(t)$. As justified in Deville (2014b), we chose this operator U to belong to the class defined by

$$U = Q\tilde{D}Q \quad (15)$$

$$\text{with} \quad \tilde{D} = \begin{bmatrix} e^{\mathrm{i}\gamma_1} & 0 & 0 & 0 \\ 0 & e^{\mathrm{i}\gamma_2} & 0 & 0 \\ 0 & 0 & e^{\mathrm{i}\gamma_3} & 0 \\ 0 & 0 & 0 & e^{\mathrm{i}\gamma_4} \end{bmatrix} \quad (16)$$

where γ_1 to γ_4 are free real-valued parameters.

The issue is then how to select adequate values of the parameters γ_1 to γ_4. From the point of view of BQSS, this ideally means values of γ_1 to γ_4 which are such that each output state $|\Phi\rangle$ of the separating system becomes equal to the corresponding unknown source state $|\psi(t_0)\rangle$. In the method proposed in Deville (2014b), this is achieved by controlling the values of γ_1 to γ_4 thanks to the resulting value of $|\Phi\rangle$, which yields a feedback loop inside the considered separating system, as shown in Fig. 1 (this has no influence on the input $|\psi(t_0)\rangle$ of the process to be identified here). More precisely, the following "separation principle" was proposed in Deville (2014b) for controlling γ_1 to γ_4. The successive values of $|\psi(t_0)\rangle$ are supposedly unknown (and not controlled by an automated loop), since we consider the *blind* configuration. However, they are known to possess a property: they are untentangled, i.e. they are product states, as shown by (3). Moreover, the "mixing" operation (in B(Q)SS terms) performed by the unknown transform (5) is known to generally destroy the above property for the state resulting from that transform, i.e. it is known to yield an *entangled* state $|\psi(t)\rangle$. This suggested us to adapt γ_1 to γ_4 so as to restore the unentanglement property in the output state $|\Phi\rangle$ of our separating system, hoping that this constraint would be sufficient for enforcing $|\Phi\rangle$ to become equal to $|\psi(t_0)\rangle$, possibly up to some indeterminacies if the above constraint is not restrictive enough (more restrictive constraints would then be appreciated but probably cannot be set, due to the limited knowledge about the source states which is available in the considered *blind* case). Indeed, we showed that the above separation principle, applied to at least two non-redundant source states, restores $|\psi(t_0)\rangle$ up to limited indeterminacies. The interested reader is referred to Deville (2014b): we do not detail this topic here because, although it is of major importance from the point of BQSS, for the BQPT task considered in this paper what is important is not the properties of $|\Phi\rangle$, but the associated properties of γ_1 to γ_4, analyzed further in this paper.

Before moving to the above-mentioned analysis of γ_1 to γ_4, we have to define how the above *theoretical* separation principle may be used to derive an associated *practical* control procedure for γ_1 to γ_4. This essentially requires one to evaluate the degree of entanglement of $|\Phi\rangle$. To this end, we proposed to perform measurements along the Oz and Ox axes for (several copies of) state $|\Phi\rangle$, thus leading to the practical measurement-based quantum control loop shown in Fig. 1. The parameters γ_1 to γ_4 of this structure are tuned by means of a two-step adaptation procedure, where each step consists of the global minimization of a cost function expressed with respect to classical-form quantities, namely the probabilities of the discrete outcomes of the above-mentioned spin component measure-

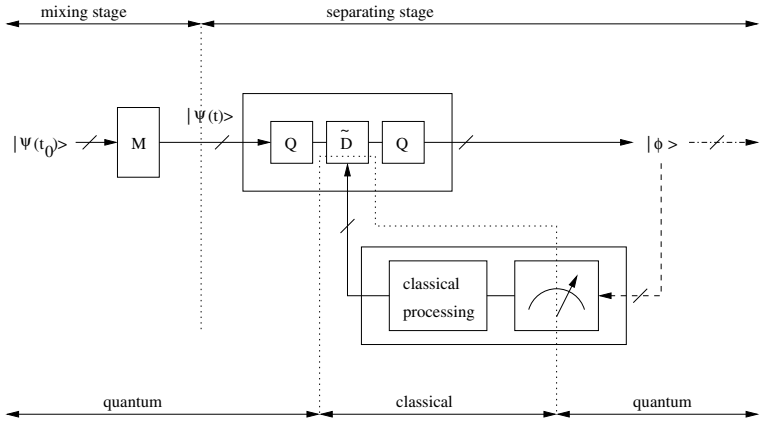

Fig. 1. Blind quantum source separation configuration. Each quantum state $|\Phi\rangle$ is used only once (no cloning): see Deville (2014b).

ments performed at the output of the inverting block. The first cost function involves $N_z \geq 2$ (non-redundant) source states, indexed by n. It is defined as

$$F_z = \sum_{n=1}^{N_z} |P_{1z}(n)P_{4z}(n) - P_{2z}(n)P_{3z}(n)|^p \tag{17}$$

with e.g. $p = 1$ or 2. $P_{1z}(n)$ to $P_{4z}(n)$ are the above-mentioned probabilities, corresponding to the case when the considered spin components are measured along the above-defined Oz axis. They read

$$\begin{aligned} P_{1z}(n) &= |c_1(n)|^2, \qquad P_{2z}(n) = |c_2(n)|^2, \\ P_{3z}(n) &= |c_3(n)|^2, \qquad P_{4z}(n) = |c_4(n)|^2 \end{aligned} \tag{18}$$

where $c_1(n)$ to $c_4(n)$ are the coefficients of (12) for the n-th source state, which is defined by (4) with corresponding parameter values $\alpha_1(n)$ to $\beta_2(n)$.

The second step of the proposed procedure then minimizes a cost function F_x which is similar to the above one, but which uses N_x source states and measurements of output spin components along the Ox axis:

$$F_x = \sum_{n=1}^{N_x} |P_{1x}(n)P_{4x}(n) - P_{2x}(n)P_{3x}(n)|^p \tag{19}$$

with associated probabilities $P_{1x}(n)$ to $P_{4x}(n)$, e.g.

$$P_{1x}(n) = \frac{1}{4}|c_1(n) + c_2(n) + c_3(n) + c_4(n)|^2. \tag{20}$$

4. FIRST PROPOSED TOMOGRAPHY METHOD

The above BQSS method was initially motivated by the desired properties of the output state $|\Phi\rangle$ of the separating system, and expressed with respect with it, as the global minimization of the cost functions (17) and (19) associated with that output state. However, this then *indirectly* yields properties which are of major interest with respect to BQPT, as will now be shown. These properties concern the tunable parameters γ_1 to γ_4 of the above separating system. We first introduce the notations

$$\gamma_{1d} = \omega_{1,1}(t - t_0) \tag{21}$$

$$\gamma_{2d} = \omega_{1,0}(t - t_0) \tag{22}$$

$$\gamma_{3d} = \omega_{0,0}(t - t_0) \tag{23}$$

$$\gamma_{4d} = \omega_{1,-1}(t - t_0). \tag{24}$$

This defines the "desired values" of γ_1 to γ_4 from the point of view of BQSS because, when setting γ_1 to γ_4 to these values, (7)-(9) and (15)-(16) show that U becomes equal to the inverse of M, and then (5) and (14) show that $|\Phi\rangle$ becomes equal to $|\psi(t_0)\rangle$, as desired. But this is also of interest from the point of view of BQPT because, if one is actually able to set U to the above desired value then, by just taking the inverse of U, one obtains the matrix M, which is the quantity to be identified in the framework of BPQT. In other words, BQSS is targeted at estimating the *inverse* of the considered mixing process but, if it succeeds, it also indirectly provides an estimate of that process itself.

Whereas the above considerations deal with the ideal behavior of BQSS and hence of BQPT, the actual capabilities of that BQSS method may be defined as follows. Denoting

$$\delta_1 = \gamma_1 - \gamma_{1d} \tag{25}$$

$$\delta_2 = \gamma_2 - \gamma_{2d} \tag{26}$$

$$\delta_3 = \gamma_3 - \gamma_{3d} \tag{27}$$

$$\delta_4 = \gamma_4 - \gamma_{4d}, \tag{28}$$

in Deville (2014b), we showed that the separation principle and criterion defined in Section 3 only guarantee that γ_1 to γ_4 are tuned so that

$$\delta_3 - \delta_2 = m\pi, \qquad \delta_1 + \delta_4 = 2\delta_2 + 2k\pi, \tag{29}$$

where m and k are arbitrary integers. The complete set of solutions of (29) therefore includes the above-defined desired case: this case is obtained when δ_1 to δ_4, m and k are equal to zero. However, the above set of solutions also includes other cases, which cannot be avoided, especially because the values of the integers m and k are not fixed. This defines the undeterminacies of the considered BQSS and resulting BQPT methods. In Deville (2014b), we only considered the BQSS problem and we therefore analyzed the solutions of (29) in terms of the resulting output states $|\Phi\rangle$ of our separating system. On the contrary, we hereafter investigate the properties that they yield for BQPT.

Combining (29) with (25)-(28), (21)-(24) and (10)-(11) allows one to show that, for any of the above solutions,

$$J_{xy} = \frac{\hbar}{2(t-t_0)}(\gamma_3 - \gamma_2 - m\pi) \quad (30)$$

$$J_z = \frac{\hbar}{2(t-t_0)}(\gamma_2 + \gamma_3 - \gamma_1 - \gamma_4 + 2k\pi - m\pi). \quad (31)$$

This defines the expressions of the *actual* principal values J_{xy} and J_z with respect to all sets of values of γ_1 to γ_4 which are *exact* solutions of (29). Now, what is obtained in practice, when adapting the tunable coefficients γ_1 to γ_4 of the separating system as explained in Section 3, is a single set of values $\widehat{\gamma}_1$ to $\widehat{\gamma}_4$ of these coefficients, which meet (29) up to estimation errors and with unknown values of m and k. One may then arbitrarily select a single couple of integers $\widehat{m}$ and $\widehat{k}$, and derive the quantities

$$\widehat{J}_{xy} = \frac{\hbar}{2(t-t_0)}(\widehat{\gamma}_3 - \widehat{\gamma}_2 - \widehat{m}\pi) \quad (32)$$

$$\widehat{J}_z = \frac{\hbar}{2(t-t_0)}(\widehat{\gamma}_2 + \widehat{\gamma}_3 - \widehat{\gamma}_1 - \widehat{\gamma}_4 + 2\widehat{k}\pi - \widehat{m}\pi), \quad (33)$$

which yields a single estimate of each of the actual values J_{xy} and J_z. When neglecting estimation errors, the ordered set $\{\widehat{\gamma}_1, \ldots, \widehat{\gamma}_4\}$ is equal to a single set of exact solutions $\{\gamma_1, \ldots, \gamma_4\}$ corresponding to fixed but unknown values of m and k. Then, taking the difference between (32)-(33) and (30)-(31), we get

$$\widehat{J}_{xy} = J_{xy} - \frac{\hbar}{2(t-t_0)}\Delta_m\pi \quad (34)$$

$$\widehat{J}_z = J_z + \frac{\hbar}{2(t-t_0)}(2\Delta_k\pi - \Delta_m\pi), \quad (35)$$

where

$$\Delta_m = \widehat{m} - m \quad (36)$$

$$\Delta_k = \widehat{k} - k. \quad (37)$$

The estimates $\widehat{J}_{xy}$ and $\widehat{J}_z$ thus obtained are therefore equal to the actual values J_{xy} and J_z up to (the above neglected estimation errors and) the additive constants which are due to the integers Δ_m and Δ_k in (34)-(35). In the general case, these constants cannot be removed, because one does not know how to select $\widehat{m}$ and $\widehat{k}$ so that they become equal to the values m and k which actually correspond to the considered single solution $\{\widehat{\gamma}_1, \ldots, \widehat{\gamma}_4\}$. These constants are then the indeterminacies of the considered blind Hamiltonian estimation problem. On the contrary, semi-blind configurations correspond to cases when some knowledge about J_{xy} and J_z is available, e.g. about the intervals of values to which they belong and/or about their signs. This knowledge may then be exploited to reduce the above indeterminacies. In particular, (34) shows that all the solutions $\frac{2(t-t_0)}{\hbar}\widehat{J}_{xy}$ provided by our procedure (depending on the selected value of $\widehat{m}$) are equal up to a multiple of π, so that only one of them is in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$. So, if $J_{xy}(t-t_0)$ is known to be small enough to guarantee that $\frac{2(t-t_0)}{\hbar}J_{xy} \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, then the only solution $\frac{2(t-t_0)}{\hbar}\widehat{J}_{xy}$ in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ provided by our procedure is guaranteed to correspond to the actual value of J_{xy} (up to estimation errors). Indeterminacies are thus completely avoided for J_{xy}. The same principle then applies to J_z.

Back to the complete set of solutions defined by (34)-(37), we now analyze the resulting BQPT capabilities. Using these estimates $\widehat{J}_{xy}$ and $\widehat{J}_z$, one derives the associated estimates of the matrix M, which defines the considered process, by inserting these values of $\widehat{J}_{xy}$ and $\widehat{J}_z$ defined by (34)-(37) into (10)-(11), then deriving the corresponding estimates $\widehat{D}$ of D by using (9) and finally using (7) and (8) to derive the associated estimates of M. These calculations especially yield

$$\widehat{D} = e^{\mathrm{i}(\Delta_k\frac{\pi}{2} - \Delta_m\frac{\pi}{4})}\begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{bmatrix} \quad (38)$$

with

$$d_1 = e^{-\mathrm{i}\frac{t-t_0}{\hbar}\left[GB - \frac{J_z}{2}\right]} \quad (39)$$

$$d_2 = e^{-\mathrm{i}\frac{t-t_0}{\hbar}\left[-J_{xy} + \frac{J_z}{2}\right]}e^{-\mathrm{i}\Delta_k\pi} \quad (40)$$

$$d_3 = e^{-\mathrm{i}\frac{t-t_0}{\hbar}\left[J_{xy} + \frac{J_z}{2}\right]}e^{-\mathrm{i}\Delta_k\pi}e^{\mathrm{i}\Delta_m\pi} \quad (41)$$

$$d_4 = e^{-\mathrm{i}\frac{t-t_0}{\hbar}\left[-GB - \frac{J_z}{2}\right]}. \quad (42)$$

If both Δ_k and Δ_m are equal to zero, all corresponding phase factors in (38)-(42) disappear and $\widehat{D}$ becomes equal to the actual matrix D, so that the estimated process becomes equal to the actual process M. Now, for any integer values of Δ_k and Δ_m, the factor $e^{\mathrm{i}(\Delta_k\frac{\pi}{2} - \Delta_m\frac{\pi}{4})}$ in (38) only yields a global phase for $\widehat{D}$ and hence for the output state of that process. Therefore, it has no physical consequence on that phenomenon and can be ignored. In (39)-(42), the factors $e^{-\mathrm{i}\Delta_k\pi}$ are equal to 1 if Δ_k is even and to -1 if it is odd. The factor $e^{\mathrm{i}\Delta_m\pi}$ yields the same phenomenon, depending on the parity of Δ_m. As an overall result, depending on the parity of Δ_k and Δ_m, $\widehat{D}$ is either exactly equal to the actual matrix D (up to the above-defined global phase and estimation errors) or equal to it up to opposite sign for some of its entries. The latter modifications cannot be avoided with this method if no additional information is available. They are therefore the indeterminacies of this BQPT method. They are quite limited. Moreover, we hereafter introduce an extended version of that method to completely remove them.

5. SECOND PROPOSED TOMOGRAPHY METHOD

As discussed e.g. in Branderhorst et al. (2009), Merkel et al. (2013), Shukla et al. (2014), Takahashi et al. (2013), quantum process tomography (and hence our blind version) may especially be used as a tool for characterizing quantum gates, which are the building blocks of quantum computers. This characterization is typically performed before using the considered gates for quantum computation, thus leading to a two-phase approach, composed of an "identification phase" and then of a "computation phase", for these quantum processes/gates. Moreover, one may consider scenarios where these processes/gates are used in coherent but somewhat different conditions during the identification and computation phases. We hereafter propose such an approach for extending the above BQPT method so as to remove its indeterminacies. We do not claim that the Heisenberg coupling model considered in this paper could be used as a suitable process/gate for

quantum computers: it is just used as an example in this section, to illustrate a possible procedure for removing BQPT indeterminacies, thus then allowing the reader to extend this procedure to other processes/gates that could be of interest in other configurations.

In the approach that we propose, the "identification phase" consists in performing BQPT as explained in Section 4, using a given time interval $(t - t_0)$ between source state preparation at an initial time, denoted as t_0, and output measurements at a final time, denoted as t. The "computation phase" then involves the same type of quantum process, but with a different time interval $(t' - t_0)$ between source state preparation at time t_0 and output measurement time denoted as t'. More precisely, we use $(t' - t_0) = 2(t - t_0)$. Thus, the identification phase here again yields the Heisenberg parameter estimates defined in (34)-(37), but the computation phase should then be analyzed as follows. During that phase, the considered actual process is defined by (7)-(11), but with $(t - t_0)$ replaced by $(t' - t_0)$. We derive its estimate by replacing J_{xy} and J_z in (10)-(11) by their estimates defined in (34)-(37), obtained during the *identification* phase. The same type of calculations as in Section 4 then shows that the estimates of the considered process M here involve the same matrix as in (38)-(42), but with $(t - t_0)$ replaced by $(t' - t_0)$, and Δ_k and Δ_m respectively replaced by $2\Delta_k$ and $2\Delta_m$. This procedure is thus equivalent to forcing Δ_k and Δ_m to be even from the point of view of the computation phase, which removes all indeterminacies in that phase, based on the same principle as in Section 4.

6. TEST RESULTS

The physical implementation of qubits is only an emerging topic, which is beyond the scope of this paper. We therefore validated the performance of the first BQPT method proposed above by means of numerical tests performed with a software simulation of the considered configuration. Each elementary test consists of the following four stages. (i) Randomly create a set of source states $|\psi(t_0)\rangle$. (ii) Process them according to (5), with the matrix M which defines the quantum process to be identified. This yields the states $|\psi(t)\rangle$. (iii) Apply the BQSS method of Section 3 to the above states $|\psi(t)\rangle$. This yields a set of four values: $\widehat{\gamma}_1$ to $\widehat{\gamma}_4$. (iv) Apply the BQPT method of Section 4 as follows. Using the above values of $\widehat{\gamma}_1$ to $\widehat{\gamma}_4$ and (32)-(33) with $\widehat{m} = 0$ and $\widehat{k} = 0$, one first gets estimates $\widehat{J}_{xy}$ and $\widehat{J}_z$ of the Hamiltonian parameters. Then inserting the latter values in (10)-(11) and (9) yields the estimate $\widehat{D}$ of the only matrix which is to be experimentally determined to estimate the quantum process defined by matrix M.

100 above-defined elementary tests were performed, with different sets of source states $|\psi(t_0)\rangle$ and with the same matrix M, in order to assess the statistical performance of the proposed BQPT method. The numerical values used in these tests were selected as follows. To create the source states $|\psi(t_0)\rangle$ with (4), we used the polar representation of the qubit parameters α_i and β_i, which reads

$$\alpha_i = r_i e^{\mathrm{i}\theta_i} \qquad \beta_i = q_i e^{\mathrm{i}\phi_i} \qquad i \in \{1, 2\} \tag{43}$$

with $0 \leq r_i \leq 1$ and

$$q_i = \sqrt{1 - r_i^2} \tag{44}$$

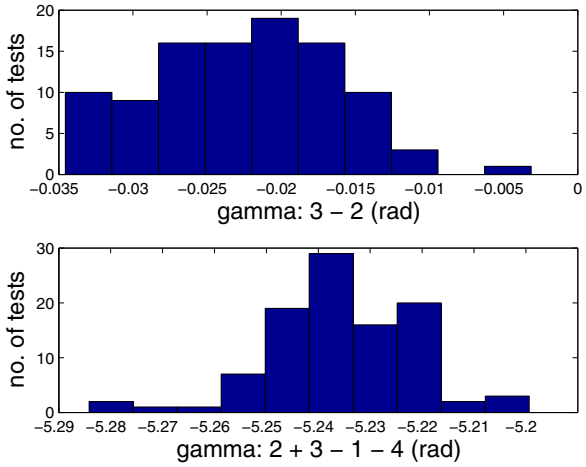

Fig. 2. Histograms of $(\widehat{\gamma}_3 - \widehat{\gamma}_2)$ (top plot) and $(\widehat{\gamma}_2 + \widehat{\gamma}_3 - \widehat{\gamma}_1 - \widehat{\gamma}_4)$ (bottom plot).

since $|\psi_i(t_0)\rangle$ has unit norm. r_1 and r_2 were randomly selected with a uniform distribution over $]0, 1[$, q_1 and q_2 were derived from (44), and θ_1, θ_2, ϕ_1 and ϕ_2 were randomly selected with a uniform distribution over $[0, 2\pi[$ (the parameters which have a physical meaning are $\phi_i - \theta_i$). The value of matrix M was set as follows. Conventional Electron Spin Resonance generally operates at X or Q bands (around 10 and 35 GHz respectively). For electron spins with $g = 2$, at 35 GHz, the resonance field is near 1.25 T. In the simulations, we used the values $g = 2$, $B = 1$ T, $(t - t_0) = 10^{-9}$ s, which yields $GB(t - t_0)/\hbar = 175.8$. Concerning the exchange coupling, we chose $J_z/k_B = 1$ K, and $J_{xy}/k_B = 0.3$ K, which lead to $J_z(t - t_0)/\hbar = 130.9$ and $J_{xy}(t - t_0)/\hbar = 39.26$. The considered BQSS method was operated with $N_z = N_x = 10$ and $p = 1$ in (17) and (19). Besides, in the sweep on γ_2 (with $\gamma_1 = \gamma_3 = \gamma_4 = 0$) and then in the sweep on γ_1 (with $\gamma_4 = 0$), we used 10^3 values of the tuned γ_j parameter. During these sweeps, γ_2 and γ_1 were respectively varied over $]0, \pi]$ and $]0, 2\pi]$. The probabilities associated with measurements of spin components along the Oz or Ox direction (see (17) or (19)) were estimated by repeatedly (10^5 times) preparing and performing measurements for each considered state.

As shown by (32)-(33), the results of BQSS which are here of importance consist of the couple of parameters $(\widehat{\gamma}_3 - \widehat{\gamma}_2)$ and $(\widehat{\gamma}_2 + \widehat{\gamma}_3 - \widehat{\gamma}_1 - \widehat{\gamma}_4)$ obtained in each elementary test. The histograms of these parameters for all 100 elementary tests are shown in Fig. 2. The ideal values of these two parameters are derived from (30)-(31) and respectively equal to $2J_{xy}(t - t_0)/\hbar$ and $2J_z(t - t_0)/\hbar$ up to the shifts due to k and m, i.e. they are equal to -0.0198 and -5.2354 rad. Their sample means are respectively -0.0211 and -5.2356 rad, whereas their standard deviations are 0.0066 and 0.0138 rad. What matters here is these absolute (not relative) values of standard deviations, since the above two combinations of $\widehat{\gamma}_1$ to $\widehat{\gamma}_4$ then essentially appear as phase parameters of the diagonal entries of $\widehat{D}$.

The performance of the associated BQPT method is then assessed by showing the histograms (real and imaginary parts, see Fig. 3) of the estimated values of the first diagonal entry of matrix D defined by (9) (the other

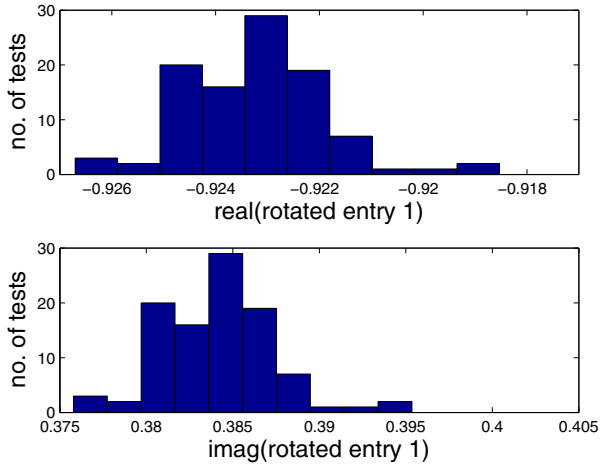

Fig. 3. Histograms of real part (top plot) and imaginary part (bottom plot) of rotated estimates of first diagonal entry of D.

diagonal entries of D yield similar figures). Due to the phase indeterminacies of this BQPT method, defined in (38)-(42), these values were first rotated by a phase factor equal to a multiple of $\pi/4$, so that they are in the same domain as the actual value of the considered entry of D, which is equal to $-0.9233 + 0.3841$ i. The difference between the sample mean of the above rotated estimates and the above actual value is $2.31 \times 10^{-5} + 3.99 \times 10^{-5}$ i, whereas the standard deviations of the real and imaginary parts of the above rotated estimates are 0.0013 and 0.0032. The proposed BQPT method is therefore very accurate.

7. DISCUSSION AND CONCLUSION

In this paper, we showed how to extend QPT to blind configurations, i.e. how to identify a quantum process with input states which have unknown values (and which are not controlled by an automated loop in our system, but they have a known property: they are unentangled in the considered configuration).

Such blind approaches especially have two potential applications. The most natural one is when the input states of the considered process indeed cannot be known. Such methods could then be of interest for characterizing quantum gates while they are operating and when only their results (output states) are available to the user who is to characterize them. This on-line characterization may be useful e.g. if the transform performed by a quantum gate slowly evolves over time (e.g. due to aging) and must be monitored, by characterizing it from time to time.

Besides, BQPT may be of even higher interest in more standard configurations, where the process input states may be prepared and known: BQPT then avoids the complexity of *accurately preparing the specific states* which are required by standard QPT methods, because BQPT can use *any* input states (unentangled in the version of BQPT that we developed so far).

We numerically validated the performance of a proposed BQPT method with a software emulation of Heisenberg-coupled spins. We plan to develop the BQPT concept beyond that specific class of processes.

REFERENCES

Abed-Meraim, K., Qiu, W., and Hua, Y. (1997). Blind system identification. *Proc. of the IEEE*, 85, 1310-1322.

Branderhorst, M. P. A., Nunn, J., Walmsley, I. A., and Kosut, R. L. (2009). Simplified quantum process tomography. *https://arxiv.org/abs/0910.4609* version 2.

Comon, P., and Jutten, C. (eds) (2010). *Handbook of blind source separation. Independent component analysis and applications.* Academic Press, Oxford.

Deville, Y., and Deville, A. (2007). Blind separation of quantum states: estimating two qubits from an isotropic Heisenberg spin coupling model. In *Proceedings of the 7th International Conference on Independent Component Analysis and Signal Separation (ICA 2007)*, 706-713. Springer-Verlag. *Erratum: replace two terms* $E\{r_i\}E\{q_i\}$ *in (33) of Deville (2007) by* $E\{r_i q_i\}$.

Deville, Y., and Deville, A. (2012). Classical-processing and quantum-processing signal separation methods for qubit uncoupling. *Quantum Information Processing*, 11, 1311-1347.

Deville, Y., and Deville, A. (2014a). Chapter 1. Quantum-source independent component analysis and related statistical blind qubit uncoupling methods. In G. R. Naik and W. Wang (eds), *Blind Source Separation: Advances in Theory, Algorithms and Applications*, 3-37. Springer, Berlin.

Deville, Y., and Deville, A. (2014b). Blind qubit state disentanglement with quantum processing: principle, criterion and algorithm using measurements along two directions. In *Proceedings of the 2014 IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP 2014)*, 6262-6266.

Deville, Y., and Deville, A. (2015). From blind quantum source separation to blind quantum process tomography In *Proceedings of the 12th International Conference on Latent Variable Analysis and Signal Separation (LVA/ICA 2015)*, 184-192, Springer Switzerland.

Deville, Y., and Deville, A. (2016a). Blind quantum computation (i.e. unsupervised quantum processing): blind quantum source separation and blind quantum process tomography. In *19th Conference on Quantum Information Processing (QIP 2016).*

Deville, Y., and Deville, A. (2016b). Fast disentanglement-based blind quantum source separation and process tomography using a feedforward quantum-classical adapting structure. In *Proceedings of the 24th European Signal Processing Conference (EUSIPCO 2016)*, 503-507, IEEE.

Merkel, S. T., Gambetta, J. M., Smolin, J. A., Poletto, S., Córcoles, A. D., Johnson, B. R., Ryan, C. A., and Steffen, M. (2013). Self-consistent quantum process tomography. *Phys. Rev. A*, 87, 062119-1 to 062119-9.

Nielsen, M. A., and Chuang, I. L. (2000). *Quantum computation and quantum information.* Cambridge University Press, Cambridge.

Shukla, A., and Mahesh, T. S. (2014). Single-scan quantum process tomography. *Physical Review A*, 90, 052301.

Takahashi, M., Bartlett, S. D., and Doherty, A. C. (2013). Tomography of a spin qubit in a double quantum dot. *Physical Review A*, 88, 022120-1 to 022120-9.