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## Hyperspectral and multispectral data fusion based on linear-quadratic nonnegative matrix factorization

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Abstract. This paper proposes three multisharpening approaches to enhance the spatial resolution of urban hyperspectral remote sensing images. These approaches, related to linear-quadratic spectral unmixing techniques, use a linear-quadratic nonnegative matrix factorization (NMF) multiplicative algorithm. These methods begin by unmixing the observable high-spectral/low-spatial resolution hyperspectral and high-spatial/low-spectral resolution multispectral images. The obtained high-spectral/high-spatial resolution features are then recombined, according to the linear-quadratic mixing model, to obtain an unobservable multisharpened high-spectral/high-spatial resolution hyperspectral image. In the first designed approach, hyperspectral and multispectral variables are independently optimized, once they have been coherently initialized. These variables are alternately updated in the second designed approach. In the third approach, the considered hyperspectral and multispectral variables are jointly updated. Experiments, using synthetic and real data, are conducted to assess the efficiency, in spatial and spectral domains, of the designed approaches and of linear NMF-based approaches from the literature. Experimental results show that the designed methods globally yield very satisfactory spectral and spatial fidelities for the multisharpened hyperspectral data. They also prove that these methods significantly outperform the used literature approaches. © 2017 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: 10.1117/1.JRS.11.025008]

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#### 1 Introduction

Hyperspectral and multispectral imaging systems are the greatest methods in the rapidly evolving field of remote sensing. These systems typically have limitations in either spectral or spatial resolution. While multispectral sensors provide high-spatial resolution data with a few relatively large spectral bands (i.e., low spectral resolution), hyperspectral sensors collect data in hundreds or thousands of contiguous narrow spectral bands. The high spectral resolution of hyperspectral sensors allows accurate detection and identification of materials present in the imaged area, but with a poor spatial resolution, which is generally lower than that of multispectral sensors. Therefore, it is desirable to improve the spatial resolution of hyperspectral data by using the spatial information of multispectral images, while keeping the spectral fidelity of hyperspectral data as much as possible. The fusion methods, commonly known as sharpening methods, are one possible approach for increasing the spatial resolution of hyperspectral data.

Over the last two decades, several sharpening methods were designed to merge multispectral or hyperspectral data with a high-spatial resolution panchromatic image.<sup>1,2</sup> The most famous of

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these classical pan-sharpening processes are component projection-substitution methods, such as those based on principal component analysis and intensity hue saturation. It should here be noted that it is natural to expect that performing pan-sharpening with hyperspectral data is more complicated than achieving it with multispectral data.<sup>1</sup> Indeed, this can be explained by the fact that the spectral band of the panchromatic image generally encompasses the spectral bands of the multispectral data, which can produce spectral distortions during the fusion process. Thus, pan-sharpening approaches were extended, in the last few years, for combining hyperspectral and multispectral data. These extended techniques are known as multisharpening approaches.

These multisharpening processes<sup>3–8</sup> aim at combining the spectral information from highspectral/low-spatial resolution hyperspectral data with spatial information obtained from highspatial/low-spectral resolution multispectral data. Applying such multisharpening processes to observable hyperspectral images allows one to perform accurate detection, identification, and classification of an imaged region at a higher spatial resolution.

Recently, other approaches,<sup>9–13</sup> via linear spectral unmixing (LSU) techniques,<sup>14</sup> and using nonnegative matrix factorization (NMF),<sup>15–17</sup> which consists of factorizing a nonnegative matrix into a product of two nonnegative matrices, were designed for increasing the spatial resolution of hyperspectral data by using a multispectral image. LSU techniques, used to solve the typical blind source separation problem,<sup>18,19</sup> consist of linearly unmixing remote sensing data into a collection of endmember spectra and their corresponding abundance fractions.

When facing nonflat landscape and/or irradiance heterogeneity in the imaged area (such as in urban environments), the linear mixing model used in LSU techniques is not valid any more, and it should be replaced by a nonlinear mixing model.<sup>20</sup> This nonlinear model can be reduced to a linear-quadratic mixing model.<sup>21,22</sup>

In this paper, new multisharpening methods are proposed for hyperspectral and multispectral remote sensing data fusion. The proposed methods are particularly useful for fusing optical urban remote sensing data. These original methods are based on linear-quadratic spectral unmixing (LQSU) techniques, which consider multiple scattering of light between endmembers in the observed area, which is the case when considering urban environments.<sup>22</sup> The multisharpening methods described below use the linear-quadratic NMF (LQNMF) multiplicative algorithm.<sup>23</sup> They first consist of unmixing the observable high-spectral/low-spatial resolution hyperspectral data and high-spatial/low-spectral resolution multispectral data. The obtained high resolution spectral and spatial parts of the information are then recombined, according to the linear-quadratic mixing model, to get unobservable multisharpened high-spectral/high-spatial resolution hyperspectral data. In the first proposed approach, called hyperspectral and multispectral data fusion based on LQNMF (HMF-LQNMF), hyperspectral and multispectral variables are independently optimized, once they have been coherently initialized.<sup>24,25</sup> These variables are alternately updated in the second proposed approach, called coupled hyperspectral and multispectral data fusion based on LQNMF (CHMF-LQNMF). In the third proposed approach, called joint hyperspectral and multispectral data fusion based on LQNMF (JHMF-LQNMF), the considered hyperspectral and multispectral variables are jointly updated.

The paper is structured as follows. Section 2 describes the linear-quadratic mixing model considered in the LQSU concept. In Sec. 3, the proposed methods are presented. The results obtained by the proposed methods and those obtained with two linear NMF-based approaches from the literature are compared in Sec. 4. These results are obtained by considering synthetic and real data. Finally, Sec. 5 concludes this paper.

#### 2 Mathematical Data Model

The linear-quadratic mixing model considers the second-order interactions between different endmember spectra and assumes that third- or higher-order interactions are insignificant. Thus, in this work, each spectral vector associated with a pixel in a remote sensing image is supposed to be a linear-quadratic mixture of different endmember spectra. Mathematically, the nonnegative reflectance spectrum  $x_i$  (column vector of size *L*), from pixel *i* of the remote sensing image, is modeled as<sup>23</sup>

$$x_{i} = \sum_{j=1}^{M} a_{j}(i)s_{j} + \sum_{j=1}^{M} \sum_{l=j}^{M} a_{j,l}(i)s_{j} \odot s_{l},$$
(1)

with 
$$\begin{cases} s_j \ge 0, & j = 1, \dots, M \\ \sum_{j=1}^{M} a_j(i) = 1, & \\ a_j(i) \ge 0, & j = 1, \dots, M \\ 0 \le a_{j,l}(i) \le 0.5, & 1 \le j \le l \le M \end{cases}$$
 (2)

where  $s_j$  (column vector of size L) is the nonnegative reflectance spectrum of endmember j,  $\odot$  corresponds to an element-wise multiplication, with  $s_j \odot s_l$  here considered as a "pseudoendmember" spectrum, and  $a_j(i)$  and  $a_{j,l}(i)$  are the abundance fractions corresponding, respectively, to the linear and quadratic parts of the mixing model. L and M correspond, respectively, to the number of spectral bands in the considered image and the number of endmembers present in the observed region. In addition, the considered mixing model is constrained according to Eq. (2). These constraints are imposed based on the physical analysis reported in Ref. 22 considering urban environments and using several synthetic data simulated with the three-dimensional advanced modeling of the atmospheric radiative transfer for inhomogeneous surfaces (AMARTIS v2) code.<sup>26</sup> The model [Eq. (1)] can be reformulated in matrix form as follows (for P pixels,  $P \ge 2$ )<sup>23</sup>

$$X = AS = A_a S_a + A_b S_b, (3)$$

with  $X = [x_1, ..., x_P]^T$  (observed pixel spectra matrix, with dimensions  $P \times L$ , where each column vector of X contains one spectral band<sup>23</sup>),  $A = [A_a A_b]$  (linear and quadratic abundance fraction matrix), and  $S = \begin{bmatrix} S_a \\ S_b \end{bmatrix}$  (endmember and pseudoendmember spectra matrix), with

$$A_a = \begin{bmatrix} a_1(1) & \cdots & a_M(1) \\ \vdots & \ddots & \vdots \\ a_1(P) & \cdots & a_M(P) \end{bmatrix},$$
(4)

$$A_{b} = \begin{bmatrix} a_{1,1}(1) & a_{1,2}(1) & \cdots & a_{M,M}(1) \\ \vdots & \ddots & \vdots \\ a_{1,1}(P) & a_{1,2}(P) & \cdots & a_{M,M}(P) \end{bmatrix},$$
(5)

$$S_a = [s_1, \dots, s_M]^{\mathrm{T}},\tag{6}$$

$$S_b = [s_1 \odot s_1 s_1 \odot s_2, \dots, s_M \odot s_M]^{\mathrm{T}}.$$
(7)

The subindices *a* and *b*, respectively, refer to the linear and quadratic parts of the considered variables. The notation  $[.]^{T}$  corresponds to the matrix transpose. The aim of hyperspectral data multisharpening is to produce unobservable fused high-spectral/high-spatial resolution hyperspectral data  $X_f \in R_+^{P_m \times L_h}$  from observable high-spectral/low-spatial resolution hyperspectral data  $X_h \in R_+^{P_m \times L_h}$  and high-spatial/low-spectral resolution multispectral data  $X_m \in R_+^{P_m \times L_m}$ .  $L_h$  (respectively  $L_m$ ) corresponds to the number of spectral bands of the hyperspectral image  $X_h$  (respectively multispectral image  $X_m$ ).  $P_h$  (respectively  $P_m$ ) corresponds to the number of pixels of the hyperspectral (respectively multispectral) image. Each column vector of the above matrices  $(X_h, X_m, \text{ and } X_f)$  contains one spectral band.<sup>23</sup> The observable hyper/multispectral images are assumed to be radiometrically corrected and geometrically coregistered.

As explained above,  $X_h$  and  $X_m$  can be reformulated in matrix form as

$$X_h = A_h S_h = A_{ha} S_{ha} + A_{hb} S_{hb}, aga{8}$$

$$X_m = A_m S_m = A_{ma} S_{ma} + A_{mb} S_{mb},$$
(9)

with  $A_h = [A_{ha}A_{hb}]$ ,  $S_h = \begin{bmatrix} S_{ha} \\ S_{hb} \end{bmatrix}$ ,  $A_m = [A_{ma}A_{mb}]$ , and  $S_m = \begin{bmatrix} S_{ma} \\ S_{mb} \end{bmatrix}$ . The subscripts *h* and *m* refer, respectively, to hyperspectral and multispectral entities.

#### 3 Proposed Methods

The proposed methods are based on the LQSU concept and consist of deriving, from the considered data, estimates  $\widetilde{S_{ha}}$  and  $\widetilde{S_{hb}}$  of the sets  $S_{ha}$  and  $S_{hb}$  of hyperspectral endmember and pseudoendmember spectra and estimates  $\widetilde{A_{ma}}$  and  $\widetilde{A_{mb}}$  of the sets  $A_{ma}$  and  $A_{mb}$  of linear and quadratic high spatial resolution abundance fractions. The desired multisharpened highspectral/high-spatial resolution hyperspectral image  $X_f$  is then obtained by

$$X_f = \widetilde{A_{ma}} \, \widetilde{S_{ha}} + \widetilde{A_{mb}} \, \widetilde{S_{hb}} \,. \tag{10}$$

The proposed methods include two LQSU processes and use the iterative LQNMF multiplicative algorithm described in Ref. 23. The first process consists of unmixing a hyperspectral image by optimizing the following Frobenius norm criterion

$$J_1 = \frac{1}{2} \| X_h - \widetilde{A_{ha}} \, \widetilde{S_{ha}} - \widetilde{A_{hb}} \, \widetilde{S_{hb}} \, \|_F^2.$$
<sup>(11)</sup>

This criterion can be formulated in scalar form as<sup>23</sup>

$$J_{1} = \frac{1}{2} \sum_{i_{h}, n_{h}} \left[ [X_{h}]_{i_{h}, n_{h}} - \sum_{j=1}^{M} \widetilde{a_{h_{j}}}(i_{h}) [\widetilde{S}_{h}]_{j, n_{h}} - \sum_{j=1}^{M} \sum_{l=j}^{M} \widetilde{a_{h_{j,l}}}(i_{h}) [\widetilde{S}_{h}]_{j, n_{h}} [\widetilde{S}_{h}]_{l, n_{h}} \right]^{2}, \quad (12)$$

where  $[X_h]_{i_h,n_h}$  is the entry of  $X_h$  with row and column indices, respectively, equal to  $i_h$  and  $n_h$ , and similar notations are used for the other matrices. Also,  $i_h$  corresponds to a hyperspectral pixel and  $n_h$  is the index of the hyperspectral spectra components, i.e., the hyperspectral wavelengths.  $[\tilde{S}_h]_{j,n_h}$  is the element  $(j, n_h)$  of matrix  $\tilde{S}_h$  (the estimate of matrix  $S_h$ ), with j = 1, ..., M.

Similarly, the second process optimizes the following Frobenius norm in order to unmix a multispectral image

$$J_2 = \frac{1}{2} \|X_m - \widetilde{A_{ma}} \widetilde{S_{ma}} - \widetilde{A_{mb}} \widetilde{S_{mb}} \|_F^2.$$
(13)

Like the  $J_1$  criterion, this second criterion can be formulated in scalar form as<sup>23</sup>

$$J_{2} = \frac{1}{2} \sum_{i_{m}, n_{m}} \left[ [X_{m}]_{i_{m}, n_{m}} - \sum_{j=1}^{M} \widetilde{a_{m_{j}}}(i_{m})[\widetilde{S_{m}}]_{j, n_{m}} - \sum_{j=1}^{M} \sum_{l=j}^{M} \widetilde{a_{m_{j,l}}}(i_{m})[\widetilde{S_{m}}]_{j, n_{m}}[\widetilde{S_{m}}]_{l, n_{m}} \right]^{2},$$
(14)

where the multispectral variables are defined in the same way as the hyperspectral variables of Eq. (12).

The designed methods work in three stages that are described hereafter.

#### 3.1 Initialization Stage

This stage is shared by the three proposed methods and aims at initializing the different hyper/ multispectral variables. The LQNMF-based methods, like the standard linear-NMF-based methods, are not assured to give a unique solution and their convergence point will possibly depend on their initialization. Therefore, and in order to derive methods that do not use random initialization, the initial estimated hyperspectral endmember spectra  $\widetilde{S}_{ha}^{(0)}$  are calculated by the linear

simplex identification via split augmented Lagrangian (SISAL) method,<sup>27</sup> which is one of the most advanced methods for endmember spectra extraction, thus allowing a better initialization of these spectra. The SISAL method requires the number M of endmembers to be known. This number can be automatically determined by using the method described in Ref. 28. The initial estimated hyperspectral pseudoendmember spectra  $\widetilde{S_{hb}}^{(0)}$  are derived from the initial matrix  $\widetilde{S_{ha}}^{(0)}$  by using element-wise multiplication. Then, the initial estimated linear low-spatial resolution abundance fractions  $\widetilde{A_{ha}}^{(0)}$  are derived from the hyperspectral image  $X_h$  and the initial estimated matrix  $\widetilde{S_{ha}}^{(0)}$ , by means of the fully constrained least squares method<sup>29</sup> (separately applied to each pixel of the hyperspectral image). From the matrix  $\widetilde{A_{ha}}^{(0)}$ , the initial estimated quadratic low-spatial resolution abundance fractions  $\widetilde{A_{hb}}^{(0)}$  are calculated, by using the Fan model<sup>30</sup> and the constraint described in Eq. (2), as follows: for a given hyperspectral pixel  $i_h$ , each element  $\widetilde{a_{h_{j,l}}}^{(0)}(i_h)$  (with  $j \leq l$ ) of matrix  $\widetilde{A_{hb}}^{(0)}$  is set to min $\{0.5, \widetilde{a_{h_i}}^{(0)}(i_h), \widetilde{a_{h_l}}^{(0)}(i_h)\}$ , where  $\widetilde{a_{h_i}}^{(0)}(i_h)$  and  $\widetilde{a_{h_i}}^{(0)}(i_h)$  are two elements of matrix  $\widetilde{A_{ha}}^{(0)}$ . The initial estimated multispectral endmember spectra  $\widetilde{S_{ma}}^{(0)}$  are derived from the initial estimated hyperspectral endmember spectra  $\widetilde{S_{ha}}^{(0)}$  by averaging the samples of the latter spectra over the wavelength domains considered in the multispectral image. The remaining multispectral variables, i.e.,  $\widetilde{S_{mb}}^{(0)}$ ,  $\widetilde{A_{ma}}^{(0)}$ , and  $\widetilde{A_{mb}}^{(0)}$ , are derived in the same manner as the above-initialized hyperspectral variables.

The above-interdependent initialization of hyperspectral and multispectral variables allows one avoiding a possible permutation [which would cause a problem when using fusion Eq. (10)] between the results of the two considered hyper/multispectral image unmixing processes. Also, the sum-to-one constraint, given in Eq. (2), allows one avoiding the known problem of the scale factor that can appear in the results of the two unmixing processes [which would also be a problem when using fusion Eq. (10)]. This constraint is enforced, for the linear parts of low/high spatial resolution abundance fractions, in the following optimization stage by adopting the method described in Ref. 28.

#### 3.2 Optimization Stage

This stage aims at updating the above-initialized variables in order to get the final estimates of these variables.

The first considered HMF-LQNMF method consists of independently updating the hyperspectral and multispectral matrices,<sup>24,25</sup> according to the LQNMF multiplicative algorithm. The hyperspectral and multispectral data are thus dependently unmixed. Practically, this first proposed method includes two successive loops during the optimization stage. The hyperspectral matrices are updated in the first loop by optimizing the criterion  $J_1$ . This first loop uses the following update rules and constraints, with the notations of Ref. 23

• Update rule (given in scalar form) of matrix  $\widetilde{S_{ha}}$ :

$$[\widetilde{S_{ha}}]_{p,n_{h}} \leftarrow [\widetilde{S_{ha}}]_{p,n_{h}} \left( \frac{[\widetilde{A_{ha}}^{T}X_{h}]_{p,n_{h}} + 2[\widetilde{S_{ha}}]_{p,n_{h}}[\widetilde{A_{hb}}^{T}X_{h}]_{(pp),n_{h}}}{[D_{h}]_{p,n} + \varepsilon} + \frac{\sum_{j=1, j \neq p}^{M} [\widetilde{S_{ha}}]_{j,n_{h}}[\widetilde{A_{hb}}^{T}X_{h}]_{(jp),n_{h}}}{[D_{h}]_{p,n_{h}} + \varepsilon} \right),$$

$$(15)$$

where

$$[D_{h}]_{p,n_{h}} = [\widetilde{A_{ha}^{T}} \widetilde{A_{h}} \widetilde{S_{h}}]_{p,n_{h}} + \sum_{j=1, j \neq p}^{M} [\widetilde{S_{ha}}]_{j,n_{h}} \times [\widetilde{A_{hb}^{T}} \widetilde{A_{h}} \widetilde{S_{h}}]_{(jp),n_{h}} + 2[\widetilde{S_{ha}}]_{p,n_{h}} \times [\widetilde{A_{hb}^{T}} \widetilde{A_{h}} \widetilde{S_{h}}]_{(pp),n_{h}}.$$
(16)

The term  $\varepsilon$  in the denominator of Eq. (15) is selected to be positive and very small (generally set to the default MATLAB epsilon value) and is intended to avoid possible division by zero, and p is an index for the endmember spectra.<sup>23</sup>

• Update rule (given in scalar form) of matrix  $\widetilde{S_{hb}}$  using  $\widetilde{S_{ha}}$ :

$$[\widetilde{S_{hb}}]_{(jp),n_h} \leftarrow [\widetilde{S_{ha}}]_{j,n_h} \times [\widetilde{S_{ha}}]_{p,n_h}.$$
(17)

• Update rule (given in matrix form) of matrix  $\widetilde{A_h}$ :

$$\widetilde{A_h} \leftarrow \widetilde{A_h} \bigcirc [(X_h \widetilde{S_h^{\mathsf{T}}}) \oslash (\widetilde{A_h} \widetilde{S_h} \widetilde{S_h^{\mathsf{T}}} + \varepsilon)],$$
(18)

where  $\oslash$  denotes element-wise division.

• Constraint on the linear part of matrix  $A_h$ :

$$[\widetilde{a_{h_1}}(i_h)\dots\widetilde{a_{h_M}}(i_h)] \leftarrow [\widetilde{a_{h_1}}(i_h)\dots\widetilde{a_{h_M}}(i_h)] / \sum_{j=1}^M \widetilde{a_{h_j}}(i_h).$$
(19)

• Constraint on the quadratic part of matrix  $\widetilde{A_h}$ 

$$\widetilde{a_{h_{j,l}}}(i_h) \leftarrow \min\{0.5, \widetilde{a_{h_{j,l}}}(i_h)\}.$$
(20)

The multispectral variables are updated in the next loop (i.e., the second loop) by optimizing the criterion  $J_2$ . This second loop uses the same update rules and constraints as above, except that they are applied to the multispectral data, instead of the hyperspectral ones. The resulting algorithm is, therefore, the same as Eqs. (15)–(20), except that the matrix indices "*h*" are replaced by "*m*."

For each loop, the updating process is stopped when the number of iterations reaches a predefined maximum value.

The second proposed method, i.e., CHMF-LQNMF, can be seen as an extension of the coupled NMF (CNMF) one<sup>9,11</sup> for a linear-quadratic mixture, since the CNMF method is designed for a linear mixture. In this proposed method, an outer loop is used, wherein the hyper-spectral and multispectral variables are alternately updated by using two inner loops.

In the first inner loop, the hyperspectral variables are updated in order to minimize the criterion  $J_1$  by using the above-defined update rules [Eqs. (15), (17), and (18)] and the constraints [Eqs. (19) and (20)]. The spectral parts of these hyperspectral variables obtained by using the update rules [Eqs. (15) and (17)] are then spectrally downsampled (by averaging the samples of the latter spectra over the wavelength domains considered in the multispectral image) and injected as the initialization of the spectra used in the second inner loop. In this loop, the multispectral variables are updated in order to minimize criterion  $J_2$  by using the above-defined counterpart of Eqs. (15)–(20) intended for the multispectral variables.

The optimized spatial parts of these multispectral variables obtained by using the multispectral counterpart of Eqs. (18)–(20) are then spatially downsampled (by means of the *k*-nearest-neighbors interpolation technique) and used, in the next iteration of the outer loop, as the initialization of the first inner loop. These alternative unmixing processes are stopped when the numbers of iterations, of the outer and inner loops, reach predefined maximum values.

The JHMF-LQNMF method can also be seen as an extension of the joint NMF (JNMF) one<sup>10</sup> for a linear-quadratic mixture. This third proposed method uses only one loop, in which the considered hyper/multispectral variables are jointly updated by using, at each iteration of the loop, the hyperspectral and multispectral update rules of the considered multiplicative LQNMF algorithm. The complete algorithm of this third proposed method includes two additional rules (in addition to Eqs. (15)–(20) and their multispectral counterpart), which are

$$A_h \leftarrow (1 - \alpha)A_h + \alpha A_{dm}, \tag{21}$$

$$\widetilde{A_m} \leftarrow (1 - \alpha)\widetilde{A_m} + \alpha \widetilde{A_{uh}}, \tag{22}$$

where the matrix  $\widetilde{A_{dm}}$  (respectively  $\widetilde{A_{uh}}$ ) represents the spatially downsampled (respectively upsampled) version of  $\widetilde{A_m}$  (respectively  $\widetilde{A_h}$ ) obtained by means of the *k*-nearest-neighbors

interpolation technique. These additional update rules force the low- and high-spatial resolution linear and quadratic fraction maps to be updated in a consistent manner by using a small positive parameter  $\alpha \in ]0, 1[$ . In this third proposed method, the stop condition used is also the maximum number of iterations.

#### 3.3 Fusion Stage

This stage consists of deriving the unobservable multisharpened high-spectral/high-spatial resolution hyperspectral image by using Eq. (10). This stage is common to the three designed methods.

#### 4 Test Results

Various experiments, based on synthetic and real images, have been conducted to assess the performance of the designed methods and compare them to those of two linear NMF-based methods from the literature.

#### 4.1 Tested Data

#### 4.1.1 Synthetic data

Two sets of eight hyperspectral endmember spectra are chosen from spectral libraries, with measurements performed from 0.4 to 2.5  $\mu$ m. The first set contains eight randomly selected spectra from the spectral library compiled by the United States Geological Survey (USGS).<sup>31</sup> The second set contains eight spectra of materials used in urban areas, selected from the spectral library compiled by the Johns Hopkins University (JHU).<sup>32</sup> From these two sets of hyperspectral spectra, two other sets of eight multispectral endmember spectra are created by averaging the samples of the latter hyperspectral spectra over the wavelength domains considered in the Landsat TM bands 1 to 5 and 7 (i.e., the 0.45 to 0.52, 0.52 to 0.60, 0.63 to 0.69, 0.76 to 0.90, 1.55 to 1.75, and 2.08 to 2.35  $\mu$ m domains).

After that, these hyperspectral and multispectral spectra are used to produce two sets of synthetic images. Each set of images corresponds to one of the above two sets of (hyperspectral and multispectral) spectra and contains two subsets of images. The first subset of images is created according to the mathematical linear mixing model, and the second subset is generated according to the linear-quadratic mixing model. Note here that the linear and linear-quadratic mixing models are considered to assess the performance of the used linear and linear-quadratic methods when these methods are applied without any information about the nature of data mixture.

Each subset of the first set contains a  $200 \times 200$ -pixel high-spectral/high-spatial resolution hyperspectral image (used as a reference image), a  $100 \times 100$ -pixel high-spectral/low-spatial resolution hyperspectral image and, a  $200 \times 200$ -pixel high-spatial/low-spectral resolution multispectral image. In the second set, each subset contains the same  $200 \times 200$ -pixel high-spectral/ high-spatial resolution reference hyperspectral image, a  $50 \times 50$ -pixel high-spectral/low-spatial resolution hyperspectral image, and a  $200 \times 200$ -pixel high-spatial/low-spectral resolution multispectral image (Figs. 1 and 2). Each hyperspectral image in each set is generated with 184 spectral bands, while each multispectral image is generated with 6 spectral bands.

The linear abundance fractions of high-spatial resolution hyperspectral and multispectral images are created from a real classification of land cover, with eight classes, by averaging pixel classification values over a nonoverlapping sliding  $2 \times 2$ -pixel window in order to create mixed pixels.

The linear abundance fractions of high-spectral/low-spatial resolution hyperspectral images are created by downsampling the above linear high-spatial resolution abundance fractions. The downsampling process is performed by means of the *k*-nearest-neighbors spatial interpolation technique and by considering, respectively, in the first and second sets of images, two scale factors (2 and 4) between the considered hyperspectral and multispectral images. The quadratic abundance fractions are generated from the linear ones in the same manner as described in the above initialization stage of the proposed methods.

#### 4.1.2 Real data

In this work, real data are also used. These real data (radiometrically corrected and geometrically coregistered), acquired on the same day (March 3, 2003) and at the same time, cover a part of the Oran urban area, Algeria. The high-spectral/low-spatial resolution hyperspectral image is from the Earth Observing-1 (EO-1) Hyperion sensor. This image, with 30-m spatial resolution, contains 125 spectral bands (after removing, from the original 242-spectral band data cube, the low signal-to-noise ratio spectral bands as well as the noncalibrated and overlapping spectral bands). Two pan-sharpened multispectral images are used. The first one, acquired by the Landsat enhanced thematic mapper plus (ETM+) sensor, is characterized by 6 spectral bands and 15-m spatial resolution. The second one [acquired by the EO-1 advanced land imager (ALI) sensor], with 10-m spatial resolution, contains 9 spectral bands.

#### 4.2 Performance Evaluation Criteria

For the tested synthetic data, the spectral and spatial qualities of the estimated unobservable high-spectral/high-spatial resolution multisharpened hyperspectral image are evaluated by comparing it with the reference hyperspectral image. To evaluate the spectral reconstruction quality,



**Fig. 1** Spectral band (in the 0.815-µm region) of the original high-spatial resolution hyperspectral image, low-spatial resolution hyperspectral image, estimated high-spatial resolution hyperspectral images—synthetic data: randomly selected spectra, linear-guadratic mixing model, scale factor: 4.



**Fig. 2** Spectral band (in the 0.815- $\mu$ m region) of the original high-spatial resolution hyperspectral image, low-spatial resolution hyperspectral image, estimated high-spatial resolution hyperspectral images—synthetic data: selected urban material spectra, linear-quadratic mixing model, scale factor: 4.

the spectral angle mapper (SAM)<sup>11</sup> criterion is used. This criterion is calculated between each pixel spectrum in the reference image and its analogue in the estimated image. A smaller angle indicates a better spectral reconstruction. In the spatial domain, the used criterion is the peak signal-to-noise ratio (PSNR).<sup>11</sup> This spatial criterion is calculated between each spectral band in the reference image and its analogue in the estimated multisharpened image. The higher the PSNR value, the better the spatial reconstruction quality.

For the tests with real data, a modified quality with no reference (mQNR) criterion is proposed and used for spatial–spectral reconstruction quality assessment. The standard QNR<sup>33</sup> has been modified in order to be used in the considered multisharpening processes. This modified spatial–spectral criterion reads

$$mQNR = (1 - D_{\lambda})^{\sigma} (1 - D_{s})^{\rho}, \qquad (23)$$

where  $\sigma$  and  $\rho$  are real-valued exponents (set to 1 in the conducted experiments), and  $D_{\lambda}$  and  $D_{s}$  are spectral and spatial distortion indices, respectively. The spectral index reads

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$$D_{\lambda} = \sqrt[\omega]{\frac{1}{L_{h}(L_{h}-1)} \sum_{j=1}^{L_{h}} \sum_{r=1, r\neq j}^{L_{h}} |\text{UIQI}(X_{f_{j}}, X_{f_{r}}) - \text{UIQI}(X_{h_{j}}, X_{h_{r}})|^{\omega}},$$
(24)

where  $\omega$  is a positive integer exponent (set to 1 in the conducted experiments).  $X_f$  is one spectral band of the multisharpened hyperspectral image.  $X_h$  is one spectral band of the hyperspectral image. The spatial distortion index  $D_s$  is calculated as follows: for each spectral band of the multispectral image, a spatial distortion subindex is calculated, by using the standard index defined in Ref. 33, between the considered multispectral band and hyperspectral bands that are covered by the considered multispectral range. The final spatial distortion index  $D_s$  represents the mean of the calculated subindices. These two distortion indices use the universal image quality index (UIQI) defined in Ref. 34. The range of mQNR,  $D_{\lambda}$ , and  $D_s$  is [0, 1]. A higher mQNR value indicates a higher spatial–spectral reconstruction quality. A smaller spatial (respectively spectral) distortion value indicates a better spatial (respectively spectral) reconstruction.

#### 4.3 Results and Discussion

The proposed methods are applied to the above defined data. The maximum number of iterations used in each of the two loops of the HMF-LQNMF method is set to 10. The same number is also considered in the JHMF-LQNMF method wherein the small positive joint parameter  $\alpha$  is set to 0.01. In the CHMF-LQNMF method, the maximum number of iterations of the outer (respectively inner) loop is set to 3 (respectively 10).

In these investigations, two linear-NMF-based fusion methods from the literature are also applied to the considered data. The first one is the CNMF method,<sup>9,11</sup> and the second one is the JNMF method.<sup>10</sup> The CNMF and JNMF methods are used in two scenarios. In the first one (Sc. 1), only the endmember spectra and linear abundance fractions are considered, whereas in the second scenario (Sc. 2), in addition to the endmember spectra and linear abundance fractions) are considered as new endmember spectra (respectively new linear abundance fractions). These two scenarios are considered in order to assess the performance of these two linear-based literature methods when they are applied without any information about the nature of data mixture.

The CPU used in the conducted experiments is an Intel<sup>®</sup> Core<sup>™</sup> i5 processor running at 1.80 GHz, with a memory capacity of 4 GB.

The computational costs and the means of the spectral and spatial criteria of the tested methods are given in Tables 1-4.

				Unmixing-based fusion method									
			CNMF		JN	MF	LQNMF-based						
			Sc. 1	Sc. 2	Sc. 1	Sc. 2	HMF-LQNMF	CHMF-LQNMF	JHMF-LQNMF				
Scale	Time (s)		17.53	21.38	2.19	3.24	7.39	21.53	10.58				
factor: 2	Criterion	Ideal											
	SAM (deg)	0	1.88	5.78	1.57	1.70	0.82	0.77	0.76				
	PSNR (dB)	$\infty$	34.80	25.16	30.62	28.23	42.03	37.44	42.27				
Scale factor: 4	Time (s)		6.48	15.64	2.04	3.61	6.04	10.69	8.01				
	Criterion	Ideal											
	SAM (deg)	0	3.78	5.79	3.31	3.38	0.82	0.77	0.79				
	PSNR (dB)	∞	29.10	25.14	29.43	27.20	42.05	37.38	42.21				

 Table 1
 Computational costs and means of the spectral and spatial criteria, for the randomly selected spectra, with a linear mixing model.

Note: Bold values refer to the best results.

				Unmixing-based fusion method								
			CNMF		JNMF		LQNMF-based					
			Sc. 1	Sc. 2	Sc. 1	Sc. 2	HMF-LQNMF	CHMF-LQNMF	JHMF-LQNMF			
Scale factor: 2	Time (s)		7.48	21.94	2.37	3.35	8.46	10.94	10.47			
	Criterion	Ideal										
	SAM (deg)	0	3.28	4.72	1.62	1.64	1.34	1.08	1.20			
	PSNR (dB)	$\infty$	30.88	27.69	31.96	29.40	38.76	35.80	38.82			
Scale	Time (s)		5.56	14.46	2.09	4.79	6.55	7.72	7.47			
factor: 4	Criterion	Ideal										
	SAM (deg)	0	3.31	4.92	3.06	3.04	1.20	1.05	1.14			
	PSNR (dB)	$\infty$	30.77	27.50	30.37	28.59	37.72	35.94	37.84			

 Table 2
 Computational costs and means of the spectral and spatial criteria, for the randomly selected spectra, with a linear-quadratic mixing model.

**Table 3** Computational costs and means of the spectral and spatial criteria, for the selected urban material spectra, with a linear mixing model.

				Unmixing-based fusion method								
			CNMF		JNMF		LQNMF-based					
			Sc. 1	Sc. 2	Sc. 1	Sc. 2	HMF-LQNMF	CHMF-LQNMF	JHMF-LQNMF			
Scale	Time (s)		12.12	23.61	2.16	3.24	8.52	10.95	10.58			
factor: 2	Criterion	Ideal										
	SAM (deg)	0	1.33	1.09	0.84	0.97	0.29	0.29	0.28			
	PSNR (dB)	$\infty$	37.00	38.04	33.27	26.23	45.76	41.54	45.56			
Scale factor: 4	Time (s)		6.30	18.80	2.03	2.38	6.23	7.96	8.09			
	Criterion	Ideal										
	SAM (deg)	0	1.43	1.14	1.67	1.74	0.29	0.28	0.28			
	PSNR (dB)	$\infty$	35.72	37.04	31.55	25.68	45.84	41.60	45.40			

Note: Bold values refer to the best results.

Globally, these tables show that the proposed approaches (HMF-LQNMF, CHMF-LQNMF, and JHMF-LQNMF) yield very good spatial and spectral fidelities for the estimated multisharpened hyperspectral images. These tables also show that the proposed methods highly outperform the tested linear-based literature methods (in both scenarios) in the linear and linear-quadratic configurations. It should be noted here that the superiority of the proposed methods in the linearquadratic configuration is quite expected, since these methods are designed for the linear-quadratic mixing model, which is not the case of the used linear-based literature ones. For spectra that are actually mixed according to the linear model, the performance improvement obtained with the three proposed methods intended for linear-quadratic mixtures, as compared with the two methods intended for linear mixtures, may be due to the following phenomena. In scenario 1, it may be due to the fact that the proposed linear-quadratic methods decompose the observed data over a higher number of spectra, which makes them able to approximate these data more accurately. In scenario 2, all considered methods use the same number of spectra for decomposing the

				Unmixing-based fusion method									
			CN	MF	JN	MF	LQNMF-based						
			Sc. 1	Sc. 2	Sc. 1	Sc. 2	HMF-LQNMF	CHMF-LQNMF	JHMF-LQNMF				
Scale factor: 2	Time (s)		11.46	24.42	2.22	3.81	7.77	10.78	10.51				
	Criterion	Ideal											
	SAM (deg)	0	1.30	1.07	1.02	1.06	0.71	0.33	0.71				
	PSNR (dB)	$\infty$	36.99	38.02	31.94	25.60	35.39	39.94	35.39				
Scale factor: 4	Time (s)		6.61	19.51	1.93	2.82	6.33	8.58	7.93				
	Criterion	Ideal											
	SAM (deg)	0	1.35	1.11	1.83	1.89	0.37	0.32	0.37				
	PSNR (dB)	$\infty$	36.04	37.21	31.24	25.57	41.63	39.79	41.24				

**Table 4** Computational costs and means of the spectral and spatial criteria, for the selected urban material spectra, with a linear-quadratic mixing model.



**Fig. 3** Histogram of the SAM criterion (deg) and PSNR criterion values (dB)—synthetic data: randomly selected spectra, linear-quadratic mixing model, scale factor: 4.



**Fig. 4** Histogram of the SAM criterion (deg) and PSNR criterion values (dB)—synthetic data: selected urban material spectra, linear-quadratic mixing model, scale factor: 4.

			Unmixing-based fusion method											
		CNMF		JN	MF	LQNMF-based								
		Sc. 1	Sc. 2	Sc. 1	Sc. 2	HMF-LQNMF	CHMF-LQNMF	JHMF-LQNMF						
Time (s)		8.32	33.24	1.92	4.27	6.35	14.11	13.73						
Criterion	Ideal													
$D_{\lambda}$	0	0.11	0.13	0.10	0.09	0.13	0.05	0.14						
$D_s$	0	0.60	0.59	0.59	0.59	0.27	0.32	0.29						
mQNR	1	0.35	0.35	0.37	0.38	0.64	0.64	0.60						

**Table 5** Values of the computational costs and of the evaluation criteria for the real data: EO-1Hyperion with Landsat ETM+.

			Unmixing-based fusion method										
		CNMF		JNMF		LQNMF-based							
		Sc. 1	Sc. 2	Sc. 1	Sc. 2	HMF-LQNMF	CHMF-LQNMF	JHMF-LQNMF					
Time (s)		11.83	42.37	2.98	7.76	10.84	26.23	25.94					
Criterion	Ideal												
$D_{\lambda}$	0	0.23	0.25	0.23	0.24	0.24	0.07	0.24					
$D_s$	0	0.71	0.70	0.69	0.69	0.44	0.48	0.45					
mQNR	1	0.23	0.22	0.24	0.23	0.43	0.48	0.42					

**Table 6** Values of the computational costs and of the evaluation criteria for the real data: EO-1Hyperion with EO-1 ALI.



**Fig. 5** Spectral band for real data (EO-1 Hyperion and EO-1 Landsat ETM+), in the 0.815- $\mu$ m region, of the low-spatial resolution hyperspectral image and the estimated (by all the tested methods) high-spatial resolution hyperspectral images.

observed data, but the spectra used in the proposed linear-quadratic methods are more constrained (and constrained in a way that is compatible with the actual nature of the observed data). This constraint may help these iterative algorithms reach a better solution (it should be remembered that standard NMF methods yield many spurious solutions). In addition,



**Fig. 6** Spectral band for real data (EO-1 Hyperion and EO-1 ALI), in the 0.815- $\mu$ m region, of the low-spatial resolution hyperspectral image and the estimated (by all the tested methods) high-spatial resolution hyperspectral images.

these tables prove that the results obtained when the scale factor is 2 between the spatial resolutions of the considered images are most often better than those obtained when the scale factor is 4. This is expected: the lower the scale factor between the spatial resolutions of the hyperspectral and multispectral data, the better the fusion results.

Also, considering the linear-quadratic configuration, and when the scale factor is 4 between the spatial resolutions of the hyperspectral and multispectral data, Figs. 1 and 2 show the spectral band, in the 0.815- $\mu$ m region,<sup>11</sup> of the original high-spatial resolution hyperspectral image, the low-spatial resolution hyperspectral image, and the obtained (by all the tested methods) high-spatial resolution hyperspectral images.

From Figs. 1 and 2, one may hardly see any differences between the spectral bands of the original and estimated images with a visual inspection. Therefore, for a better discrimination between the tested methods (proposed methods and NMF-based literature ones considering the first scenario), Figs. 3 and 4 show the histogram over all pixels of the SAM criterion, and the PSNR criterion values for all hyperspectral wavelength regions, for the used synthetic data in the linear-quadratic mixture configuration, and when the scale factor is 4.

From these figures, it is clear that the proposed methods globally yield less spectral and spatial distortion than the tested literature methods.

For the real data, the obtained results with all the tested methods are given in Tables 5 and 6. Note here that the number of endmembers estimated by the method described in Ref. 28 is 10.

These tables also confirm the good overall performance of the designed methods in comparison with the linear-based literature JNMF and CNMF methods. It should be noted here that the proposed methods provide better results than the methods from the literature for both  $D_s$  and  $D_\lambda$  measures. The improvement obtained for  $D_s$  may be explained by the fact that the considered real data are related to an urban environment, and therefore, with probable nonlinearities (of linear-quadratic type), in these observed data, which generate nonzero linear-quadratic abundance coefficients that represent the spatial aspect of the considered data model. Thus, taking into account these linear-quadratic coefficients may explain the improvement of the spatial distortion index  $D_s$  for the sharpened data with the three proposed linear-quadratic methods. These tables also prove that the results improve when the scale factor between the spatial resolutions of the hyperspectral and multispectral data is lower. Indeed, the results obtained with the fusion of real data from Hyperion EO-1 with Landsat ETM+ (where the above scale factor is 2) are significantly better than those obtained with the fusion of the real data from Hyperion EO-1 with ALI (where the above scale factor is 3). These tables also show that CHMF-LQNMF is the method that globally provides the best results.

Also, Figs. 5 and 6 show the spectral band for real data, in the 0.815- $\mu$ m region, of the lowspatial resolution hyperspectral image and the estimated (by all the tested methods) high-spatial resolution hyperspectral images. These figures show that our proposed methods give rather good visual results in comparison with tested literature methods, especially when considering the fusion of Hyperion EO-1 and ALI data.

Finally, it should here be noted that the given computational costs (for all conducted experiments) are only indicative, and are not used as comparison criteria. These computational costs are given so that readers may just have an idea about the execution time of each used method. Indeed, it is difficult to compare, using the computational costs, differently constructed methods (with different numbers of used loops).

#### 5 Conclusion

In this paper, new methods, called HMF-LQNMF, CHMF-LQNMF, and JHMF-LQNMF, were proposed for fusing observable high-spectral/low-spatial resolution hyperspectral and high-spatial/low-spectral resolution multispectral images. These methods, related to LQSU techniques, are based on the LQNMF multiplicative algorithm.

The proposed methods consist of first unmixing the hyperspectral and multispectral data. The obtained high resolution spectral and spatial parts of information are then recombined, according to the linear-quadratic mixing model, in order to obtain multisharpened high-spectral/high-spatial resolution hyperspectral data. In the first method, the hyperspectral and multispectral variables are independently optimized, once they have been coherently initialized. These variables are alternately updated in the second method, and jointly updated in the third method.

The proposed methods were applied to various synthetic and real data, and their performance, in spatial and spectral domains, were evaluated with established performance criteria.

Experimental results show that the proposed LQNMF-based methods yield good overall spectral and spatial fidelities for the multisharpened hyperspectral data, and significantly outperform the linear-based JNMF and CNMF multisharpening literature methods.

The proposed methods are easy to implement, and the high qualities of the obtained multisharpened data can certainly contribute to the accurate identification and classification of an observed region at a finer spatial resolution.

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