Convergence properties of Cichocki's extension of the Hérault-Jutten source separation neural network

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Abstract

An extended source separation neural network was recently derived by Cichocki et al. [1] from the classical Hérault-Jutten network. It was claimed to have several advantages, but its convergence properties were not described. In this paper, we exhaustively define the equilibrium points of the standard version of this network and analyze their stability. We prove that the stationary independent sources that this network can separate are the globally sub-gaussian signals. As the Hérault-Jutten network applies to the same sources, we show that the advantages of the new network are not counterbalanced by a reduced field of application, which confirms its attractiveness.

1 Introduction

Blind source separation is a generic signal (and data) processing technique, which applies e.g. to antenna or microphone array processing [2]. In its "simplest configuration", two sensors provide measured signals $x_1(t)$ and $x_2(t)$, which are unknown linear instantaneous mixtures of two unknown independent source signals $s_1(t)$ and

$s_2(t)$, i.e. (see Fig. 1):

$$x_1(t) = a_{11}s_1(t) + a_{12}s_2(t) \quad (1)$$

$$x_2(t) = a_{21}s_1(t) + a_{22}s_2(t), \quad (2)$$

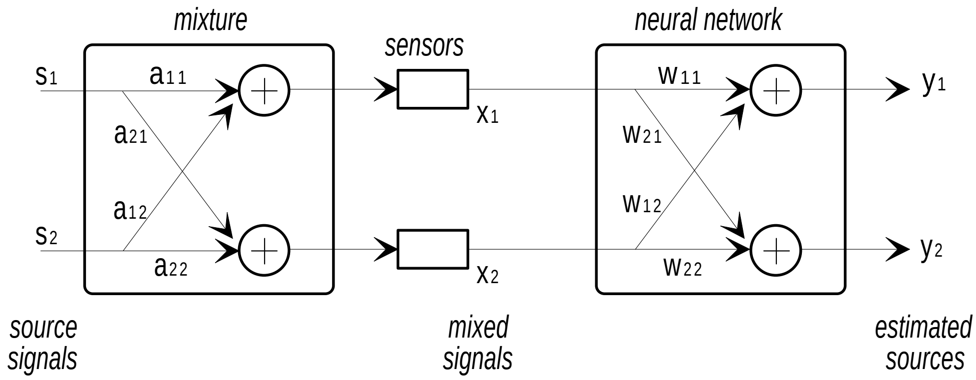

Figure 1: Basic source separation configuration and direct Cichocki network.

where the terms a_{ij} are unknown real-valued constant (non-zero[1]) mixture coefficients such that[2] $a_{11}a_{22} - a_{12}a_{21} \neq 0$. The problem is then to estimate the source signals $s_j(t)$ from the measured signals $x_i(t)$, up to a permutation and scaling factor.

Hérault and Jutten are considered as having proposed the first solution to this problem (see e.g. [2],[3]). Their approach is based on a recurrent artificial neural network. This network was introduced about a decade ago and its convergence properties were analytically studied a few years later. Several papers were thus published by independent authors about its convergence in the "simplest configuration" defined above. Sorouchyari [4] and Fort [5] used almost the same method, which was then revisited and somewhat extended by Moreau and Macchi [6],[7]. Comon et al. [8] presented another method which yields different results. An approach bridging the gap between these two methods was then proposed by Deville [9], so that the convergence properties of the simplest versions of this network are now well defined.

Two classes of structures related to the Hérault-Jutten network were then also proposed for performing linear instantaneous source separation. On the one hand, Moreau and Macchi [6],[7],[10] introduced a direct (i.e. non-recurrent) version of

[1]The case when at least one mixture coefficient a_{ij} is zero is not detailed here for two reasons. On the one hand, it is of no importance in this paper, because it implies that at least one of the measured signals $x_1(t)$ and $x_2(t)$ is not a mixed signal, so that the initial source separation configuration reduces to a classical adaptive filtering (or gain control) problem, which should be treated with a simpler signal processing structure than the one considered here. On the other hand, not considering that useless case somewhat simplifies the stability analysis, as shown in Section 4.

[2]The latter condition corresponds to the invertibility of the mixing matrix A consisting of the mixture coefficients a_{ij}. This condition is required, otherwise the two measured signals are identical (up to a scalar factor): in the latter case, the mixing matrix cannot be inverted by using these measured signals, meaning that the source separation problem cannot be solved.

the Hérault-Jutten network, based on the same adaptation rule. This network is attractive because it avoids the matrix inversion which must be performed with the recurrent version in order to derive the outputs from the inputs and network weights. Moreau and Macchi also studied the convergence properties of this network in the "simplest configuration", using the same type of method as Sorouchyari and Fort. They also proposed and studied a mixed version of this network [6],[7].

On the other hand, Cichocki et al. [1] defined neural networks which may be considered as extensions of the above-mentioned ones. These new networks contain additional self-adaptive weights, which are updated so as to normalize the "scales" of the network outputs. These networks were claimed to be thus able to process ill-conditioned mixtures and badly-scaled source signals to which the Hérault-Jutten network would not apply. Both the direct and recurrent versions of this type of neural networks were described, and it was also proposed to cascade then in a multilayer neural network in order to improve performance. To our knowledge, the papers published up to now only describe the principles of these networks as well as their empirical performance derived from numerical simulations. On the opposite, no theoretical proof has been provided about their convergence properties. This may result from the fact that such analyses are significantly more complex than for the Hérault-Jutten and Moreau-Macchi networks as will be shown in this paper, due to the normalization scheme introduced by Cichocki in his networks.

Therefore, at the current stage, the Cichocki networks seem to be more powerful than the simpler formely proposed structures, but despite these advantages one is entitled to hesitate to use them, as their behaviour is not well defined. Especially, it is not known whether these networks have spurious stable equilibrium points, that is weight values towards which they may converge without providing separated signals at their outputs. Similarly, it is not known whether they are able to separate a limited or large class of source signals. Such restrictions have been shown to exist for the Hérault-Jutten and Moreau-Macchi networks. Similar limitations are therefore expected to arise with the Cichocki networks, and they should be determined. This paper therefore aims at precisely defining the conditions of operation of these networks. The convergence analysis needs only be performed for the single-layer versions of the Cichocki networks, because the overall properties of multilayer networks are directly derived from those of the individual layers that compose them. Moreover, only the direct version of these networks is considered hereafter, because it is more attractive than the recurrent one as explained above, and because similar results are expected for both versions, based on the similarity observed between the properties of the Hérault-Jutten and Moreau-Macchi networks.

The remainder of this paper is organized as follows. The considered source separation network is defined in Section 2. Its equilibrium points are studied in Section 3, while the stability of these points is investigated in Section 4. Section 5 presents the conclusions drawn from this investigation and outlines potential extensions.

2 Definition of the considered network

This section briefly describes the principles of the direct Cichocki network which is analyzed in the subsequent sections. This investigation is performed for the "simplest configuration" defined in Section 1, and the source signals $s_1(t)$ and $s_2(t)$ are assumed to be stationary, zero-mean and statistically independent. At each time step t, the network (shown in Fig. 1) receives both mixed signals $x_1(t)$ and $x_2(t)$ defined by (1)-(2) and processes them as follows:

1. It computes its output signals $y_1(t)$ and $y_2(t)$ corresponding to the current input signals and internal weight values w_{ij}, i.e.[3]:

$$y_1(t) = w_{11}x_1(t) + w_{12}x_2(t) \tag{3}$$
$$y_2(t) = w_{21}x_1(t) + w_{22}x_2(t). \tag{4}$$

2. It also updates its four real-valued weights w_{ij} according to the following adaptation rules:

$$\frac{dw_{ii}(t)}{dt} = -\mu[f[y_i(t)]g[y_i(t)] - 1], \quad i \in \{1,2\} \tag{5}$$
$$\frac{dw_{ij}(t)}{dt} = -\mu f[y_i(t)]g[y_j(t)], \quad i \neq j \in \{1,2\}, \tag{6}$$

   where μ is a small positive adaptation gain and f and g are distinct odd functions, called the "separating functions" below.

The principles of the adaptation rules (5) and (6) may be summarized as follows:

- Rule (6) is used for updating the cross-coupling weights w_{12} and w_{21}. It aims at making the outputs independent and thus resp. proportional to each source. It is the same rule as that of the Hérault-Jutten and Moreau-Macchi networks.

- Rule (5) is used for updating the direct weights w_{11} and w_{22}. It aims at normalizing the "scales" of the output signals $y_1(t)$ and $y_2(t)$ respectively. The scaling coefficients w_{11} and w_{22} are meant to self-adapt to the processed signals. This rule is specific to the Cichocki network (whereas in the Hérault-Jutten and Moreau-Macchi networks, w_{11} and w_{22} are fixed to 1). The above-mentioned advantages of the Cichocki network result from the adaptation of these weights.

The last parameters which must be defined in order to fully specify the considered network are the selected separating functions f and g. These parameters appear in the Cichocki, Hérault-Jutten and Moreau-Macchi networks. Various functions have been considered in the papers related to all these networks. The most commonly used set of functions is:

$$f(x) = x^3 \quad \text{and} \quad g(x) = x. \tag{7}$$

The current paper only concerns this specific set of functions.

[3]The weight values w_{ij} depend on t. For readability, this is often omitted in the notations used below.

3 Equilibrium points

Hereafter, we denote resp. A and W the 2x2 matrices with terms a_{ij} and w_{ij}. We define $P = WA$ and we denote p_{ij} its terms. (1)-(4) then yield:

$$y_1(t) = p_{11}s_1(t) + p_{12}s_2(t) \quad (8)$$

$$y_2(t) = p_{21}s_1(t) + p_{22}s_2(t). \quad (9)$$

The equilibrium points of the adaptation rules (5)-(6) of the network correspond to all the constant weight matrices W (or to the associated matrices P) such that:

$$< \frac{dw_{ij}(t)}{dt} >= 0, \quad i,j \in \{1,2\}, \quad (10)$$

where $<>$ stands for mathematical expectation. (10) may be rewritten by using (5)-(6) and by inserting (8)-(9) in the latter equations. Developing and solving these equations yields the equilibrium points defined in Table 1. The four points $F_{\epsilon_1,\epsilon_2}$ correspond to source separation without any permutation, i.e. each network output $y_i(t)$ is proportional to ($\propto$) the source having the <u>same</u> index i. The four points $G_{\epsilon_1,\epsilon_2}$ correspond to source separation with a permutation, i.e. $y_1(t) \propto s_2(t)$ and $y_2(t) \propto s_1(t)$. So, at any of these points (called the "separating equilibrium points" below), the network does achieve the source separation function defined in Section 1. On the contrary, at any of the eight points $H_{\epsilon_1,\epsilon_2,\epsilon_3}$, its outputs are mixtures of the source signals as all terms p_{ij} are different from zero, i.e. it fails to separate the sources. Therefore, this network indeed has "spurious equilibrium points", $H_{\epsilon_1,\epsilon_2,\epsilon_3}$, so that one must determine which equilibrium points are stable, or at least which types of sources can be separated by this network as a result of some stability properties of its equilibrium points.

four points: $F_{\epsilon_1,\epsilon_2}$	four points: $G_{\epsilon_1,\epsilon_2}$	eight points: $H_{\epsilon_1,\epsilon_2,\epsilon_3}$
$p_{11} = \frac{\epsilon_1}{< s_1^4 >^{1/4}}$	$p_{11} = 0$	$p_{11} = \frac{\epsilon_3}{\left[2 < s_1^4 > \left(1 + 3\frac{<s_1^2><s_2^2>}{\sqrt{<s_1^4><s_2^4>}}\right)\right]^{1/4}}$
$p_{12} = 0$	$p_{12} = \frac{\epsilon_2}{< s_2^4 >^{1/4}}$	$p_{12} = \frac{\epsilon_2 p_{11}}{R}$, with $R = \left(\frac{< s_2^4 >}{< s_1^4 >}\right)^{1/4}$
$p_{21} = 0$	$p_{21} = \frac{\epsilon_1}{< s_1^4 >^{1/4}}$	$p_{21} = \epsilon_1 \epsilon_2 p_{11}$
$p_{22} = \frac{\epsilon_2}{< s_2^4 >^{1/4}}$	$p_{22} = 0$	$p_{22} = \frac{-\epsilon_1 p_{11}}{R}$
$y_1(t) = p_{11}s_1(t)$ $y_2(t) = p_{22}s_2(t)$	$y_1(t) = p_{12}s_2(t)$ $y_2(t) = p_{21}s_1(t)$	see (8) see (9)

Table 1: Equilibrium points of the network, consisting of three subsets. Each point of a subset is defined by ϵ_1, ϵ_2 (and possibly ϵ_3), with: $\epsilon_1, \epsilon_2, \epsilon_3 \in \{-1, 1\}$.

4 Stability of the equilibrium points

The local stability of any equilibrium point E is analyzed by: i) considering the mean adaptation algorithm obtained by replacing the right side of (5)-(6) by its mathematical expectation, and ii) deriving a first-order development of this mean algorithm at point E [4]. This yields, in matrix form:

$$\frac{d\Delta V}{dt} = J\Delta V, \tag{11}$$

where J is the Jacobian matrix of the system at point E, and ΔV is the column vector defined as: $\Delta V = (\Delta w_{11}, \Delta w_{12}, \Delta w_{21}, \Delta w_{22})^T$, where each component Δw_{ij} is the difference between two values of w_{ij}, resp. at a considered point in the neighbourhood of E and at point E itself. A necessary and sufficiency condition for point E to be locally stable is (C1): the real parts of all the eigenvalues of J are negative [4]. An equivalent condition is (C2): the real parts of all the roots of $Q(\Lambda) = det(J - \Lambda I)$ are negative. For the considered network, it may be shown that $Q(\Lambda) = r^4 P(\lambda)$ with:

$$\Lambda = r\lambda \tag{12}$$

$$r = -\frac{\mu}{D} \tag{13}$$

$$D = w_{11}w_{22} - w_{12}w_{21} \tag{14}$$

$$P(\lambda) = p_4\lambda^4 + p_3\lambda^3 + p_2\lambda^2 + p_1\lambda + p_0 \tag{15}$$

$$p_4 = 1 \tag{16}$$

$$p_3 = -(w_{11} + w_{22})(3 < y_1^2 y_2^2 > +4) \tag{17}$$

$$p_2 = < y_1^2 y_2^2 >^2 9w_{11}w_{22} + < y_1^2 y_2^2 > 12(w_{11}^2 + w_{22}^2 + 2D) + 15w_{11}w_{22} \tag{18}$$

$$p_1 = -4(w_{11} + w_{22})D[9 < y_1^2 y_2^2 >^2 +12 < y_1^2 y_2^2 > -1] \tag{19}$$

$$p_0 = 16D^2[9 < y_1^2 y_2^2 >^2 -1]. \tag{20}$$

where w_{ij} are the weight values at point E. A natural method would then be to determine the real parts of the roots of $P(\lambda)$ and $Q(\Lambda)$, and to study their signs depending on the considered equilibrium point E and on $< y_1^2 y_2^2 >$ (and hence on the source statistics, due to (8)-(9)). This could be done for the Hérault-Jutten network, because its associated polynomial $P(\lambda)$ is only of order 2 and thus yields simple computations[4]. On the contrary, this method is very impractical for the 4th-order polynomial (15)-(20) that we determined for the network considered in this paper, as the expressions of its roots turn out to be very complicated. To solve this problem, we developed an original method, by focusing on specific stability properties that enable us to determine which types of sources this network is able to separate. The first step of this method is based on the following theorem.

Theorem 1: for any equilibrium point, a necessary stability condition is (C3): $< y_1^2 y_2^2 > > 1/3$.

Proof: if $P(0) < 0$, $P(\lambda)$ has at least one real positive and one real negative roots because $P(\lambda)$ is a continuous function of λ and $P(\lambda) \to +\infty$ when $\lambda \to \pm\infty$. So has $Q(\Lambda)$, whatever the sign of r. Then (C2) is not met. Similarly, if $P(0) = 0$, one of the roots of $P(\lambda)$, and therefore of $Q(\Lambda)$, is zero. Then (C2) is not met. In other words, stability requires $P(0) > 0$, and therefore requires (C3), due to the expression of $P(0)$ derived from (15)-(20).

[4] $P(\lambda)$ is of order 2 because the Hérault-Jutten network contains 2 adaptive weights (w_{11} and w_{22} are fixed to 1).

For any separating equilibrium point $F_{\epsilon_1,\epsilon_2}$ or $G_{\epsilon_1,\epsilon_2}$, if may be shown that (C3) is equivalent to: $< s_1^4 >< s_2^4 > < \ 9 < s_1^2 >^2 < s_2^2 >^2$, which is the definition of globally sub-gaussian sources. Similarly, for any spurious equilibrium point $H_{\epsilon_1,\epsilon_2,\epsilon_3}$, (C3) is equivalent to: $< s_1^4 >< s_2^4 > > \ 9 < s_1^2 >^2 < s_2^2 >^2$, which is the definition of globally super-gaussian sources. In other words, the network is not able to separate globally gaussian or super-gaussian sources (because no separating equilibrium points are stable in this case). This completes our stability analysis for such sources. As for globally sub-gaussian sources, the above discussion shows that no spurious equilibrium points are stable. Moreover, we will show below that at least one separating equilibrium point is then stable. Therefore, the network can only converge to such a point[5], and thus it does achieve source separation. The above-mentioned type of stable point is defined as follows: Theorem 2 below shows the existence of a type of point to be considered, while Theorem 3 shows its stability.

Theorem 2: Among the separating equilibrium points (i.e. $F_{\epsilon_1,\epsilon_2}$ and $G_{\epsilon_1,\epsilon_2}$), at least one is such that $w_{11} > 0$, $w_{22} > 0$ and $D > 0$.

Proof : This directly results from the expressions of the weight matrix at these points, i.e. $W = PA^{-1}$, where the terms p_{ij} of P are provided in Table 1 (this uses the above assumption: all $a_{ij} \neq 0$).

Theorem 3: For globally sub-gaussian sources, if a separating equilibrium point (i.e. $F_{\epsilon_1,\epsilon_2}$ or $G_{\epsilon_1,\epsilon_2}$) is such that $w_{11} > 0$, $w_{22} > 0$ and $D > 0$, then it is stable.

Proof : As a first step, consider the specific case $w_{21} = 0$ (in addition to $w_{11} > 0$, $w_{22} > 0$ and $D > 0$). In this case, the expressions of the roots of $P(\lambda)$ are derived easily, and Theorem 3 follows directly by using (C2). As a second step, consider the general case $w_{11} > 0$, $w_{22} > 0$ and $D > 0$. It is possible to evolve continuously from the specific case to the corresponding general case by varying w_{21}, and it may be shown that the real parts of the roots of $P(\lambda)$ cannot thus reach zero. Therefore, they have the same sign in the general case as in the specific case. This also applies to $Q(\Lambda)$ because the sign of r thus remains constant. Therefore, the considered equilibrium point remains stable in the general case.

5 Conclusions and prospects

The Cichocki network has been claimed to have significant advantages over the previously published Hérault-Jutten network, but up to now its exact conditions of operation had not been described. In this paper, we have shown that the stationary independent sources that can be separated by the standard version of this network are the globally sub-gaussian signals. As the corresponding Hérault-Jutten network applies to the same sources, we have thus shown that the new features provided by the Cichocki network are not obtained at the expense of a degradation of the field of application. From this point of view, this paper confirms the attractiveness of the Cichocki network on the basis of objective criteria. Our future investigations will especially concern the convergence properties of a modified version of this network, targetted at the separation of globally super-gaussian sources.

[5] This may require an adequate initialization point and a low enough adaptation gain μ for the network to remain in the attraction domain of this point.

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