A SOURCE SEPARATION CRITERION BASED ON SIGNED NORMALIZED KURTOSIS

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Abstract

In this paper, we first consider a linear instantaneous combination of two independent source signals. We show that the cancellation of either source component in this combined signal corresponds exactly to all the extrema of given types of its normalized kurtosis, where these types depend on the signs of the source kurtosis. From this, we then derive a source separation method based on the optimization (i.e. maximization and/or minimization, depending on the nature of the sources) of the normalized kurtosis of a linear instantaneous combination of the observed mixtures of the sources. Finally, we discuss the advantages obtained by taking into account the signs of the kurtosis of the sources and system outputs.

1 Introduction

Blind source separation is becoming a classical signal processing problem, which may be summarized as follows [1]-[3]. In its standard version, n observed signals are available and they are linear instantaneous mixtures of n unknown statistically independent source signals. The mixture coefficients are also unknown. The goal is then to restore the source signals. This is achieved by designing a separating system which recombines the observed signals in a linear instantaneous way, with coefficients which are most often adapted so as to let the outputs of this system become independent. Various practical (approximate) independence criteria have been proposed, based on this principle. Especially, several methods consist in optimizing combinations of higher-order moments or cumulants associated with the outputs of the separating system. Among them, we here consider approaches based on a criterion which concerns a moment or cumulant specific to each considered output (see e.g [1],[2]). This criterion is most often optimized under a constraint [1],[2], which may lead to numerical instability problems [3]. In this paper, we propose a different method, which is based on the optimization of the normalized kurtosis of the considered system output. To ease our analysis, we introduce a two-step approach, which makes it possible to split the overall problem into two simpler sub-problems.

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2 Normalized kurtosis of a superposition of signals

As a first step, we consider a signal $z(t)$ which is a linear instantaneous combination of two independent centered stationary signals $x_1(t)$ and $x_2(t)$:

$$z(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t), \tag{1}$$

where α_1 and α_2 are real constant coefficients. Denoting $k(u) = cum_4(u)/[cum_2(u)]^2$ the normalized kurtosis of any centered signal u (expressed vs. the cumulants of this signal), our calculations yield the following relationship (after simplifications) :

$$k(z) = \frac{k(x_1) + k(x_2)p^2}{(1+p)^2}, \tag{2}$$

where p is defined as:

$$p = \frac{cum_2(\alpha_2 x_2)}{cum_2(\alpha_1 x_1)}. \tag{3}$$

Let us note that, due to (1), $z(t)$ only contains a component resulting from $x_1(t)$ if and only if (iff) $\alpha_2 = 0$ and therefore[1] iff $p = 0$. Similarly, $z(t)$ only depends on $x_2(t)$ iff $\alpha_1 = 0$, i.e. iff $p = +\infty$. We here investigate the influence of the coefficients α_1 and α_2 on $k(z)$ for fixed (but unknown) signals $x_1(t)$ and $x_2(t)$. Due to (2)-(3), this influence is completely taken into account by the corresponding value of the variable p that we introduced. We therefore study the variations of $k(z)$ vs. p. Our calculations show that they depend on the signs of $k(x_1)$ and $k(x_2)$. In the case when $k(x_1) > 0$ and $k(x_2) > 0$, they are defined by Table 1. The values of p which are of interest to us are the ones for which $z(t)$ only depends on <u>one</u> of the signals $x_1(t)$ and $x_2(t)$, i.e. $p = 0$ and $p = +\infty$ as shown above. Table 1 shows that these values are exactly the ones which maximize $k(z)$ in the considered case. The other cases lead to the following results[2]. If $k(x_1) < 0$ and $k(x_2) < 0$, $k(z)$ increases when p is increased from 0 to p_s and then decreases when p is increased from p_s to $+\infty$. If $k(x_1) < 0$ and $k(x_2) > 0$, $k(z)$ is an increasing function of p. Finally, if $k(x_1) > 0$ and $k(x_2) < 0$, $k(z)$ is a decreasing function of p. In each case, the values of p of interest therefore correspond exactly to all the extrema of $k(z)$ of given types (i.e. maxima and/or minima), where these types depend on the considered case. This property suggests to use the optimization (i.e. maximization and/or minimization, depending on the nature of the sources) of the normalized kurtosis of a linear instantaneous combination of the observed mixtures of the sources as a source separation criterion (note that this criterion contains no normalization constraint on the magnitude of the weighing coefficients nor on the estimated sources, and does not require to first whiten these sources). However, this idea requires to be further refined, because the final separation criterion should be based on the variations of the normalized kurtosis of this signal vs. a parameter c which is controlled in the actual separating system (and not only on the variations vs. p). This refinement of the proposed approach requires to define more precisely the considered mixing conditions and separating system. This corresponds to the second step of our investigation, which will now be presented.

[1] $x_1(t)$ and $x_2(t)$ are assumed to have non zero average power.

[2] The cases when $k(x_1) = 0$ or $k(x_2) = 0$ lead to very similar results and are omitted here and in the next section for the sake of brevity.

p	0		p_s		$+\infty$
$\frac{dk(z)}{dp}$		< 0	0	> 0	0
$k(z)$	$k(x_1)$	$\searrow$	k_s	$\nearrow$	$k(x_2)$

Table 1: Variations of $k(z)$ vs. p, in the case: $k(x_1) > 0$ and $k(x_2) > 0$. Notations: 1) $p_s = k(x_1)/k(x_2)$, 2) $k_s = k(z)$ for $p = p_s$.

3 Application to a separating system

We assume that the following observed signals are available:

$$y_1(t) = a_{11}x_1(t) + a_{12}x_2(t) \tag{4}$$
$$y_2(t) = a_{21}x_1(t) + a_{22}x_2(t), \tag{5}$$

where x_i are the source signals and a_{ij} are the real constant mixture coefficients. The basic unit of the separating system that we propose then provides the signal:

$$s(t) = y_1(t) - cy_2(t), \tag{6}$$

where c is a tunable real coefficient. The values of c which are of interest to us are those for which $s(t)$ only depends on one of the signals $x_1(t)$ and $x_2(t)$, i.e. resp. $c = c_2 = a_{12}/a_{22}$ and $c = c_1 = a_{11}/a_{21}$. We then have to investigate the variations of $k(s)$ vs. c. This analysis is simplified by taking advantage of the preliminary calculations that we presented above. To this end, we split the variations of $k(s)$ into two aspects:

$$\frac{dk(s)}{dc} = \frac{dk(s)}{dp}\frac{dp}{dc}. \tag{7}$$

$\frac{dk(s)}{dp}$ was studied above (z here corresponds to s). We therefore only have to : i) determine the expression of p vs. c, which is specific to the considered separating system (to this end, combine (4)-(6) and identify the resulting expression with those of the first step of this analysis), ii) study $\frac{dp}{dc}$, iii) combine these results with those of the first step. Taking into account Table 1, we thus get Table 2. This table shows that the maxima of $k(s)$ for a finite c exactly correspond to the values c_1 and c_2, i.e. to the two solutions of the source separation problem, in the considered case (this also holds if $p_s > p_\infty$). This allows us to introduce a new source separation criterion, which consists in maximizing the normalized kurtosis of the output signal of the considered system (for a finite c; this is not repeated for the other cases hereafter). Similar criteria are derived for the other cases, based on the following results. If $k(x_1) < 0$ and $k(x_2) < 0$ the two source separation solutions correspond to the two minima of $k(z)$. In the case of a negative-kurtosis source and a positive-kurtosis one, their extractions respectively correspond to the only maximum and the only minimum of $k(z)$. The latter result is of importance because some practical applications require to extract a positive-kurtosis useful signal from mixtures of it together with a negative-kurtosis interfering signal ("noise"). In such a situation, many traditional source separation methods cannot take advantage of that a priori knowledge and therefore

c	$-\infty$			c_1			c_{21}		c_2		c_{22}		$+\infty$
$\frac{dp}{dc}$		>0		$\Vert$			<0		0		>0		
p	p_∞	$\nearrow$	$+\infty$		$+\infty$		$\searrow$		0		$\nearrow$		p_∞
$\frac{dk(s)}{dc}$		>0		$\Vert$		<0	0	>0	0	<0	0	>0	
$k(s)$		$\nearrow$		$k(x_2)$		$\searrow$		$\nearrow$	$k(x_1)$	$\searrow$		$\nearrow$	

Table 2: Variations of $k(s)$ vs. c, in the case: $k(x_1) > 0$, $k(x_2) > 0$ and $p_s \leq p_\infty$ (and $c_2 > c_1$, but one can always get in this case by renumbering the sources). Notations: 1) c_{21} and c_{22} are the two values of c surrounding c_2 and such that $p = p_s$, 2) p_∞ is the value of p for $c = \pm\infty$.

have to extract both signals, leaving it up to the designer to decide which output of the separating system provides the signal of interest. On the contrary, by maximizing $k(z)$ the proposed approach then only extracts a single signal (thus reducing computational load) and guarantees that this is the signal of interest. Moreover, if both signals are now of interest, the proposed method can extract them in parallel by adapting two above-defined basic units resp. so as to maximize and minimize $k(z)$. This is to be contrasted with the deflation approach which is often used in the related methods mentioned in Section 1: in the latter approach, the sources are extracted serially, which may decrease convergence speed and accuracy.

4 Conclusions

In this paper, we derived a source separation criterion based on the optimization of the (signed[3]) normalized kurtosis (and which does not require any whitening nor any coefficient or source normalization constraints). This approach may be seen as a generalization of the classical average power minimization criterion which only applies to the case when a reference (i.e. unmixed) signal is available. Our future work will esp. concern extensions of the proposed approach to more than two sources and convolutive mixtures.

References

[1] N. Delfosse, P. Loubaton, "Adaptive blind separation of independent sources: a deflation approach", Signal Processing, vol. 45, no. 1, pp. 59-84, July 1995.

[2] S. C. Douglas, S.-Y. Kung, "An ordered-rotation KuicNet algorithm for separating arbitrarily-distributed sources", Proceedings of the International Workshop on Independent Component Analysis and Blind Signal Separation (ICA'99), pp. 81-86, Aussois, France, Jan. 11-15, 1999.

[3] S. C. Douglas, S.-Y. Kung, S. Amari, "On the numerical stabilities of principal, minor, and independent component analysis algorithms", Proceedings of the International Workshop on Independent Component Analysis and Blind Signal Separation (ICA'99), pp. 419-424, Aussois, France, Jan. 11-15, 1999.

[3] After this paper was accepted, we discovered that J. K. Tugnait recently published papers about a related approach. However, that approach uses the absolute value of the normalized kurtosis and cannot therefore take advantage of the features related to the signs of the kurtosis discussed in Section 3.