Blind separation of nonstationary sources by spectral decorrelation

Shahram Hosseini and Yannick Deville

Université Paul Sabatier
Laboratoire d'Acoustique, Métrologie, Instrumentation
Bat. 3R1B2, 118 route de Narbonne, 31062 Toulouse Cedex, France
hosseini@cict.fr , ydeville@cict.fr

Abstract. This paper demonstrates and exploits some interesting frequency-domain properties of nonstationary signals. Considering these properties, two new methods for blind separation of linear instantaneous mixtures of mutually uncorrelated, nonstationary sources are proposed. These methods are based on spectral decorrelation of the sources. The second method is particularly important because it allows the existing time-domain algorithms developed for stationary, temporally correlated sources to be applied to nonstationary, temporally uncorrelated sources just by mapping the mixtures in the frequency domain. Moreover, it sets no constraint on the variance profile, unlike previously reported methods.

1 Introduction

Blind source separation can be achieved by exploiting nonGaussianity, time correlation or nonstationarity [1]. In this paper, our goal is to propose new approaches using the nonstationarity of the sources. A few authors have studied this problem [2]-[9]. In many of these works, the nonstationarity of the variance of the sources is used. In [2], separation of nonstationary signals is achieved by computing output components which are uncorrelated *at every time point*. The method requires the joint diagonalization of N covariance matrices, where N represents the number of samples. In [3], the signals are divided in only two subintervals. Then, the joint diagonalization of two covariance matrices, estimated on the two subintervals, allows one to separate the sources. Another approach, presented in [4], is based on the maximization of the nonstationarity, measured by the cross-cumulant, of a linear combination of the observed mixtures. Several methods use the time-frequency diversity of the sources. Some of them [5] are based on a time-frequency version of joint-diagonalization source separation techniques. Others [6]-[8] assume that each source occurs alone in a small time-frequency area and identify the corresponding columns of the scaled mixing matrix in these areas. Pham and Cardoso have developed novel approaches based on the principles of maximum likelihood and minimum mutual information [9].

The methods proposed in the present paper are based on spectral decorrelation of the signals. They result from some interesting frequency-domain properties of nonstationary signals, and may be used for separating linear instantaneous mixtures of Gaussian or nonGaussian nonstationary, mutually uncorrelated signals. For the sake of simplicity, in this paper we only study the case of two mixtures of two sources. However, the method may be extended to more sources and mixtures.

2 Some mathematical preliminaries

We here introduce some interesting statistical properties of the Fourier transforms of real random signals. Their proofs are given in Appendix A.

1. Let $u_1(t)$ and $u_2(t)$ be two zero-mean, mutually uncorrelated real signals, *i.e.* such that $E[u_1(t)u_2(t)] = 0$. Then, denoting their Fourier transforms[1] by $U_1(\omega)$ and $U_2(\omega)$, we have $E[U_1(\omega)U_2(\omega)] = E[U_1(\omega)U_2^*(\omega)] = 0$.
2. Let $u(t)$ be a real stationary signal with Fourier transform $U(\omega)$. Then, $E[U^2(\omega)] = 0$ for $\omega \neq 0$.
3. If $u_1(t)$ and $u_2(t)$ are two stationary, mutually uncorrelated, real, zero-mean signals with Fourier transforms $U_1(\omega)$ and $U_2(\omega)$, and if $V_1(\omega)$ and $V_2(\omega)$ are two linear combinations of $U_1(\omega)$ and $U_2(\omega)$, then $E[V_1^2(\omega)] = E[V_2^2(\omega)] = E[V_1(\omega)V_2(\omega)] = 0$ for $\omega \neq 0$.
4. If $u(t)$ is a temporally uncorrelated, real, zero-mean signal with a nonstationary variance $q(t)$, *i.e.* if $E[u(t_1)u(t_2)] = q(t_1)\delta(t_1 - t_2)$, then its Fourier transform, $U(\omega)$ is a stationary[2], correlated process with autocorrelation $Q(\omega)$, the Fourier transform of $q(t)$.

3 Source separation in the frequency domain

Given N samples of two linear instantaneous mixtures $x_1(t)$ and $x_2(t)$ of two mutually uncorrelated, nonstationary, real, zero-mean sources $s_1(t)$ and $s_2(t)$, our objective is to estimate $s_1(t)$ and $s_2(t)$ up to a scaling factor and a permutation. Let's denote $\mathbf{s}(t) = [s_1(t), s_2(t)]^T$ and $\mathbf{x}(t) = [x_1(t), x_2(t)]^T$ so that $\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t)$ where $\mathbf{A}$ is the mixing matrix. Taking the Fourier transform of $\mathbf{x}(t)$, we obtain:

$$\mathbf{X}(\omega) = \mathbf{A}\mathbf{S}(\omega) \tag{1}$$

where $\mathbf{S}(\omega) = [S_1(\omega), S_2(\omega)]^T$, $\mathbf{X}(\omega) = [X_1(\omega), X_2(\omega)]^T$, and $S_1(\omega)$, $S_2(\omega)$, $X_1(\omega)$ and $X_1(\omega)$ are respectively the Fourier transforms of $s_1(t)$, $s_2(t)$, $x_1(t)$ and $x_2(t)$. The spectra $\mathbf{Y}(\omega) = [Y_1(\omega), Y_2(\omega)]^T$ of the estimated sources $\mathbf{y}(t) = [y_1(t), y_2(t)]^T$ may be obtained by multiplying $\mathbf{X}(\omega)$ by a real separating matrix $\mathbf{B}$, *i.e.* $\mathbf{Y}(\omega) = \mathbf{B}\mathbf{X}(\omega)$. It is well known that because of the indeterminacies involved in the problem, this matrix has only two degrees of freedom. Hence, we need at least two equations for estimating it. In the following, we propose two

[1] The Fourier transform of a stochastic process $u(t)$ is a stochastic process $U(\omega)$ given by [10] $U(\omega) = \int_{-\infty}^{\infty} u(t)e^{-j\omega t}dt$. The integral is interpreted as a Mean Square limit.

[2] In the sense that $E[U(\Omega + \omega)U^*(\Omega)] = Q(\omega)$, *i.e.* its autocorrelation depends only on ω, not on Ω.

alternative ideas for obtaining such equations in the frequency domain, using the properties mentioned in Section 2, and knowing that the estimated sources $y_1(t)$ and $y_2(t)$ must be mutually uncorrelated.

3.1 First source separation method, using Property 1

To avoid the indeterminacy due to the scaling factor, let's fix the entries of the second column of the separating matrix $\mathbf{B}$ to one, so that $\mathbf{Y}(\omega) = \begin{pmatrix} b_1 & 1 \\ b_2 & 1 \end{pmatrix} \mathbf{X}(\omega)$. Following Property 1, the uncorrelatedness of $y_1(t)$ and $y_2(t)$ implies that

$$\begin{aligned} E[Y_1(\omega)Y_2^*(\omega)] &= E[(b_1X_1(\omega) + X_2(\omega))(b_2X_1^*(\omega) + X_2^*(\omega))] = 0 \\ E[Y_1(\omega)Y_2(\omega)] &= E[(b_1X_1(\omega) + X_2(\omega))(b_2X_1(\omega) + X_2(\omega))] = 0 \end{aligned} \tag{2}$$

Solving these two equations with respect to b_1 and b_2, it can be shown (see Appendix B) that b_1 and b_2 are the two real solutions of the following second-order equation:

$$Az^2 + Bz + C = 0 \tag{3}$$

where

$$\begin{aligned} A &= -E[X_1(\omega)X_2(\omega)]E[X_1(\omega)X_1^*(\omega)] + E[X_1^2(\omega)]E[X_1(\omega)X_2^*(\omega)] \\ B &= -E[X_1(\omega)X_1^*(\omega)]E[X_2^2(\omega)] + E[X_1^2(\omega)]E[X_2(\omega)X_2^*(\omega)] \\ C &= -E[X_1(\omega)X_2^*(\omega)]E[X_2^2(\omega)] + E[X_1(\omega)X_2(\omega)]E[X_2(\omega)X_2^*(\omega)] \end{aligned} \tag{4}$$

These equations are of interest only if $s_1(t)$ and/or $s_2(t)$ are nonstationary, because, from Property 3, if $s_1(t)$ and $s_2(t)$ are stationary, $E[X_1^2(\omega)] = E[X_2^2(\omega)]$ $= E[X_1(\omega)X_2(\omega)] = 0$ for $\omega \neq 0$ so that the coefficients A, B and C are equal to zero for $\omega \neq 0$. Moreover, since the Fourier transform of a real signal is real at $\omega = 0$, we can write $E[X_1(0)X_2(0)] = E[X_1(0)X_2^*(0)]$, $E[X_1^2(0)] = E[X_1(0)X_1^*(0)]$, and $E[X_2^2(0)] = E[X_2(0)X_2^*(0)]$, so that at $\omega = 0$, $A = B = C = 0$ too, and the sources cannot be separated. This result is not surprising because it is well known that the mutual decorrelation of two sources (which is a second-order statistical parameter) is not a strong enough hypothesis for separating stationary sources[3]. It is therefore necessary to suppose that at least one of the sources is nonstationary for achieving source separation only using mutual decorrelation.

Discussion. From (4), the implementation of the above method requires the computation of the expected values of some spectral functions. Three different cases may be considered.

a) Several realizations of the mixtures $x_1(t)$ and $x_2(t)$ are available. In this case, the expected values may be approximated by averaging the spectral functions on these realizations (for a particular frequency).

b) Only one realization of the mixtures is available but the spectra are ergodic so that the expected values in (4) can be estimated by frequency averages. A

[3] Except for temporally correlated sources by exploiting the time correlation.

necessary condition for the ergodicity is the stationarity of the spectral functions, *i.e.*, the expected values in (4) must be independent from ω. However, it seems difficult to find signals satisfying this condition.

c) Only one realization of the mixtures is available but each mixture has nearly the same spectral shape in different time frames (for example, the mixtures are cyclostationary). In this case, the expected values may be estimated by dividing the mixtures in several time frames, computing the Fourier transforms and the spectral functions over each frame, and averaging the results on different frames (for a particular frequency).

3.2 Second source separation method, using Property 4

If we also suppose that $s_1(t)$ and $s_2(t)$ are temporally uncorrelated, from Property 4, $S_1(\omega)$ and $S_2(\omega)$ are stationary and correlated processes. Moreover, from (1), $\mathbf{X}(\omega)$ is a linear mixture of these two processes. Many algorithms have been proposed for separating such mixtures [11]-[16]. Although these algorithms were originally developed for time-domain stationary, time-correlated processes, nothing prohibits us from applying them to frequency-domain stationary, frequency-correlated processes. Thus, only by mapping the nonstationary temporally uncorrelated mixtures in the frequency domain, they can be separated using one of the numerous methods developed previously for time-correlated stationary mixtures.

4 Simulation results

In the first experiment, we consider the example used in [2]. The following stationary and nonstationary Gaussian signals are used: $s_1(t) = n_1(t)$, $s_2(t) = \mu_2(t)n_2(t)$, where $n_1(t)$ and $n_2(t)$ are mutually independent Gaussian i.i.d. signals with zero mean and unity variance, and $\mu_2(t) = 2\sin(\omega_0 t)$. The mixing matrix is $\mathbf{A} = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$. It can be easily shown that $E[s_1(t_1)s_1(t_2)] = \delta(t_1 - t_2)$ and $E[s_2(t_1)s_2(t_2)] = 4\sin^2(\omega_0 t_1)\delta(t_1 - t_2)$. Thus, using the same notations as in Property 4, $q_1(t) = 1$ and $q_2(t) = 4\sin^2(\omega_0 t)$, so that $Q_1(\omega) = 2\pi\delta(\omega)$ and $Q_2(\omega) = 2\pi[2\delta(\omega) - \delta(\omega - 2\omega_0) - \delta(\omega + 2\omega_0)]$.

In the first step, we want to separate the sources using the method proposed in Subsection 3.1. The coefficients A, B and C in (4) depend on $E[S_1^2(\omega)]$, $E[S_2^2(\omega)]$, $E[S_1(\omega)S_1^*(\omega)]$ and $E[S_2(\omega)S_2^*(\omega)]$. Using the method employed in the proof of Property 2, it can be shown that $E[S_1^2(\omega)] = Q_1(2\omega)$, $E[S_2^2(\omega)] = Q_2(2\omega)$, $E[S_1(\omega)S_1^*(\omega)] = Q_1(0)$ and $E[S_2(\omega)S_2^*(\omega)] = Q_2(0)$. Since $E[S_1^2(\omega)]$ and $E[S_2^2(\omega)]$ depend on ω, they cannot be considered as ergodic processes so that the coefficients A, B and C in (4) cannot be estimated by frequency averages. However, as $s_1(t)$ is stationary and $s_2(t)$ is cyclostationary, we can estimate the expected values in (4) using the method proposed in part (c) of the discussion of Subsection 3.1.

The experiment was done using 1 second of the sources $s_1(t)$ and $s_2(t)$ containing 8192 samples. The frequency $\omega_0 = 2\pi.256$ of $\mu_2(t)$ was chosen so that each period of $\mu_2(t)$ contains 32 points. Hence, the signal $s_2(t)$ includes 256 periods of $\mu_2(t)$. Then, the 32-point Discrete Fourier Transforms of the mixtures $x_1(t)$ and $x_2(t)$ were computed on each period and the expected values in (4) were estimated by averaging the spectral functions (at $\omega = \omega_0$) on 256 periods. The experiment was repeated 100 times corresponding to 100 different seed values of the random variable generator. For each experiment, the output Signal to Noise Ratio (in dB) was computed by $SNR = 0.5 \sum_{i=1}^{2} 10 \log_{10} \frac{E[s_i^2]}{E[(y_i - s_i)^2]}$, after normalizing the estimated sources, $y_i(t)$, so that they have the same variances as the source signals, $s_i(t)$. The mean and the standard deviation of SNR on the 100 experiments were 27.0 dB and 8.9 dB .

In the second step, we want to separate the sources using the method proposed in Subsection 3.2. This time, we compute the Fourier transforms of $x_1(t)$ and $x_2(t)$ on the whole signals. The autocorrelation function of $X_1(\omega)$ is shown in Figure 1 which presents three peaks at $\omega = 0$ and $\omega = \pm 2\omega_0$, and confirms the theoretical calculus mentioned above (see the expression of $Q_2(\omega)$). The separating matrix may be estimated using the following equations: $E[Y_1(\omega)Y_2^*(\omega)] = 0$, and $E[Y_1(\omega + 2\omega_0)Y_2^*(\omega)] = 0$. We used a modified version of the AMUSE

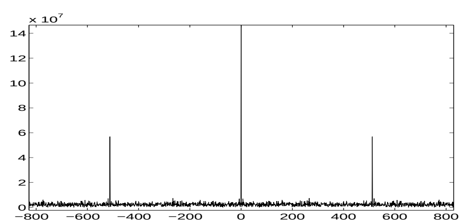

Fig. 1. Autocorrelation function of $X_1(\omega)$.

algorithm [11] for this purpose. This simple and fast algorithm, originally developed for separating time-correlated stationary sources in the time domain, here works as follows. (a) Spatially whiten the data $\mathbf{X}(\omega)$ to obtain $\mathbf{Z}(\omega)$. (b) Compute the eigenvalue decomposition of $\overline{\mathbf{C}_{2\omega_0}^{\mathbf{Z}}} = \frac{1}{2}[\mathbf{C}_{2\omega_0} + \mathbf{C}_{2\omega_0}^T]$, where $\mathbf{C}_{2\omega_0} = E[\mathbf{Z}(\omega + 2\omega_0)\mathbf{Z}^*(\omega)]$ is the covariance matrix corresponding to lag $2\omega_0$. (c) The rows of the separating matrix $\mathbf{B}$ are given by the eigenvectors of $\overline{\mathbf{C}_{2\omega_0}^{\mathbf{Z}}}$. Using the same signals as in the first step, the mean and the standard deviation of SNR were 41.6 dB and 7.2 dB. Other experiments with different profiles of nonstationary variance for the sources $s_1(t)$ and $s_2(t)$ led to similar results.

In the second experiment, the above algorithm based on AMUSE was used for separating mixtures of speech signals. Three tests using three couples of 44100-sample speech signals led to an average SNR of 40.6 dB. This experiment shows that although Property 4 is derived for temporally uncorrelated signals, the proposed method works well also for temporally correlated signals.

5 Conclusion

A major objective of this paper was to demonstrate and exploit some theoretically interesting frequency-domain properties of signals which are nonstationary in the time domain. These properties provide sufficient second-order constraints in the frequency domain for separating instantaneous linear mixtures of nonstationary sources.

Two separating methods were proposed based on Properties 1 and 4. The first method is theoretically interesting but its implementation is difficult unless either many realizations of the mixtures are available or the sources are cyclostationary. The second method is very simple and powerful because it allows the time-domain algorithms developed for stationary time-correlated signals to be applied to temporally uncorrelated sources which are nonstationary in the time domain, just by mapping them in the frequency domain. It should be remarked that this algorithm does not require the variance of the sources to be constant over subintervals, while this hypothesis is necessary in the majority of the source separation algorithms based on the nonstationarity of variance which have been reported in the literature.

A Proofs of the properties of Section 2

Proof of Property 1: Consider two mutually uncorrelated zero-mean real signals $u_1(t)$ and $u_2(t)$, with Fourier transforms $U_1(\omega)$ and $U_2(\omega)$. We can write:

$$E[U_1(\omega)U_2(\omega)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[u_1(t_1)u_2(t_2)]e^{-j\omega(t_1+t_2)}dt_1dt_2 = 0$$

$$E[U_1(\omega)U_2^*(\omega)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[u_1(t_1)u_2(t_2)]e^{-j\omega(t_1-t_2)}dt_1dt_2 = 0$$

because $E[u_1(t)u_2(t)] = 0$.

Proof of Property 2: Let $u(t)$ be a real stationary signal with Fourier transform $U(\omega)$. We want to show that $E[U^2(\omega)] = 0$, for $\omega \neq 0$. Using the definition of the Fourier transform, we can write

$$E[U^2(\omega)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[u(t_1)u(t_2)]e^{-j\omega(t_1+t_2)}dt_1dt_2$$

Since $u(t)$ is stationary, its autocorrelation function depends only on $t_1 - t_2$: $E[u(t_1)u(t_2)] = R(t_1 - t_2)$. Denoting the auxiliary variable $\tau = t_1 - t_2$,

$$\begin{aligned} E[U^2(\omega)] &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R(\tau)e^{-j\omega(2t_2+\tau)}d\tau dt_2 \\ &= \int_{-\infty}^{\infty} e^{-j2\omega t_2}\int_{-\infty}^{\infty} R(\tau)e^{-j\omega\tau}d\tau dt_2 \end{aligned}$$

The inner integral represents the power spectral density of $u(t)$, denoted by $\Gamma(\omega)$. Thus $E[U^2(\omega)] = \Gamma(\omega)\int_{-\infty}^{\infty} e^{-j2\omega t_2} dt_2 = 2\pi\Gamma(\omega)\delta(2\omega)$, which yields $E[U^2(\omega)] = 0$ for $\omega \neq 0$.

Proof of Property 3: Suppose $V_1(\omega) = a_{11}U_1(\omega) + a_{12}U_2(\omega)$ and $V_2(\omega) = a_{21}U_1(\omega) + a_{22}U_2(\omega)$. We can write

$$\begin{aligned} E[V_1^2(\omega)] &= a_{11}^2 E[U_1^2(\omega)] + a_{12}^2 E[U_2^2(\omega)] + 2a_{11}a_{12}E[U_1(\omega)U_2(\omega)] \\ E[V_2^2(\omega)] &= a_{21}^2 E[U_1^2(\omega)] + a_{22}^2 E[U_2^2(\omega)] + 2a_{21}a_{22}E[U_1(\omega)U_2(\omega)] \\ E[V_1(\omega)V_2(\omega)] &= a_{11}a_{21}E[U_1^2(\omega)] + a_{12}a_{22}E[U_2^2(\omega)] + (a_{11}a_{22} + a_{12}a_{21}) \\ &\qquad E[U_1(\omega)U_2(\omega)] \end{aligned}$$

Since $u_1(t)$ and $u_2(t)$ are real, zero-mean, uncorrelated and stationary, the first two terms of the right side of all the above equations vanish for $\omega \neq 0$ following Property 2, and the third term of all the equations vanishes whatever ω following Property 1.

Proof of Property 4: (see also [10]) If $E[u(t_1)u(t_2)] = q(t_1)\delta(t_1 - t_2)$, where $\delta(t_1 - t_2)$ is a Dirac distribution, then

$$\begin{aligned} E[U(\Omega+\omega)U^*(\Omega)] &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[u(t_1)u(t_2)]e^{-j(\Omega+\omega)t_1}e^{j\Omega t_2}dt_1dt_2 \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} q(t_1)\delta(t_1-t_2)e^{-j\Omega(t_1-t_2)}e^{-j\omega t_1}dt_1dt_2 \end{aligned} \tag{5}$$

Since $\delta(t_1 - t_2)e^{-j\Omega(t_1-t_2)} = \delta(t_1 - t_2)$,

$$\begin{aligned} E[U(\Omega+\omega)U^*(\Omega)] &= \int_{-\infty}^{\infty} q(t_1)e^{-j\omega t_1}\int_{-\infty}^{\infty}\delta(t_1-t_2)dt_2dt_1 \\ &= \int_{-\infty}^{\infty} q(t_1)e^{-j\omega t_1}dt_1 = Q(\omega) \end{aligned} \tag{6}$$

B Derivation of Equation (3)

For the sake of simplicity, we omit the parameter ω in the following notations. Developing Equations (2), we obtain[4]:

$$b_1b_2E[X_1X_1^*] + (b_1+b_2)E[X_1X_2^*] + E[X_2X_2^*] = 0 \tag{7}$$

$$b_1b_2E[X_1^2] + (b_1+b_2)E[X_1X_2] + E[X_2^2] = 0 \tag{8}$$

From (8), $b_2 = \frac{-b_1E[X_1X_2]-E[X_2^2]}{b_1E[X_1^2]+E[X_1X_2]}$. Replacing b_2 in (7), we obtain:

$$\frac{-b_1E[X_1X_2] - E[X_2^2]}{b_1E[X_1^2] + E[X_1X_2]}(b_1E[X_1X_1^*] + E[X_1X_2^*]) + (b_1E[X_1X_2^*] + E[X_2X_2^*]) = 0$$

[4] Note that $E[X_1X_2^*] = E[X_2X_1^*]$, because X_1 and X_2 are linear combinations of two spectra S_1 and S_2, and $E[S_1S_2^*] = E[S_1^*S_2] = 0$, following Property 1.

which yields:

$$(-b_1 E[X_1X_2] - E[X_2^2])(b_1 E[X_1X_1^*] + E[X_1X_2^*]) + (b_1 E[X_1X_2^*] + E[X_2X_2^*])$$
$$(b_1 E[X_1^2] + E[X_1X_2]) = 0$$

Developing the above equation leads to the second-order equation (3), for which b_1 is a real solution. Note that the two equations (7) and (8) are symmetrical with respect to b_1 and b_2. This implies that b_2 is also a real solution of (3). This result is not surprising because the sources may be estimated only up to a permutation.

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