Fresnel Lenses - why not ?

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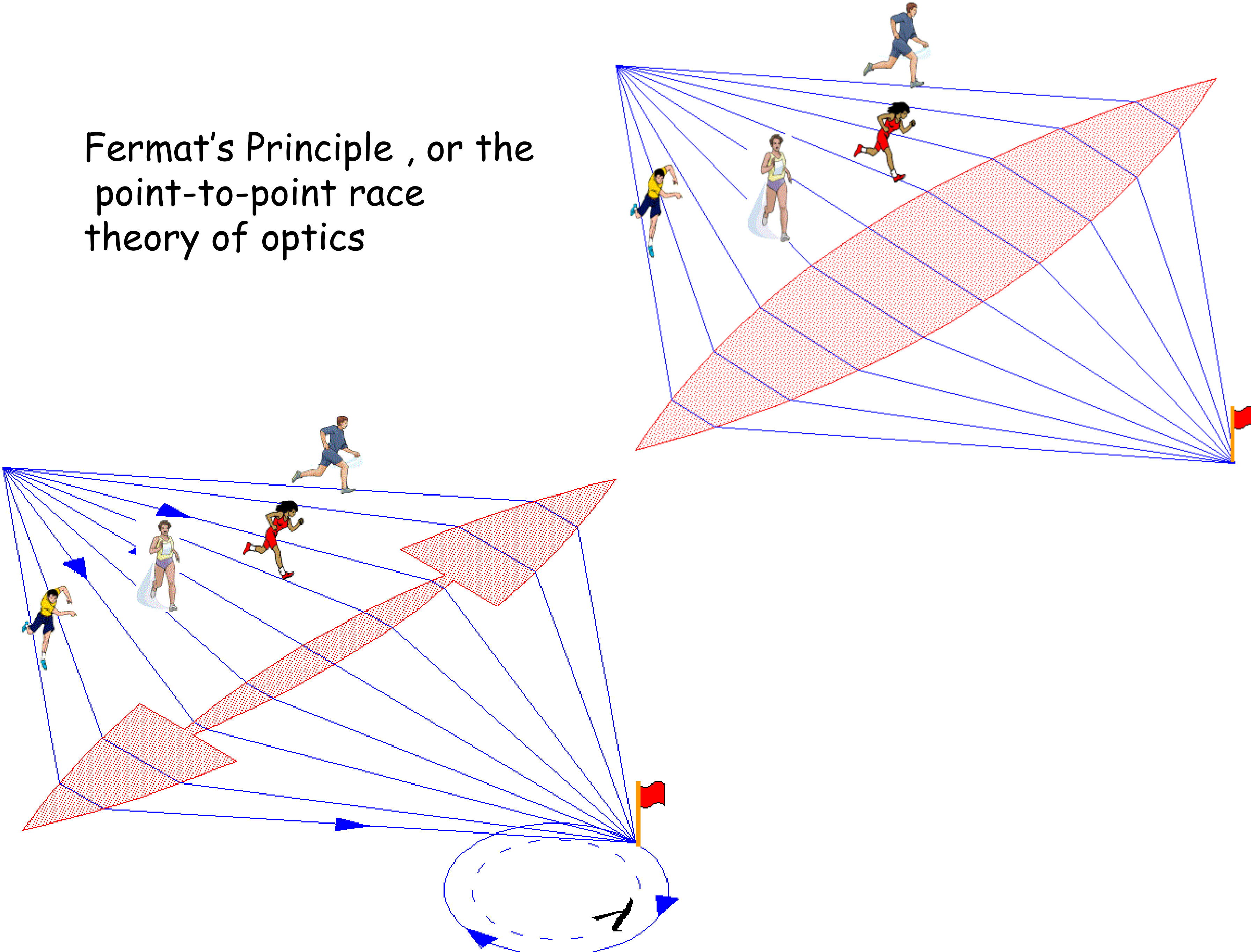
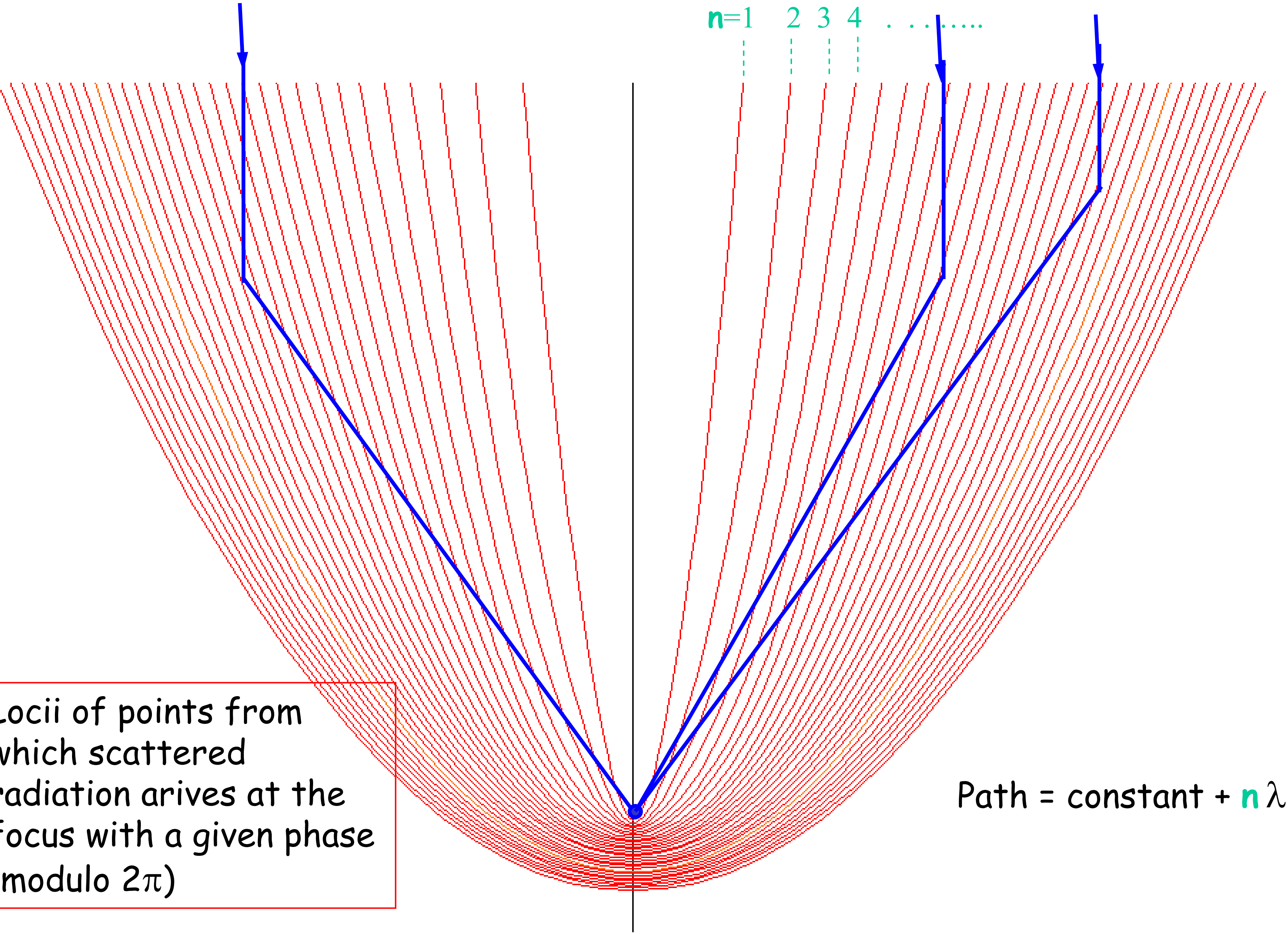
1) Review of focusing
2) Focusing by control of phase
3) What does a Fresnel lens for astronomy look like
4) Pros and cons
5) Overcoming the cons to take advantage of the pros

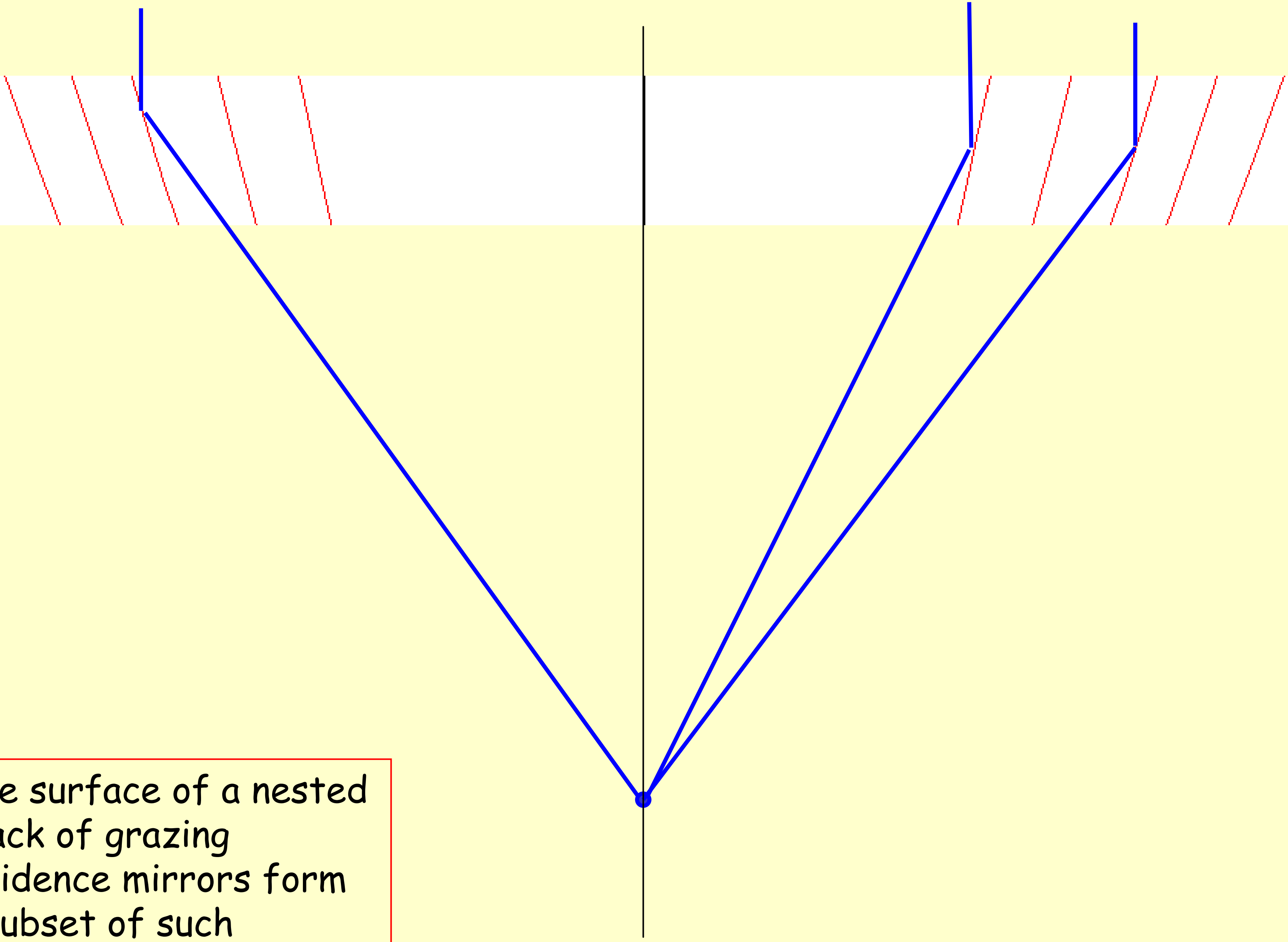
Fermat's Principle , or the point-to-point race theory of optics



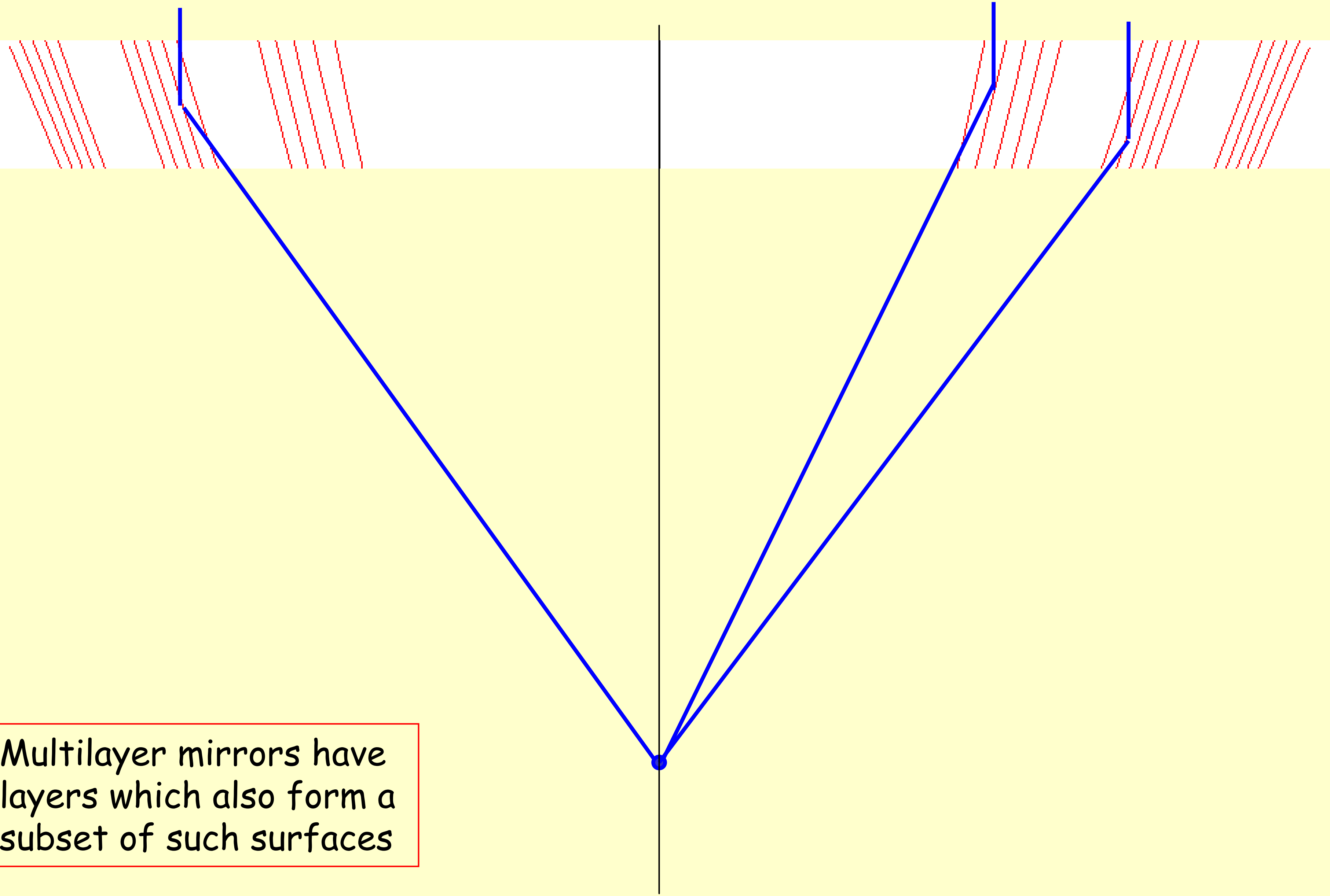


Locii of points from
which scattered
radiation arives at the
focus with a given phase
(modulo 2π)

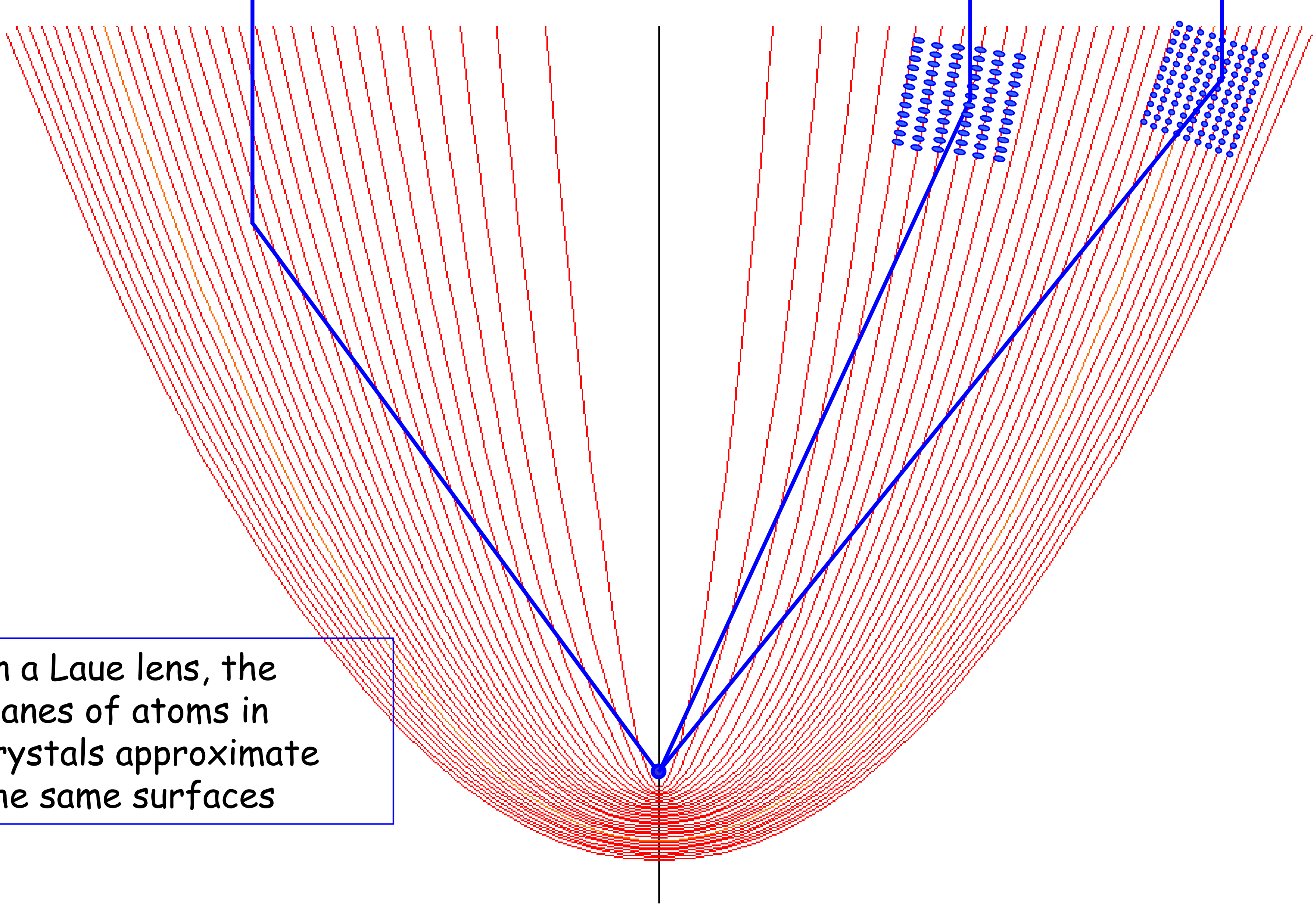
Path = constant + **n** λ



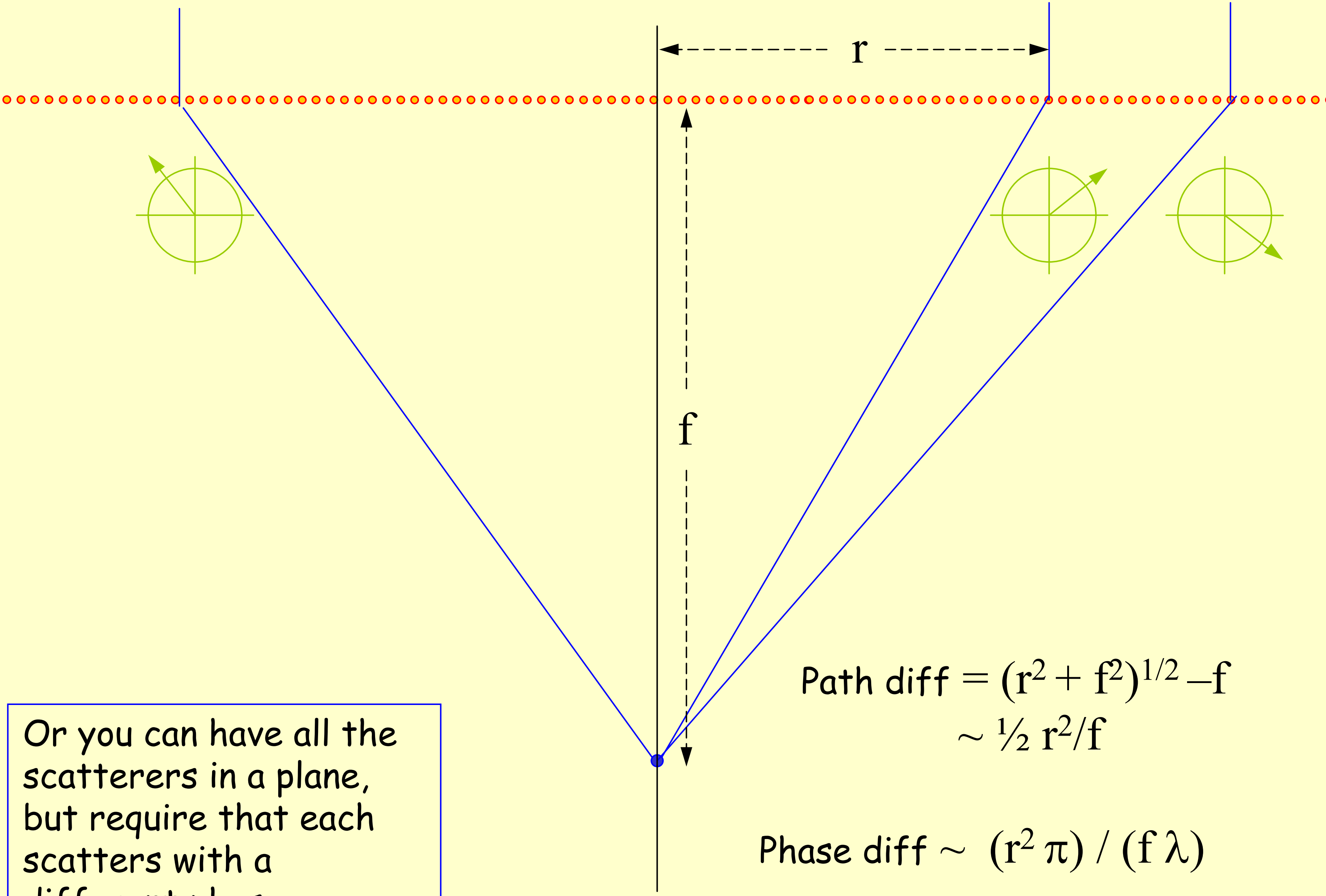
The surface of a nested stack of grazing incidence mirrors form a subset of such surfaces



Multilayer mirrors have layers which also form a subset of such surfaces



n a Laue lens, the
lanes of atoms in
rystals approximate
he same surfaces



Or you can have all the scatterers in a plane, but require that each scatters with a different phase

Path diff $= (r^2 + f^2)^{1/2} - f$

$\sim \frac{1}{2} r^2/f$

Phase diff $\sim (r^2 \pi) / (f \lambda)$

(Modulo 2π)

Real part of the refractive indices

$$\mu = (1-\delta)$$

$$\boxed{\delta \approx 2\times10^{-10}\left(\frac{\rho}{1\ gcm^{-3}}\right)\left(\frac{E}{1MeV}\right)^{-2}}$$

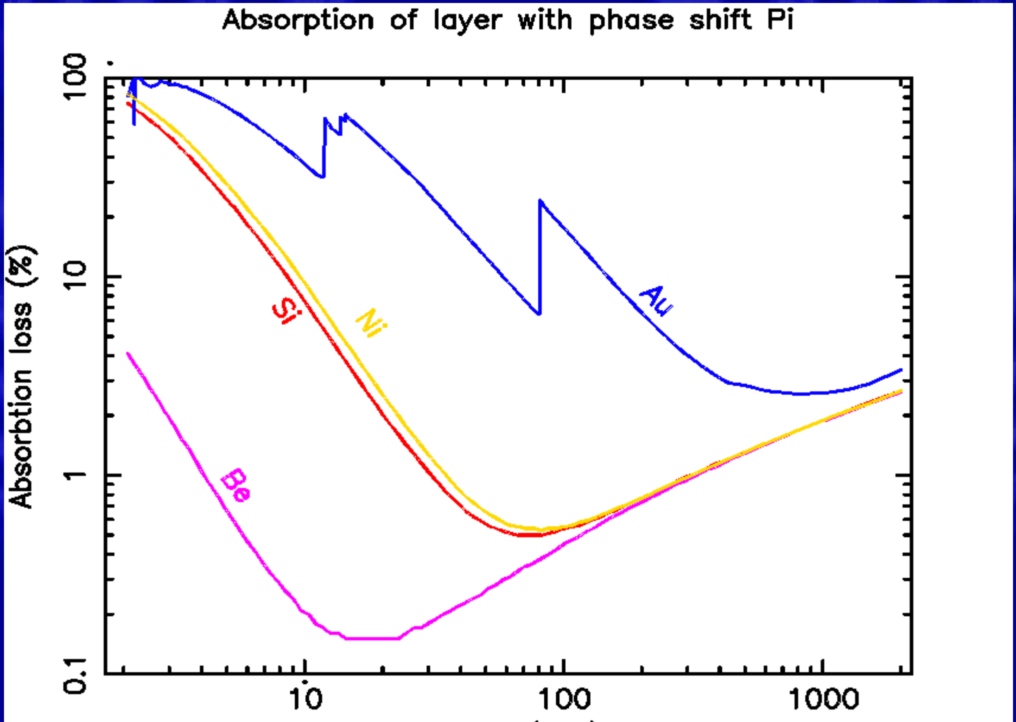
$$\boxed{\frac{\lambda}{\delta} = t_{2\pi} = \left(\frac{\rho}{1\ gcm^{-3}}\right)^{-1}\left(\frac{E}{1\ MeV}\right)6mm}$$

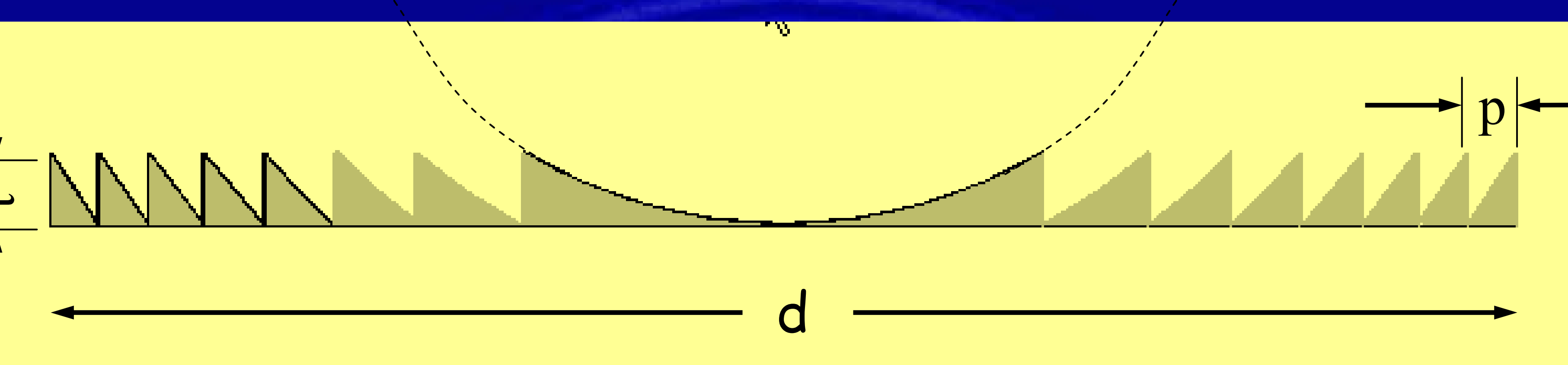
E = Energy

ρ = density

Some examples of the thickness necessary for a phase change of 2π :

	Energy	$t_{2\pi}$	Transmission
Plastic	6 keV	30 microns	96 %
Aluminium	100 keV	225 microns	99 %
Titanium	500 keV	0.7 mm	97 %





Example parameters

$d \sim$ 5 m

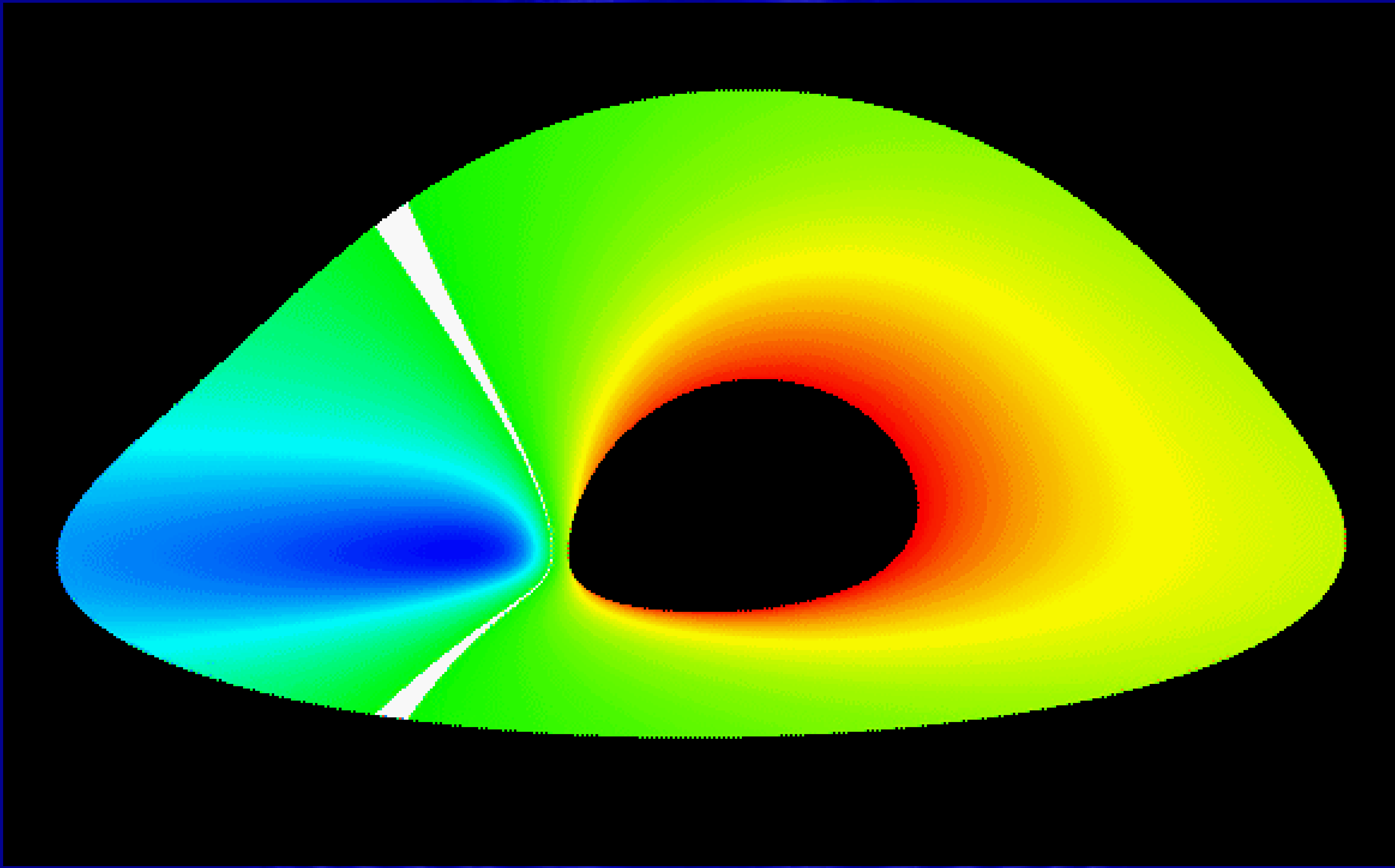
$t_{2\pi} \sim$ 1 mm (for E= 500 keV in aluminium)

$p \sim$ 1 mm

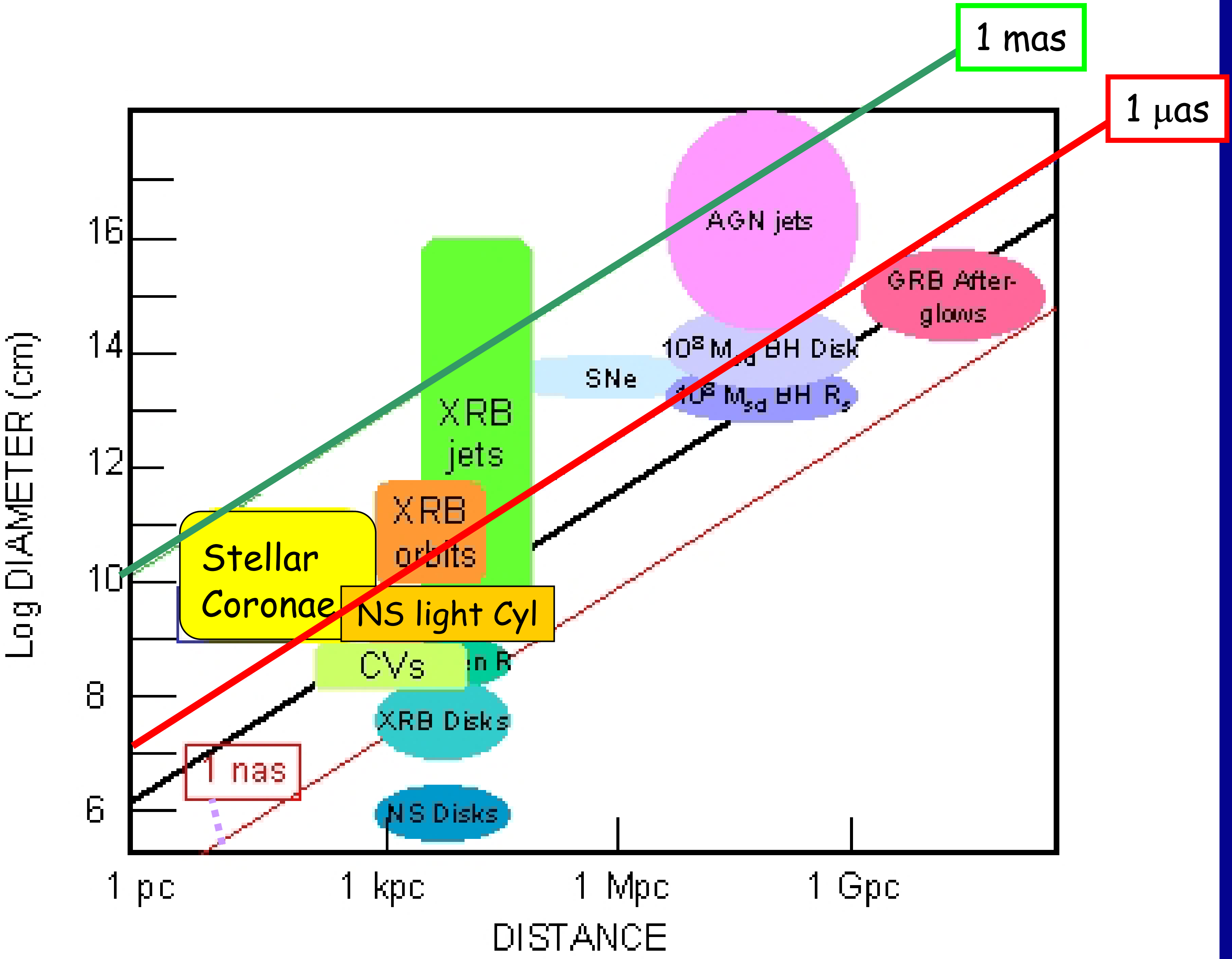
⇒ {

Effective area 15 m^2

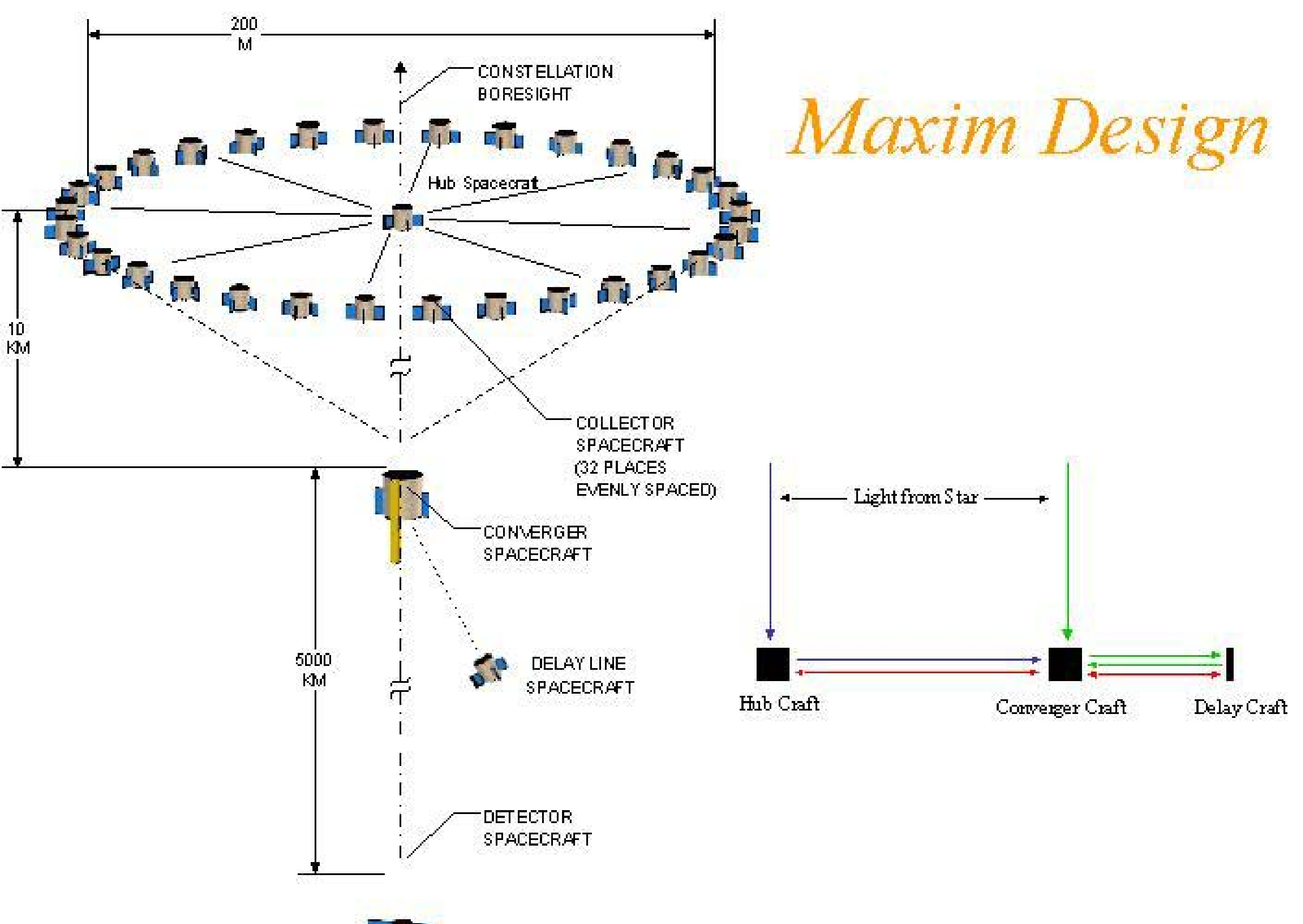
Angular resolution 0.12 micro arc sec



Accretion disk around an extreme Kerr Black Hole
Ben Bromley, CFA

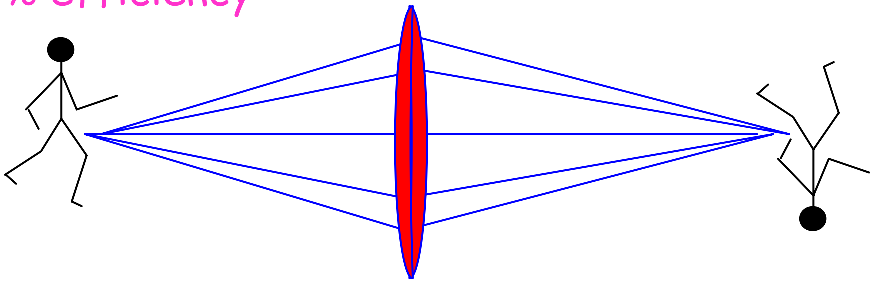

1 mas
1 μas
AGN jets
GRB After-glows
$10^8 M_{sd}$ BH Disk
SNe
$10^8 M_{sd}$ BH R_s
XRB jets
XRB orbits
Stellar Coronae
NS light Cyl
CVs
n R
XRB Disks
1 nas
NS Disks
16
14
12
10
8
6
Log DIAMETER (cm)
1 pc
1 kpc
1 Mpc
1 Gpc
DISTANCE

A « *Black Hole Imager* » is foreseen as part of NASA's 'beyond Einstein' planning. Ambitious systems are being considered for this rôle.

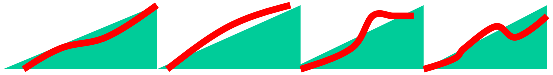


Advantages of Fresnel lenses

- At every radius in the lens, you can adjust the phase to be ideal
  → perfect lens with close to 100% efficiency

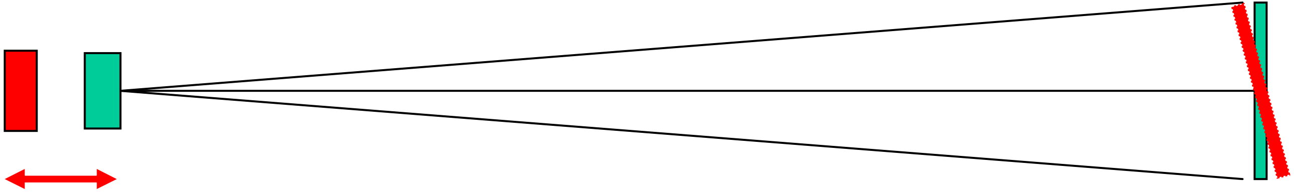


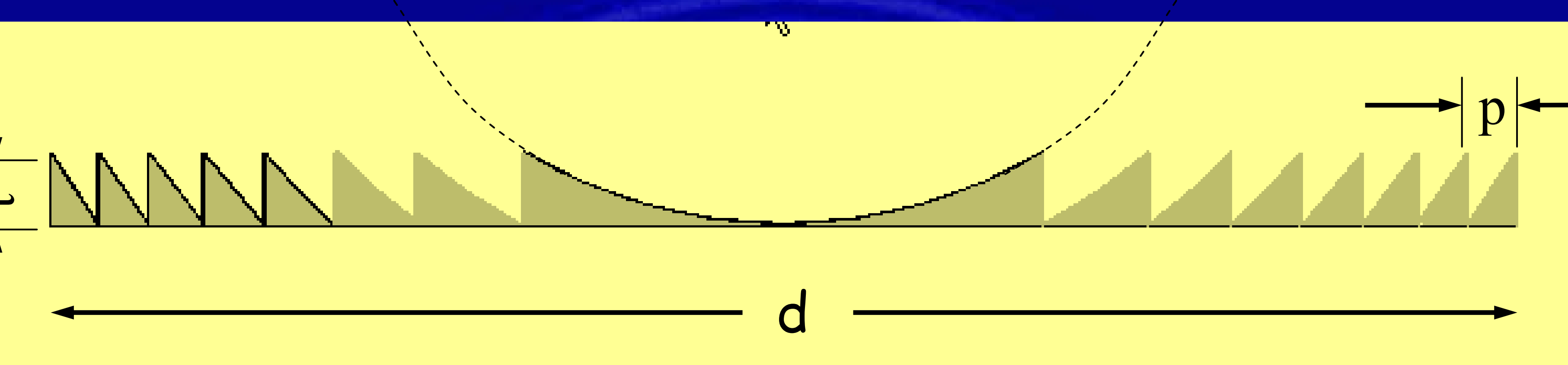
- Its a true imaging system
  Geometrical aberrations are negligible



- Because the refractive index is close to 1,
  surface profile errors have little effect
  eg at 500 keV, $\lambda/50$ optical precision ⇔ 20 micron tolerances - lens 'polishing' can be done with any decent machine tool

- It is a low f-number, 'thin', lens
  Lens 'tilt' is relatively unimportant (*eg* 1° tolerance)
  Focal depth is very large





Example parameters

$d \sim$ 5 m

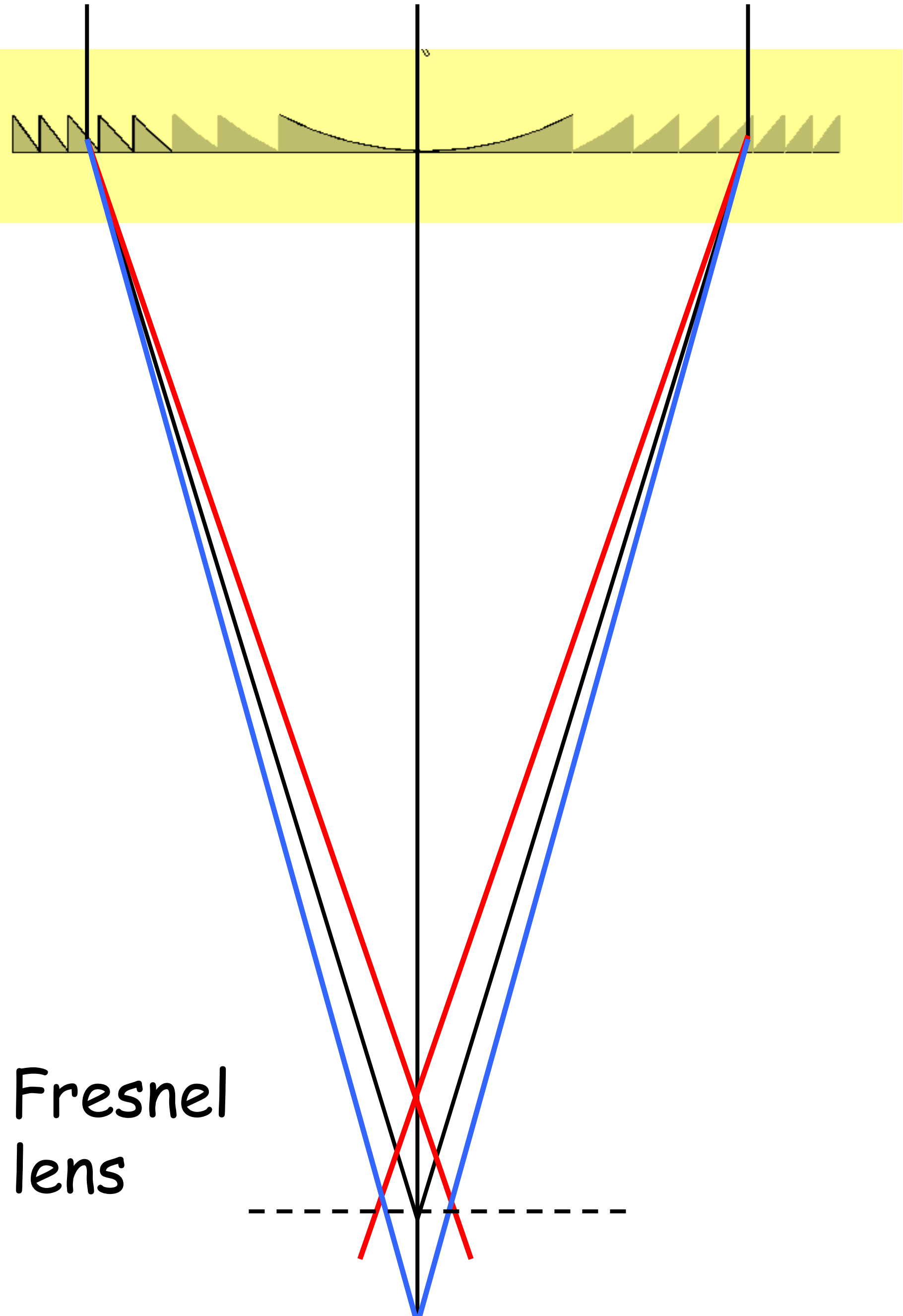
$t_{2\pi} \sim$ 1 mm (for E= 500 keV in aluminium)

$p \sim$ 1 mm

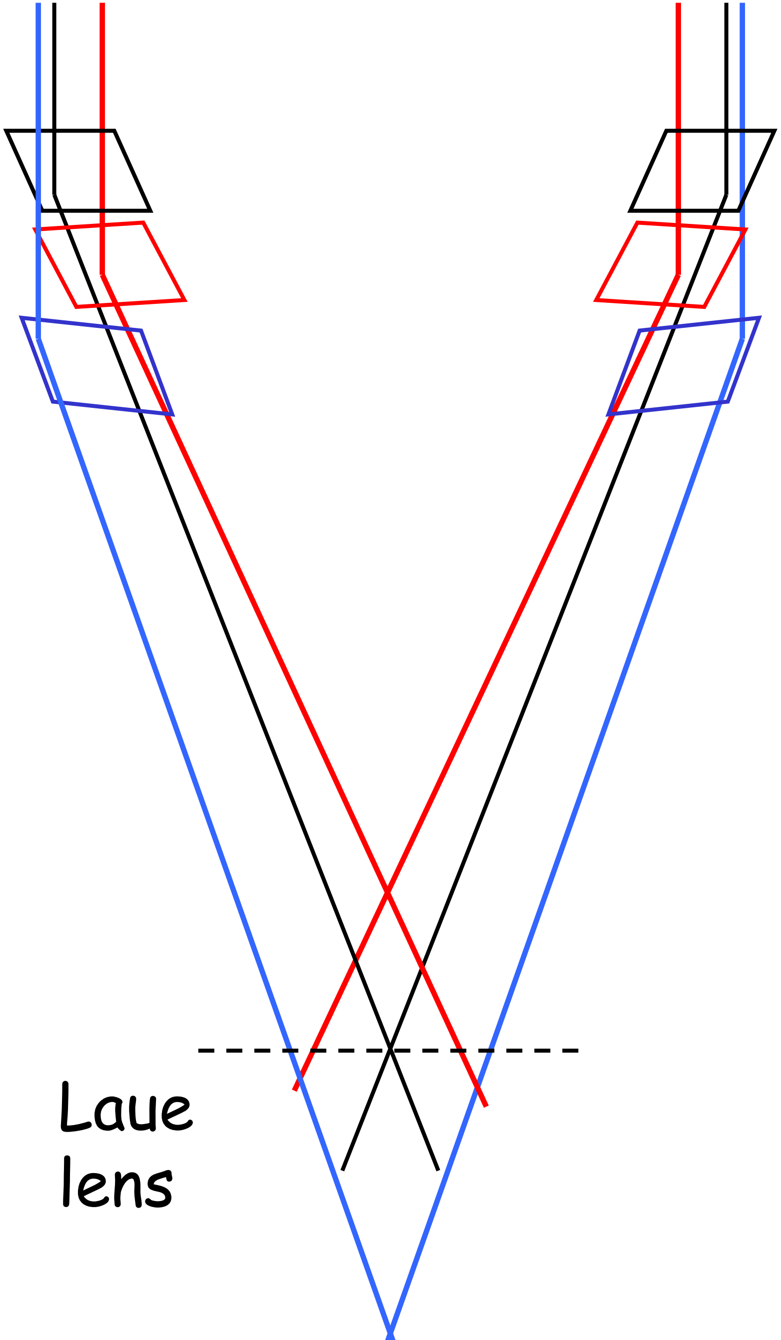
Effective area 15 m^2

Angular resolution 0.12 micro arc sec

$$\left\{ \begin{array}{l} \Delta E \sim 0.1 \text{ keV} \\ f \sim 10^6 \text{ km} \end{array} \right.$$



Fresnel lens

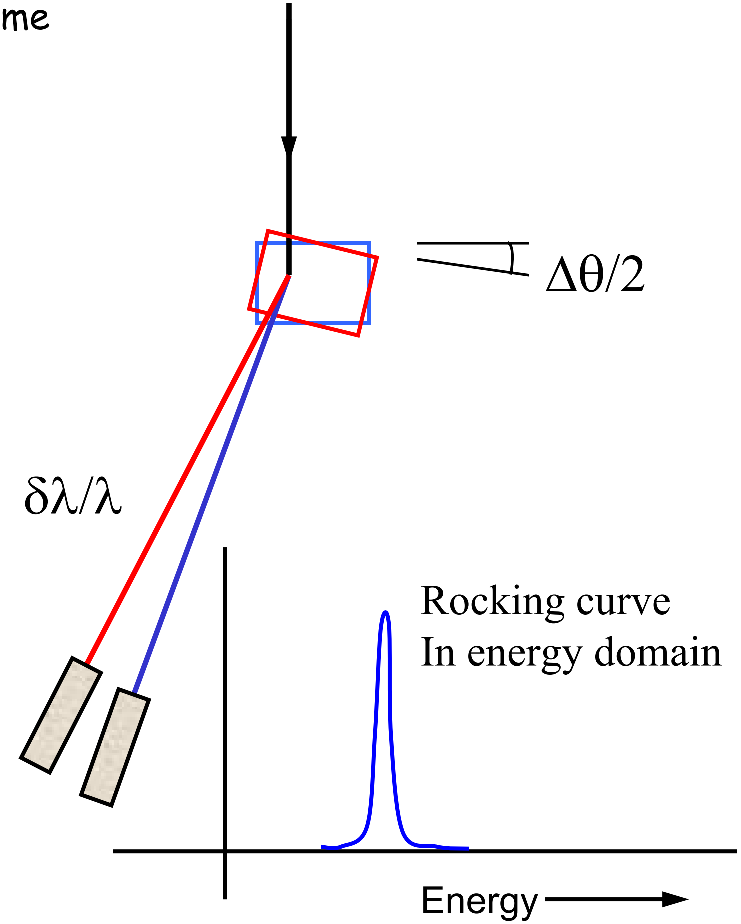


Laue lens

What is the width of the rocking curve of (a sample of) a Fresnel lens?

Dispersion by a mosaic Crystal

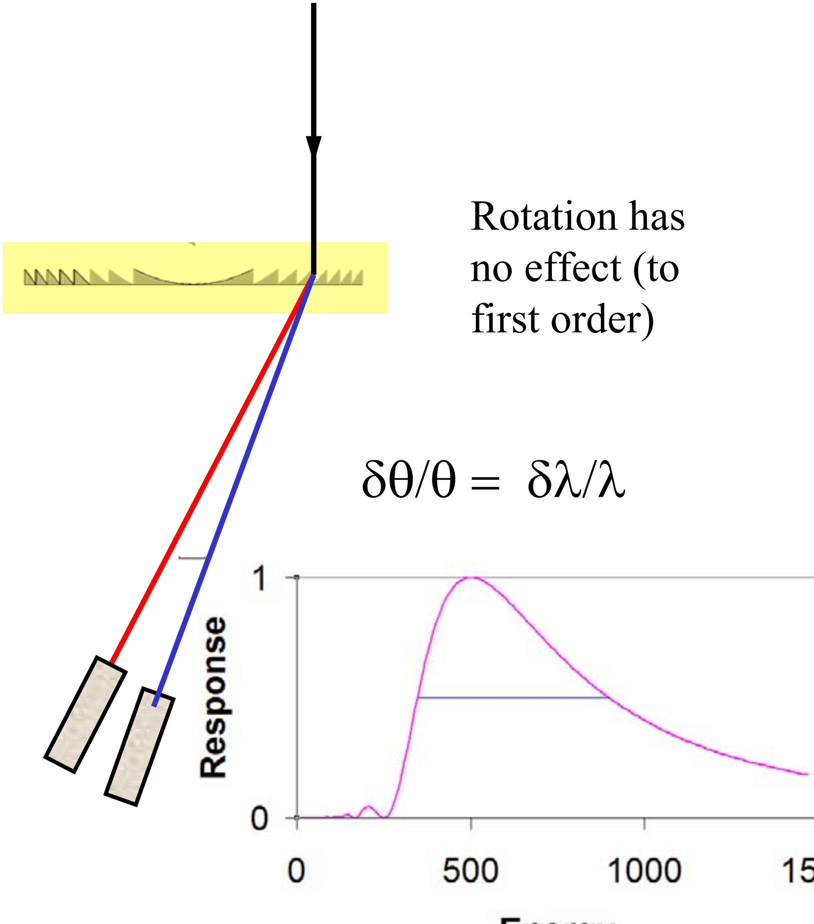
Different crystallites within the rocking curve do indeed diffract different energies, but they diffract them through angles which are not quite the same



$\delta\theta/\theta = \delta\lambda/\lambda$

Fresnel lens equivalent

No need to rotate the sample
One moves the detector
One measures a slightly different waveleng



Rotation has no effect (to first order)

$\delta\theta/\theta = \delta\lambda/\lambda$

Some useful formulae

For diameter d and finest pitch p :

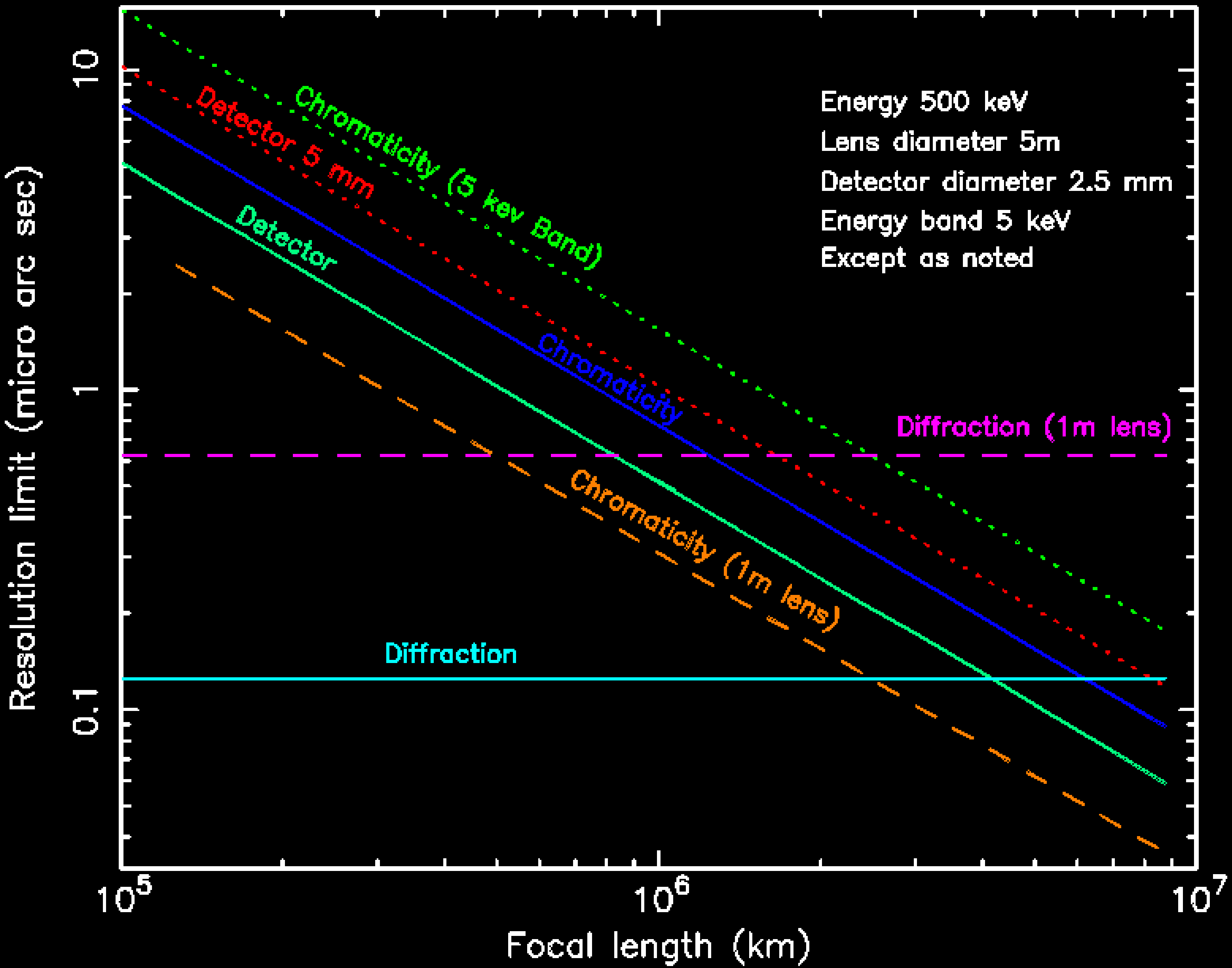
Focal length $f = p\,d\,/\,2\lambda$

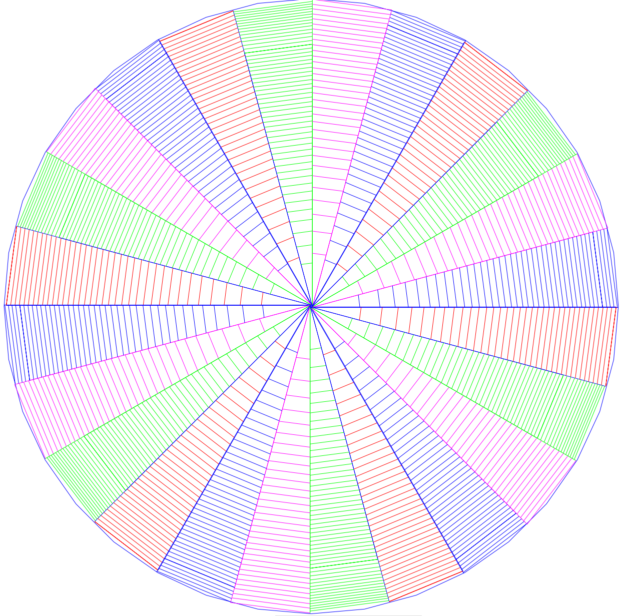
Focal spot size $= 0.66\,p$

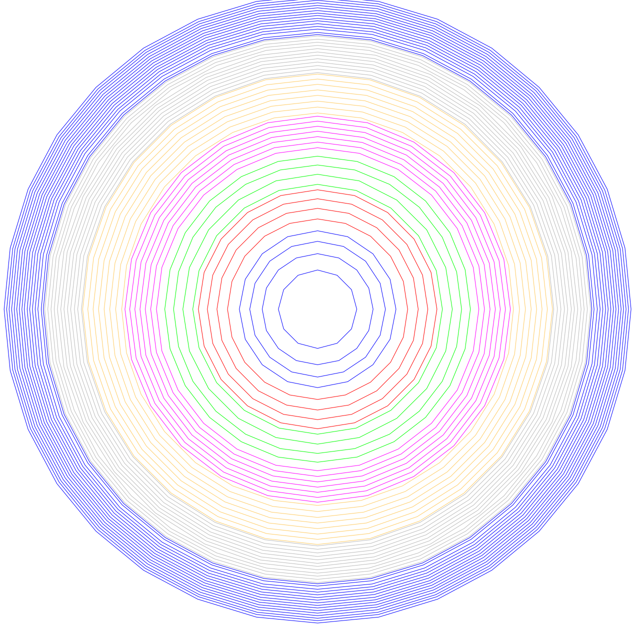
Angular Resolution limits

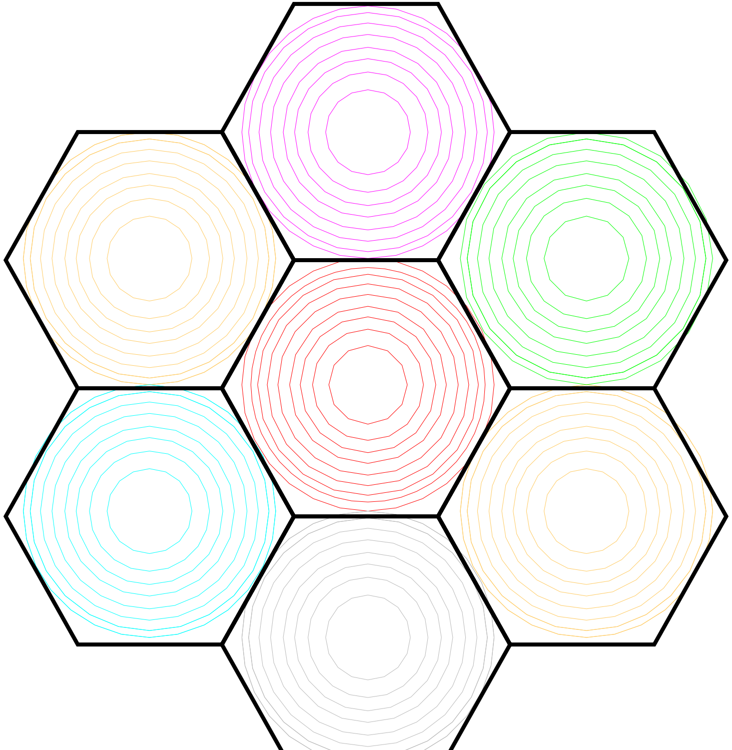
Diffraction	$1.22\,\lambda/d$
Detector spatial resolution	$\Delta x/f$
Chromatic Aberration	$0.15\,(d/f)\,(\Delta\lambda/\lambda)$

Angular Resolution limits for a Fresnel gamma ray lens









Chromatic aberration

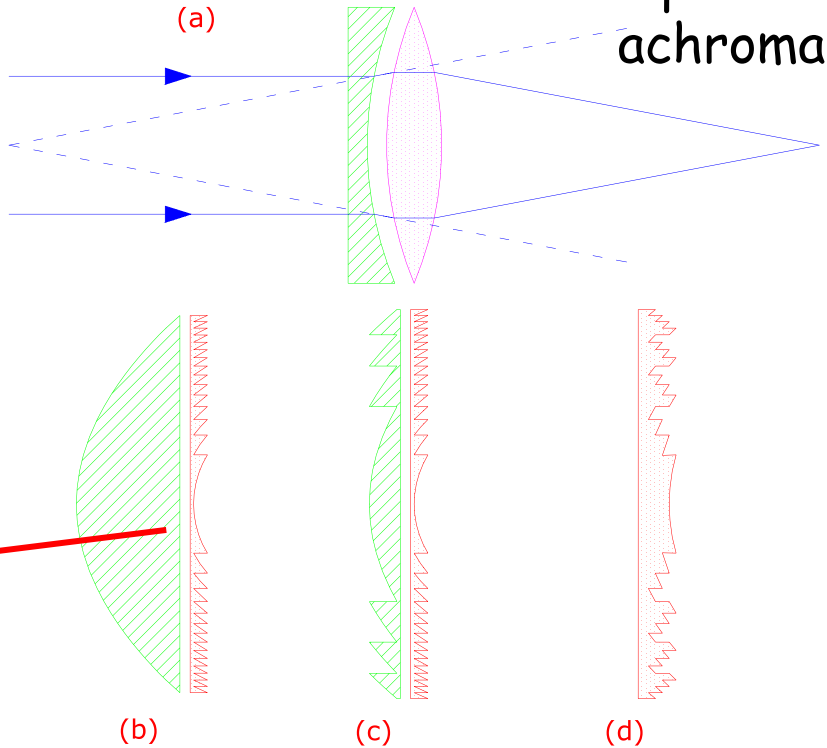
1) Increase of bandpass by subdividing the surface

Subdivision of the surface area, azimuthally, radially, or by unit, can provide zones optimised for different energies

Chromatic aberration

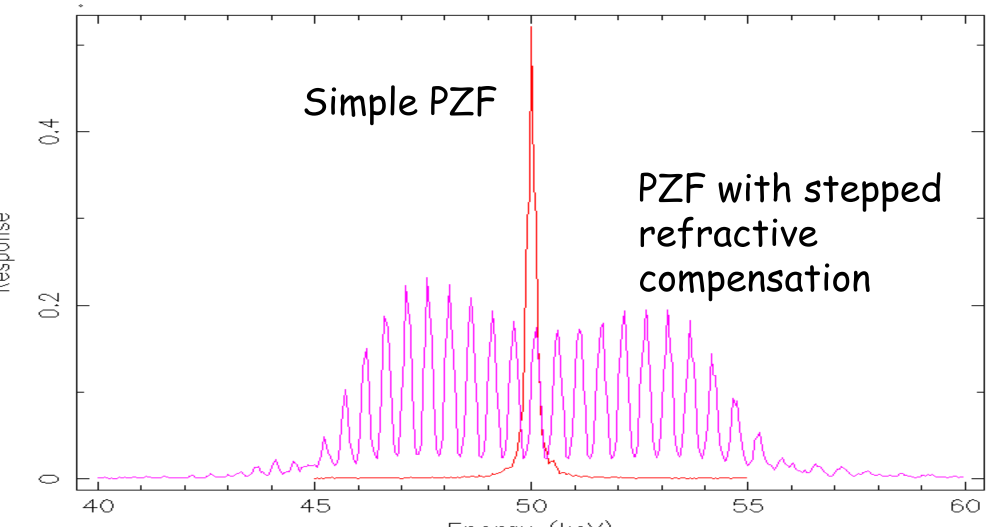
2) One can reduce the effect

Achromatic Pairs



Optical
achroma

X-ray /Gamma-ray
Equivalent :



Simple PZF

PZF with stepped
refractive
compensation

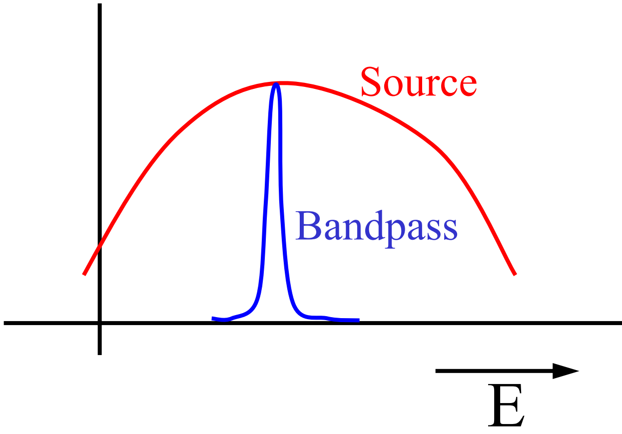
Chromatic aberration

3) Is it such a bad thing to have a narrow passband ?

a) Radio astronomers frequently make images or characterise emission by measurements at one or a few spot frequencies.

b) Sensitivity can actually be better using a narrow band,

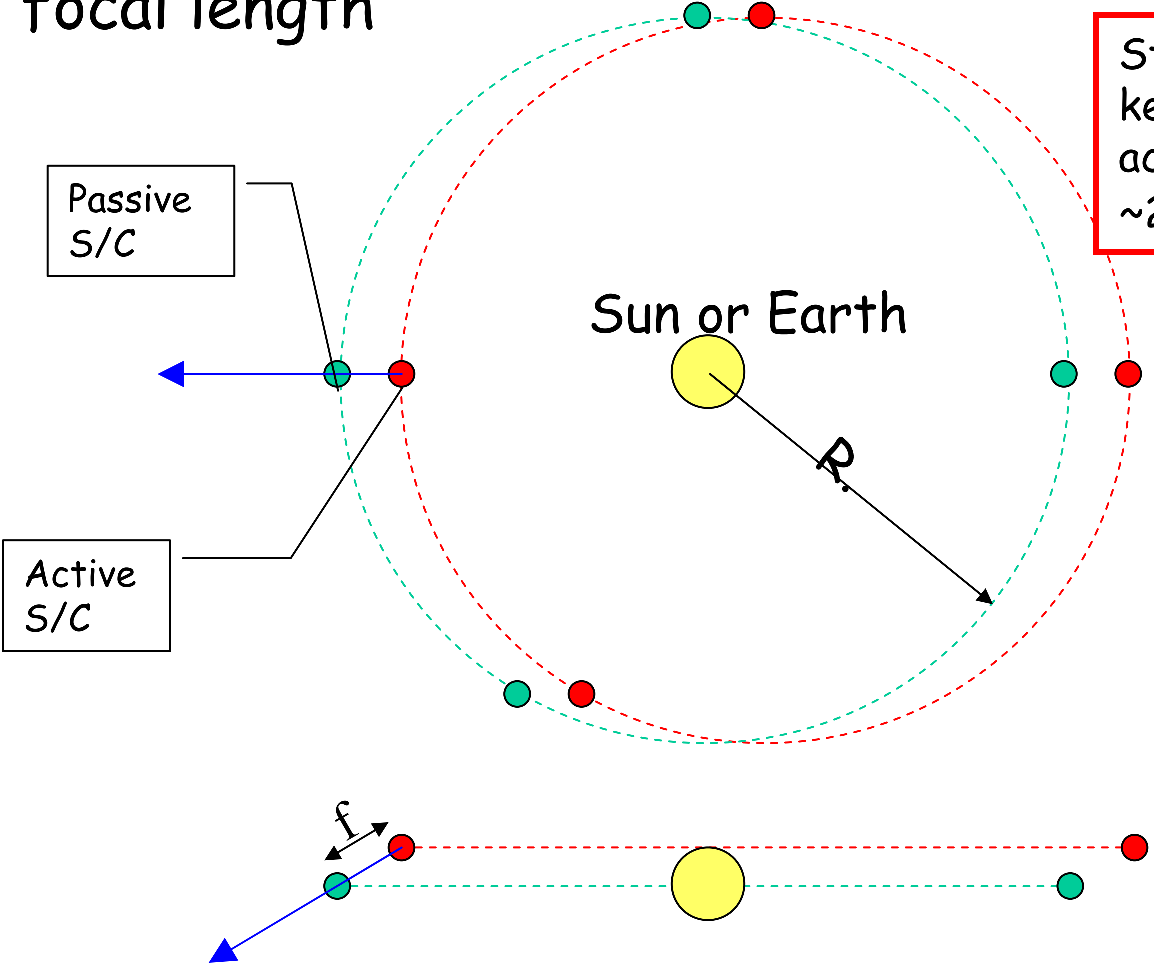
- even for broad lines or continuum emission



	Proportional to
Signal	ΔE
BG in a given volume of the detector	ΔE
Size of chromaticity-limited focal spot	ΔE
Volume of detector used	ΔE^2
Overall BG	ΔE^3
Signal/Noise (BG limited)	$\frac{\Delta E}{\sqrt{\Delta E^3}} = \frac{1}{\sqrt{\Delta E}}$

Problem 2

Extreme focal length

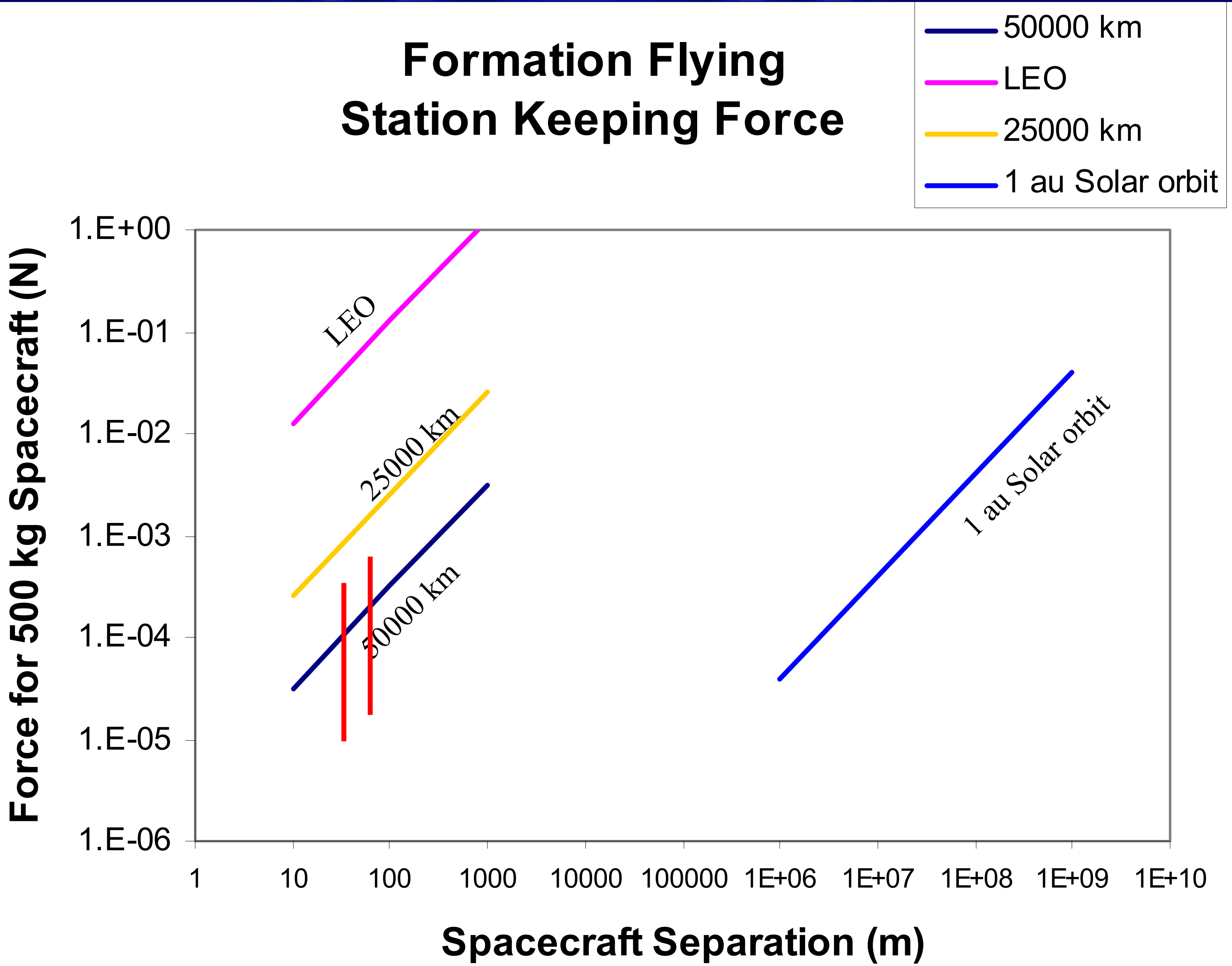


Passive S/C

Active S/C

Station-keeping acceleration ~$2GMf/R^3$

Formation Flying
Station Keeping Force



Example System Parameters

Lens

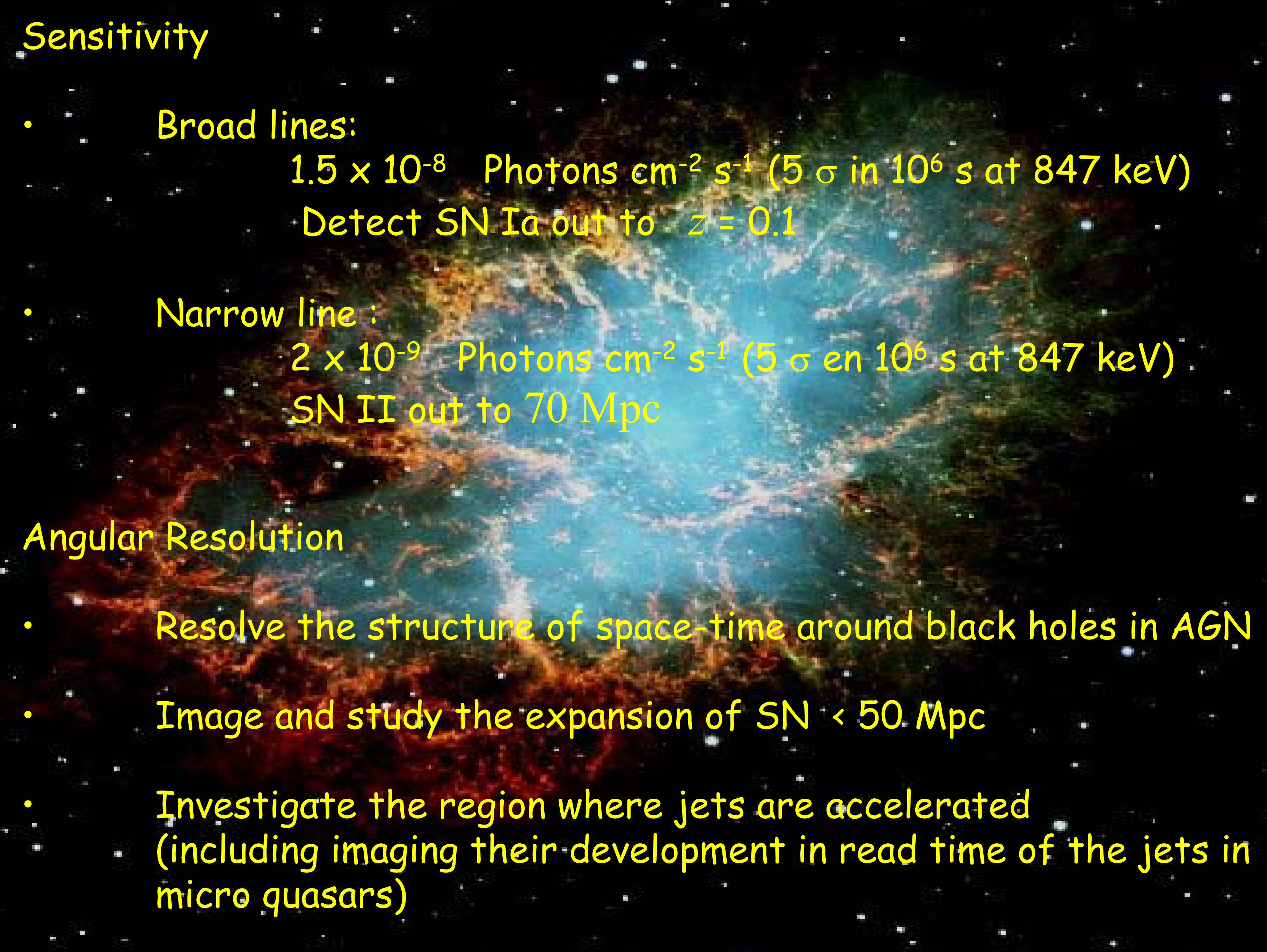
Energy	500 keV	
Diameter	5	m
Material	Aluminium (e.g.)	
Thickness	1.7	mm
Minimum pitch	2	mm
Efficiency	98%	
Focal Length	2×10^6	km

Angular Resolution (μ arc sec)

Diffraction limit	0.12	
Detector resolution limit	0.1	
Chromatic Aberration	0.5	($\Delta E/E = 0.5\%$)
Net	0.6	

Effective area

$\Delta E = 2.5$ keV, 5mm spot		15 m^2	
$\Delta E = 20$ keV	70 mm detector	15 m^2	
$\Delta E = 100$ keV	"	3 m^2	by dividing surface



Sensitivity

- Broad lines:
  - 1.5×10^{-8} Photons cm^{-2} s^{-1} ($5\ \sigma$ in 10^6 s at 847 keV)
  - Detect SN Ia out to $z = 0.1$
- Narrow line :
  - 2×10^{-9} Photons cm^{-2} s^{-1} ($5\ \sigma$ en 10^6 s at 847 keV)
  - SN II out to 70 Mpc

Angular Resolution

- Resolve the structure of space-time around black holes in AGN
- Image and study the expansion of SN < 50 Mpc
- Investigate the region where jets are accelerated (including imaging their development in read time of the jets in micro quasars)

The End

Or is it the beginning
of the era of imaging
at pico metre wavelengths
with micro arc second resolution
using giga metre focal lengths