

# Bound states of positron with nitrile species with several multi-component molecular theories

“International Workshop on **Positrons in Astrophysics**”  
@Hotel Alpin Palace, Murren, Switzerland,  
on 20-23, March 2012

Masanori Tachikawa  
(Yokohama-city University)

# Today's talk

## 1. Introduction

### 1.1. Positronic compounds

## 2. Theory

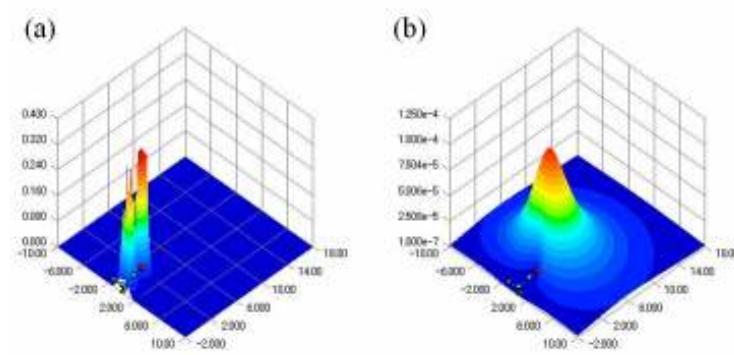
### 2.1. Multi-Component Molecular Orbital (MC\_MO)

### 2.2. Multi-Component Quantum Monte Carlo (MC\_QMC)

## 3. Results and discussion

### 3.1 Positronic nitrile compounds

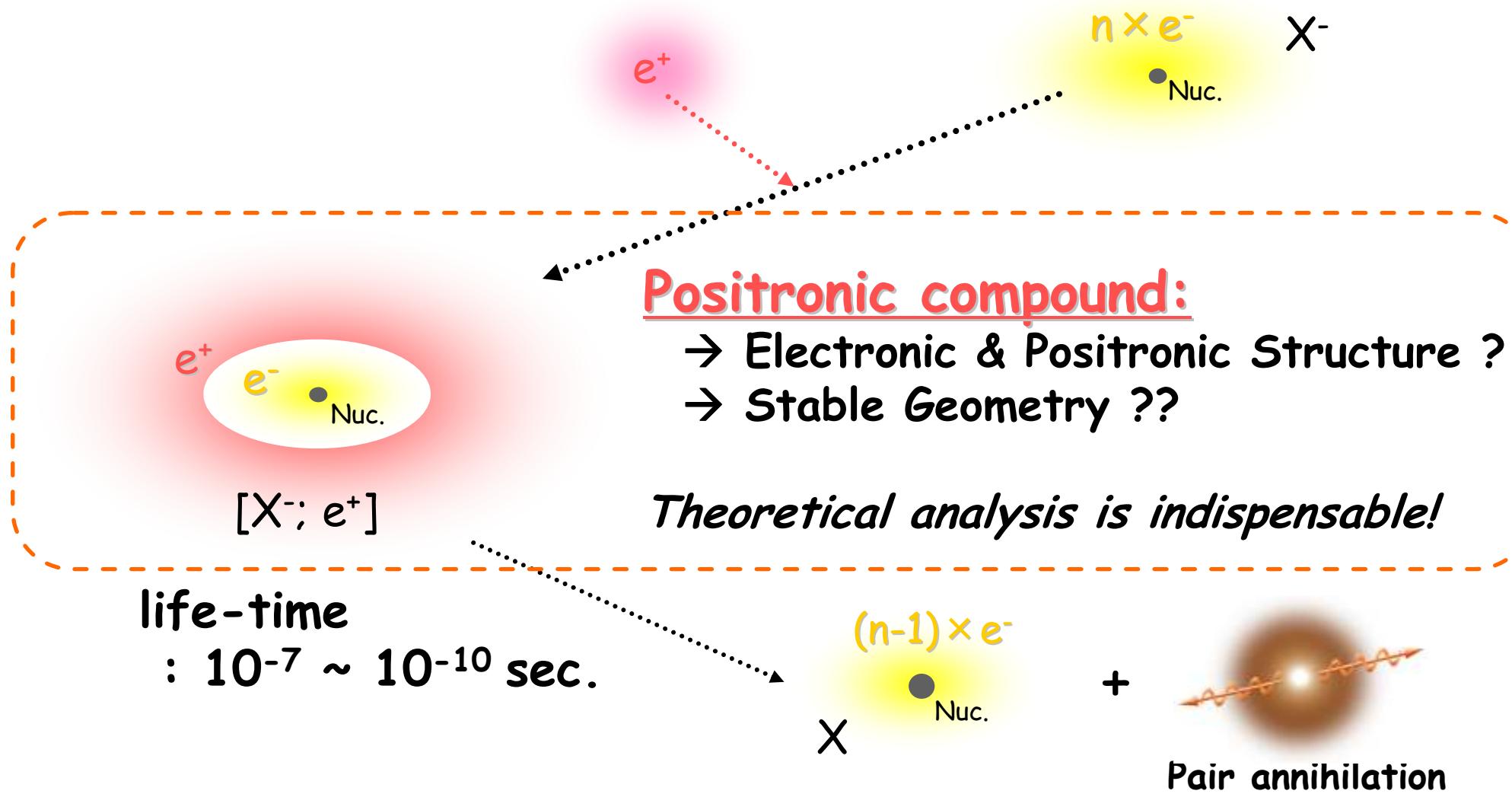
## 4. Summary & Future plans



- Y. Kita, R. Maezono, M. Tachikawa, M. Towler, and R. J. Needs, *J. Chem. Phys.* **131**, 134310 (2009).
- M. Tachikawa, Y. Kita, and R. J. Buenker, *Phys. Chem. Chem. Phys.*, **13**, 2701 (2011).

# 1.1. Introduction: Positronic compounds

When a positron interacts with atom/molecule, they can form a meta-stable “**positronic compound**” before pair annihilation.



# 1.2. Experimental PA values !

## Dipole enhancement of positron binding to molecules

J. R. Danielson, J. J. Gosselin, and C. M. Surko

*Department of Physics, University of California at San Diego, La Jolla, CA 92093*

(Dated: May 12, 2010)

Measurements of positron-molecule binding energies are made for molecules with large permanent dipole moments ( $> 2.7$  debye), by studying vibrational-Feshbach-mediated annihilation resonances as a function of incident positron energy. The binding energies are relatively large (e.g.,  $\geq 90$  meV) as compared to those for similar sized molecules studied previously and analogous weakly bound electron-molecule (negative ion) states. Comparisons with existing theoretical predictions are discussed.

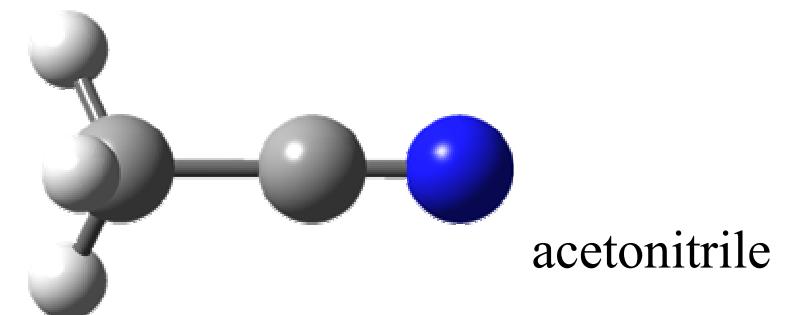
TABLE I: Measured and predicted positron- and electron-molecule binding energies  $e_b$  (meV), permanent dipole moments  $\mu$  (D) and dipole polarizabilities  $\alpha$  ( $\text{\AA}^3$ ) for selected molecules. Data for  $\mu$  and  $\alpha$  taken from Ref. [26]. Data from Figs. 1-4 in bold.

Molecule	Formula	$\mu$ (D)	$\alpha$ ( $\text{\AA}^3$ )	$e_b$ (meV)			
				positrons		electrons <sup>g</sup>	
				meas.	pred.	meas.	pred.
carbon disulfide	$\text{CS}_2$	0	8.7	75		0.7	
butane	$\text{C}_4\text{H}_{10}$	0	8.2	40 <sup>a</sup>			
methanol	$\text{CH}_3\text{OH}$	1.7	3.3	2 <sup>a</sup>			
meth.-chloride	$\text{CH}_3\text{Cl}$	1.9	5.4	25 <sup>a</sup>			
formaldehyde	$\text{H}_2\text{CO}$	2.3	2.8	19 <sup>b</sup>		0.02	
acetaldehyde	$(\text{CH}_3)\text{HCO}$	2.8	4.6	90		0.6	0.95
acetone	$(\text{CH}_3)_2\text{CO}$	2.9	6.4	173	4 <sup>c</sup>	2.6	1.6
propanal	$(\text{C}_2\text{H}_5)\text{HCO}$	2.7	6.5			1.0	0.6
hydr. cyanide	$\text{HCN}$	3.0	2.5	35 <sup>d</sup>	4	3.3	
acetonitrile	$\text{CH}_3\text{CN}$	3.9	4.4	180	123 <sup>e</sup>	19	15.5
lith. hydride	$\text{LiH}$	5.9	3.8	1000 <sup>f</sup>	342	330	

<sup>a</sup>Ref. [11], <sup>b</sup>Ref. [5], <sup>c</sup>Ref. [8], <sup>d</sup>Ref. [6, 7], <sup>e</sup>Ref. [15], <sup>f</sup>Ref. [3],

<sup>g</sup>Meas. from Refs. [25, 27–29], pred. from Refs. [25, 30].

J. R. Danielson, J. J. Gosselin, and C. M. Surko,  
Phys. Rev. Lett. **104**, 233201 (2010).



$$\text{PA}(\text{CH}_3\text{CN}) = 180 \text{ meV}$$

$$\left. \begin{aligned} &\text{PA (Positron Affinity)} \\ &= E(X) - E([X; e^+]) \end{aligned} \right\}$$

# 2.1. Multi-Component MO (MC\_MO)

- Total Hamiltonian (N electron, 1 positron, M fixed nuclei):

$$H_{tot} = \sum_{i=1}^N h^e(i) + \sum_{i=1}^N \sum_{j>i}^N \frac{1}{r_{ij}} + h^p(p) - \sum_{i=1}^N \frac{1}{r_{pi}}$$

$$h^e(i) = -\frac{1}{2} \nabla_i^2 - \sum_{A=1}^M \frac{Z_A}{r_{iA}}, \quad h^p(p) = -\frac{1}{2} \nabla_p^2 + \sum_{A=1}^M \frac{Z_A}{r_{pA}},$$

- Total wavefunction:

$$|\Psi_{tot}\rangle = C_{00} |\Phi_0^e\rangle |\Phi_0^p\rangle + \sum_{IJ} |C_{IJ}\rangle |\Phi_I^e\rangle |\Phi_J^p\rangle \quad |\Phi_0^e\rangle |\Phi_0^p\rangle \longrightarrow \text{Hartree-Fock}$$

$$\sum_{IJ} |C_{IJ}\rangle |\Phi_I^e\rangle |\Phi_J^p\rangle \longrightarrow \text{Configuration Interaction}$$

## 1. Hartree-Fock (HF) equations:

Electrons: $f^e \phi_i^e = \varepsilon_i^e \phi_i^e$	$f^e = h^e + \sum_i^N (J_i - K_i) - J_p$
Positron : $f^p \phi_p^p = \varepsilon_p^p \phi_p^p$	$f^p = h^p - \sum_i^N J_i$

# 2.1. Multi-Component MO (MC\_MO)

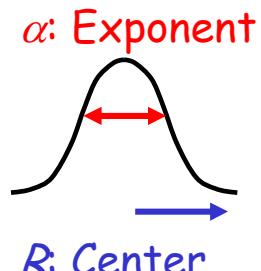
## 1. LCAO expansion → HF Roothaan equations:

Electrons:

$$\varphi_i^e = \sum_{\mu} \chi_{\mu}^e C_{\mu i}^e \quad \mathbf{F}^e \mathbf{C}^e = \mathbf{S}^e \mathbf{C}^e \boldsymbol{\varepsilon}^e$$

Positron:

$$\varphi_p^p = \sum_{\mu} \chi_{\mu}^p C_{\mu p}^p \quad \mathbf{F}^p \mathbf{C}^p = \mathbf{S}^p \mathbf{C}^p \boldsymbol{\varepsilon}^p$$



[\*] [STF](#): I. L. Thomas, PR (1969). [ECG](#): L. Adamowicz, PRL (2002). [GTF](#): H. Nagao, IJQC (1996), Y. Shigeta, JCP (1999), H. Nakai, JCP (2003), S. Hammes-Schiffer, JCP (2005), C.D. Sherrill, MP (2004). [DFT](#): E. K. U. Gross, PRL (2001).

## 2. Correlation: Configuration Interaction (CI)

$$H_{tot} = \sum_{i=1}^N h^e(i) + \underbrace{\sum_{i=1}^N \sum_{j>i}^N \frac{1}{r_{ij}}}_{E_{ee}} + h^p(p) - \underbrace{\sum_{i=1}^N \frac{1}{r_{pi}}}_{E_{ep}}$$

$$|\Psi_{tot}\rangle = C_{00} |\Phi_0^e\rangle |\Phi_0^p\rangle + \sum_{IJ} C_{IJ} |\Phi_I^e\rangle |\Phi_J^p\rangle$$

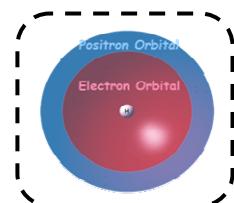
I-electron excitation [IO]  
 |IJ> : J-positron excitation [OJ]  
 I-elec & J-pos excitation [IJ]

[\*] [MP2](#): M. Tachikawa, K. Iguchi, and K. Suzuki, J. Chem. Phys. 101, 5925 (1995). [Gradient](#): M. Tachikawa, K. Mori, H. Nakai, and K. Iguchi, Chem. Phys. Lett. 290, 437 (1998). [CI](#): M. Tachikawa, Chem. Phys. Lett. 350, 269 (2001), Chem. Phys. Lett. 360, 494 (2002). R. J. Buenker, M. Tachikawa, M. Kimura, et al., Phys. Rev. A73, 022705 (2006).

# 2.1. Multi-Component MO (MC\_MO)

Correlation between  $e^-$  and  $e^+$  is indispensable

$[H^-; e^+]$



MC\_MO(HF) -0.666950 (hartree)  
[ $e^-/e^+ = 10s/10s$ ]

↓ -0.769167 (85%)

MC\_MO(Full CI) [1]

[ $e^-/e^+ = 6s3p2d1f/6s3p2d1f$ ]

-0.789196705

ECG [2]

[1] M. Tachikawa; Chem. Phys. Lett. 350 269 (2001)

[2] J. Mitroy; Phys. Rev. A 73 054502 (2006).

[3] J. Mitroy, M. W. J. Bromley and G. G. Ryzhikh, J. Phys. B35 81 (2002), -0.7750785(L=3, STF), -0.7867761(L=9, STF)

## 2.2. Multi Component QMC (MC\_QMC)

Trial Wave function  $\Psi_T$ :

$$\Phi = \Psi_{\text{trial}} = e^{J(\mathbf{R})} \Psi_{\text{HF}}(\mathbf{R})$$

### (A) MC MO

$$f^e = h^e + \sum_i^{Ne} (J_i - K_i) - \sum_p^{Np} J_p$$

$$f^p = h^p + \sum_p^{Np} (J_p - K_p) - \sum_i^{Ne} J_i$$

### (B) MC QMC (Jastrow factor)

$$\begin{aligned} J(\mathbf{R}) &= J(\mathbf{R}_e, \mathbf{R}_I, \mathbf{r}_p) \\ &= \sum_i^N \sum_{j(>i)}^N \frac{u_{ij}(r_{ij})}{e^- - e^-} + \sum_i^N \sum_I^M \frac{\chi_I(r_{iI})}{e^- - \text{Nuc.}} + \sum_i^N \sum_{j(>i)}^N \sum_I^M \frac{f_{ijI}(r_{iI}, r_{jI}, r_{ij})}{e^- - e^- - \text{Nuc.}} \\ &\quad + \sum_i^N \frac{u_p(r_{ip})}{e^- - e^+} + \sum_i^M \frac{\chi_p(r_{pI})}{e^+ - \text{Nuc.}} \end{aligned}$$

### Variational Monte Carlo [VMC] method:

- Energy is obtained by using Metropolis sampling.
- Parameters in trial WF can be optimized under variational principles

### Diffusion Monte Carlo [DMC] method :

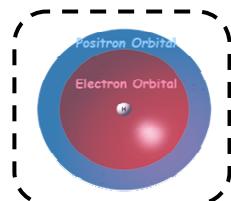
- Projection to ground state by using imaginary-time dependent Schrödinger equation

$$\lim_{\tau \rightarrow \infty} \Phi(\mathbf{R}, \tau=it) \rightarrow \Psi_{\text{Exact}}(\mathbf{R})$$

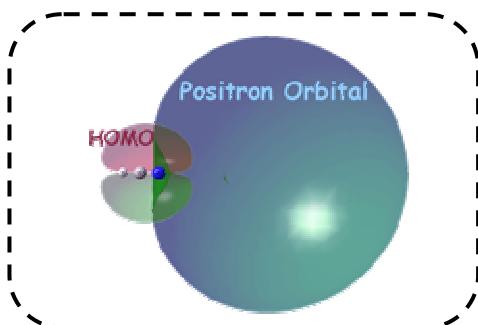
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$[HCN; e^+]$



MC\_MO(HF) -0.666950 (hartree)

$[e^-/e^+ = 10s/10s]$

VMC  
-0.78675(6)

DMC  
-0.78920(11)

-0.769167 (85%)

MC\_MO(Full CI) [1]  
 $[e^-/e^+ = 6s3p2d1f/6s3p2d1f]$

-0.789196705  
ECG [2]

MC\_MO(HF) -92.90074 (hartree)

$[e^-/e^+ = 6-311++G(2d,2p)/15s\ 15p\ 6d\ 2f]$

VMC  
-93.2591(5)

DMC  
-93.40121(13) [5]

CISD [4]  
 $[e^-: 6-311++G(2d,2p)\ e^+: 6-311++G(2d,2p) + 10s\ GTF\ (\text{off-atom})]$

PA(meV)  
HF + 1.8  
DMC +38(5)

[1] M. Tachikawa; Chem. Phys. Lett. 350 269 (2001)

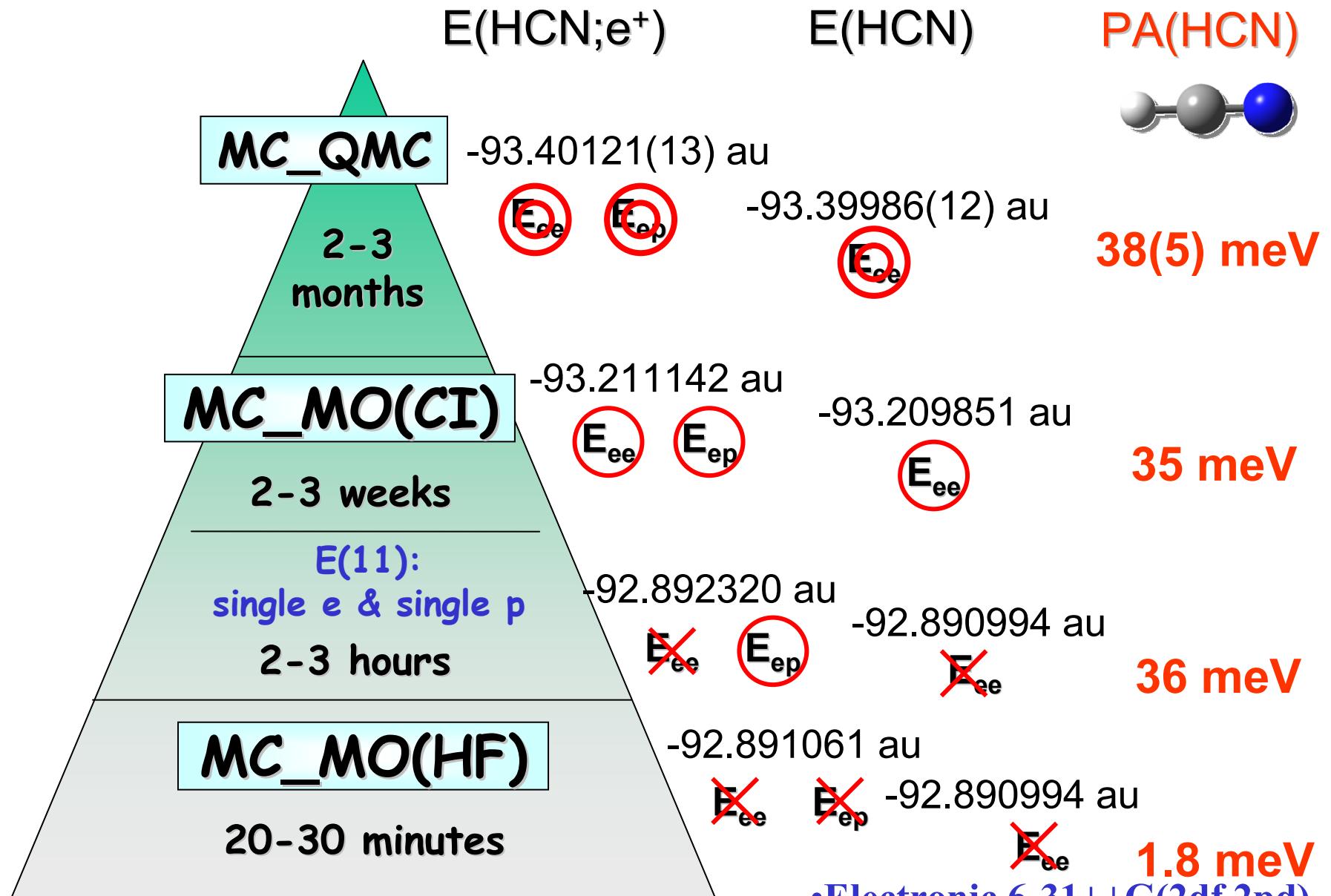
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[4] H. Chojnacki and K. Strasburger, Mol. Phys. 104 2273 (2006)

[5] Y. Kita, R. Maezono, M. Tachikawa, M. Towler, and R. J. Needs, J. Chem. Phys. 131 134310 (2009)

## 2.3. Hierarchie of computational methods



# 1.2. Experimental PA values !

## Dipole enhancement of positron binding to molecules

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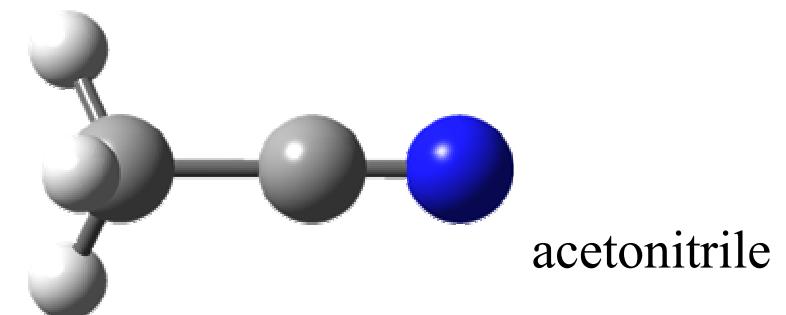
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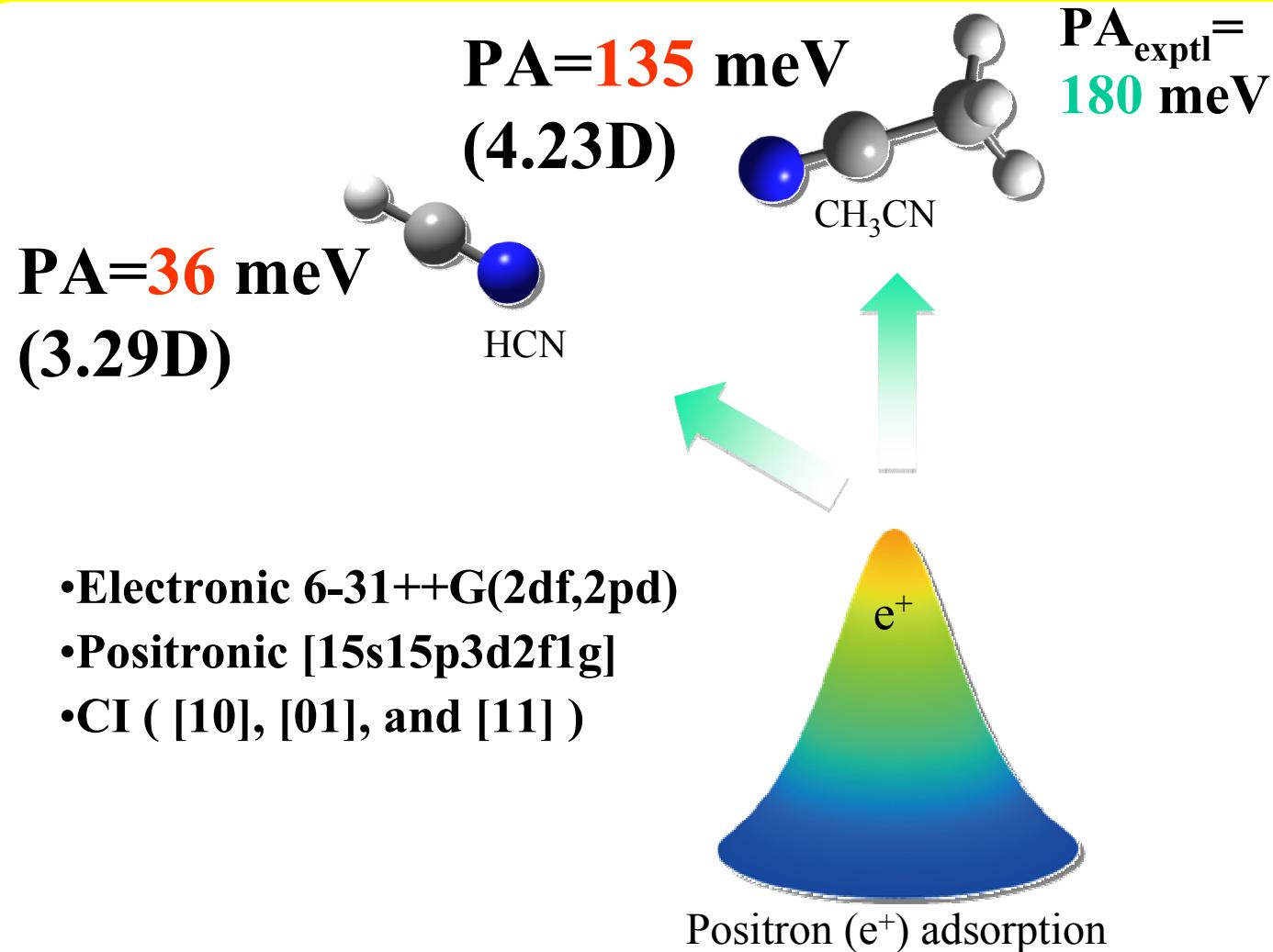
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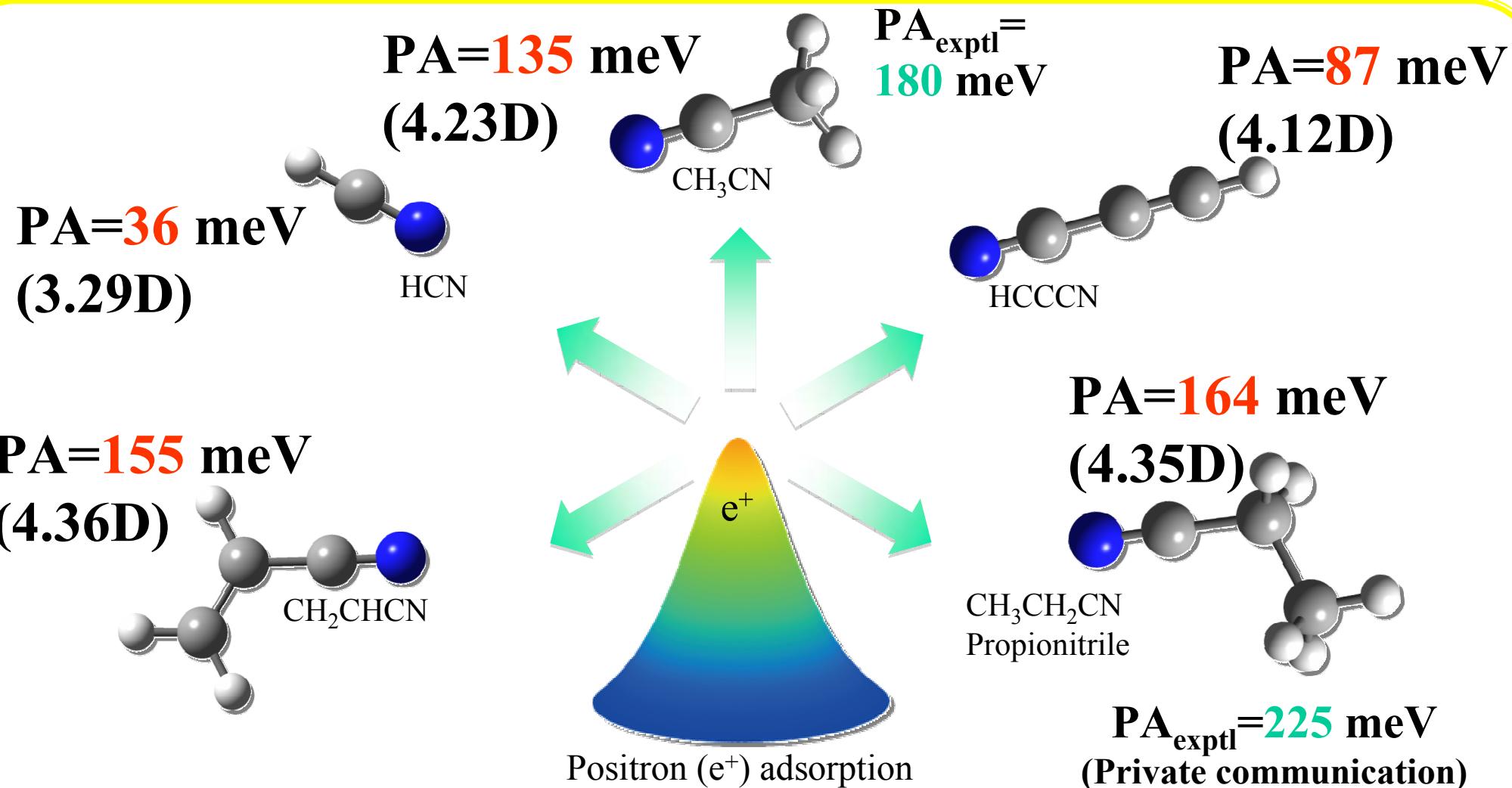
$$\text{PA (Positron Affinity)} \\ = E(X) - E([X; e^+])$$

## 3.1. PAs of nitrile species



Within 25% of the recent experimental value of 180 meV  
by Danielson et al. [Phys. Rev. Lett. 104, 233201 (2010)].

# 3.1. PAs of nitrile species



**The PA values are strongly correlated with the dipole moments.**

M. Tachikawa, Y. Kita, and R. J. Buenker, Phys. Chem. Chem. Phys., 13, 2701 (2011).

## 4. Summary

- Development of Multi-Component molecular theory:

- Multi component molecular orbital (MC\_MO) from Hartree-Fock (HF) to Configuration Interaction (CI)
- Multi component quantum Monte Carlo (MC\_QMC)

- Application of positronic nitrile species:

- We have obtained 135 meV as PA( $\text{CH}_3\text{CN}$ ), which is 75% of the recent experimental value of 180 meV.
- The PA values are strongly correlated with the dipole moments.

- (Near) future Plans:

- Vibrational effect on positronic molecules
- PAs for non-polar molecules

# Acknowledgement

- Prof. R.J Buenker (Wuppatal Univ.)
- Prof. R.J Needs (Cambridge Univ.)
- Dr. Yukumi Kita (Assistant Prof.)
- Dr. Taro Udagawa (Gifu Univ.)
- Mr. Katsu Koyanagi (M1 student)

• TOKYU Car (東急車輛)  
for electric power  
of our computer systems



• JST  
PREST(03-06), CREST(07-08)

• KAKENHI  
KIBAN(B), Sakaki and Fujii  
Tokutei, Material-Design, etc

• YCU  
Strategic Research Project