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# Low-energy positron interactions with atoms and molecules

#### Gleb Gribakin

Department of Applied Mathematics and Theoretical Physics Queen's University Belfast





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John Ludlow Louise Dunlop Chris Lee Dermot Green UCSD Cliff Surko Koji Iwata Levi Barnes Joan Marler Jason Young James Danielson Adric Jones

#### Positron collisions





#### **Positron collisions**



#### Positron collisions



Positron collisions



Positron collisions



Positron collisions



Low

Positron collisions



Low

**Positron collisions** 



Positron collisions



Negative

Positron collisions



#### Main points

- Low-energy positron interaction with atoms and molecules is attractive
- For mildly attractive targets this results in low-lying positron virtual levels
- Elastic scattering and annihilation are enhanced
- Positrons can form bound states with many neutral species
- For molecules positron binding gives rise to vibrational Feshbach resonances
- Vibrational Feshbach resonances can lead to very large annihilation rates
- Resonant annihilation is enhanced by intramolecular vibrational energy redistribution (a ubiquitous effect!)
- Observation of vibrational Feshbach resonances in annihilation allows measurement of positron binding energies for many molecules
- Larger molecules can possess multiple positron bound states



Massey & Mohr Gaseous reactions involving Ps Proc. Phys. Soc. Lond. A 67 695

1954



"It is not surprising that I was interested in positrons from the outset. Indeed I had been fascinated with Dirac's wave equation from my M.Sc. days in Melbourne and can remember being sharply reprimanded by the Senior Demonstrator when I overtly studied Dirac's paper in the Proceedings of the Royal Society while supposedly demonstrating to a practical class."

H. S. W. Massey, Can. J. Phys. 60, 461 (1982)

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"... The annihilation studies [...] are completely mysterious at present, in almost all substances, and particularly for instance in xenon and krypton and methane and so on."

Round-table discussion on future directions in positron-gas research Can. J. Phys. **60**, 565 (1982)

Bhatia, Temkin, Drachman & Eiserike Positron-hydrogen scattering Phys. Rev. A 3 1328





Bhatia, Temkin, Drachman & Eiserike Positron-hydrogen scattering Phys. Rev. A <b>3</b> 1328	J.W. Humberston Scattering/annihilation of low-energy positrons by helium J. Phys. B & L305	Amusia, Cherepkov, Chernysheva, Shapiro Elastic scattering of slow positrons by He J. Phys. B 10 663
1971	1973	1976
McEachran, Ryman, Stauffer et al. Positron scattering from noble gases J. Phys. B 10 663		
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1977-80	1983	



#### Positron scattering from alkali atoms

lonization potential I < 6.8 eV



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lonization potential  $I < 6.8 \ {
m eV}$ 



### Positron scattering from alkali atoms lonization potential I < 6.8 eV



#### Physics of positron-atom interaction

#### Physics of positron-atom interaction

Small distances - repulsive electrostatic potential

 $\sim Z/r$  at  $r \ll a_B$
Small distances - repulsive electrostatic potential

 $\sim Z/r$  at  $r \ll a_B$ 

Large distances



Small distances - repulsive electrostatic potential

$$\sim Z/r$$
 at  $r \ll a_B$ 

#### Large distances

1



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Large distances

 $\boldsymbol{\mathcal{C}}$ 





Small distances - repulsive electrostatic potential

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 at  $r \ll a_B$ 

Large distances







Small distances - repulsive electrostatic potential

$$\sim Z/r$$
 at  $r \ll a_B$ 

Large distances

Atomic-size distances

n



# **Dyson equation** $H_0\varphi_{\varepsilon}(\mathbf{r}) + \int \Sigma_{\varepsilon}(\mathbf{r}, \mathbf{r}')\varphi_{\varepsilon}(\mathbf{r}')d\mathbf{r}' = \varepsilon\varphi_{\varepsilon}(\mathbf{r})$

# $\begin{array}{l} \textbf{Dyson equation} \\ H_0\varphi_{\varepsilon}(\mathbf{r}) + \int \Sigma_{\varepsilon}(\mathbf{r},\mathbf{r}')\varphi_{\varepsilon}(\mathbf{r}')d\mathbf{r}' = \varepsilon\varphi_{\varepsilon}(\mathbf{r}) \end{array}$

Oth-order Hamiltonian (Hartree-Fock)

# Dyson equation $H_0\varphi_{\varepsilon}(\mathbf{r}) + \int \Sigma_{\varepsilon}(\mathbf{r},\mathbf{r}')\varphi_{\varepsilon}(\mathbf{r}')d\mathbf{r}' = \varepsilon\varphi_{\varepsilon}(\mathbf{r})$

Oth-order Hamiltonian (Hartree-Fock) Positron wavefunction

$$H_0\varphi_{\varepsilon}(\mathbf{r}) + \int \Sigma_{\varepsilon}(\mathbf{r},\mathbf{r}')\varphi_{\varepsilon}(\mathbf{r}')d\mathbf{r}' = \varepsilon\varphi_{\varepsilon}(\mathbf{r})$$

Oth-order Hamiltonian Positron wavefunction (Hartree-Fock) Nonlocal energy-dependent correlation potential

$$H_0\varphi_{\varepsilon}(\mathbf{r}) + \int \Sigma_{\varepsilon}(\mathbf{r},\mathbf{r}')\varphi_{\varepsilon}(\mathbf{r}')d\mathbf{r}' = \varepsilon\varphi_{\varepsilon}(\mathbf{r})$$

Oth-order Hamiltonian f Positron wavefunction (Hartree-Fock) Nonlocal energy-dependent correlation potential

Solving the equation for the positron energy  $\varepsilon = k^2/2$  (k momentum) Radial wavefunction  $\varphi_{\varepsilon l}(r) \propto \sin(kr - l\pi/2 + \delta_l)$ 

$$H_0\varphi_{\varepsilon}(\mathbf{r}) + \int \Sigma_{\varepsilon}(\mathbf{r},\mathbf{r}')\varphi_{\varepsilon}(\mathbf{r}')d\mathbf{r}' = \varepsilon\varphi_{\varepsilon}(\mathbf{r})$$

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Scattering phaseshift

$$H_0\varphi_{\varepsilon}(\mathbf{r}) + \int \Sigma_{\varepsilon}(\mathbf{r},\mathbf{r}')\varphi_{\varepsilon}(\mathbf{r}')d\mathbf{r}' = \varepsilon\varphi_{\varepsilon}(\mathbf{r})$$

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#### Scattering phaseshift

Low-energy  $kR_a \ll 1$ e.g. thermal at 300 K k = 0.05 a.u. s-wave (l = 0) dominates  $\varphi \propto \sin(kr + \delta_0)$  $\delta_0 \simeq -Qk$ 

scattering length



polar. orbital is the *polarised orbital calculation*by McEachran et al. (J. Phys. B 1977-1980)
Kohn variat. is the calculation by P. Van Reeth
and J.W. Humberston, J. Phys. B 32, 3651 (1999)



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Much stronger attraction in heavier noble gases



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Much stronger attraction in heavier noble gases

Correlations change the scattering length from positive (static repulsion) to negative



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Higher partial waves also show attraction

#### Comparison between theory and experiment



Sullivan et al. J. Phys. B 41 081001 (2008)

#### Comparison between theory and experiment



Sullivan et al. J. Phys. B 41 081001 (2008)

Cross sections at low energies increase greatly from He to Xe. Positronatom attraction creates virtual s-levels in heavier atoms, at  $\varepsilon = 1/2a^2$ 

	He	Ne	Ar	Kr	Xe
scattering length (a.u.)	-0.42	-0.43	-4.4	-10.1	-81
virtual level energy (eV)	—	_	0.7	0.13	0.002

#### Comparison between theory and experiment



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## Positron-atom binding



#### **Positron-atom binding**

#### Strong attraction



#### BINDING

Many-body theory calculations predicted bound states for 4 atoms V.A. Dzuba, V.V. Flambaum, G. F. Gribakin and W.A. King, Phys. Rev. A 52, 4541 (1995)

1																	1	2
H																	H	He
3	4												5	6	7	8	9	10
Li	Be												B	C	N		)   F	Ne
11	12												13	14	15	16	17	18
Na	Mg											ŀ	<b>\</b> 1	Si	P			.   A1
19	20	21	22	23	24	25	26	27	28	3 29	30	)	31	32	33	34	35	36
K	Ca	Sc	Ti		Cr	Mn	$   F\epsilon$	e   C c	5   N	i   C	l Z	n   (	Ja	Ge	A	s   Se	e   B1	:   K1
37	38	39	40	41	42	43	44	45	46	5 47	48	3 4	49	50	51	52	53	54
Rb	Sr	Y	Zr	Nb	Mo	Tc	Rı	ı   Rl	n   Pe	d   A	g C	d ]	n	Sn	St	5   Te	e   I	Xe
55	56	57	72	73	74	75	76	77	78	3 79	80	)	81	82	83	84	. 85	86
Cs	Ba	La	Hf	Ta	W	Re	Os	s   Ir	·   P	t A	u H	g [	Γ1	Pb	B	i   Po	$\mathbf{b} \mid \mathbf{A}$	t   Rr
87	88	89	104	105	106	107	108	109	) 110	0 11	1 112	2		114		110	5	118
Fr	Ra	Ac	Rf	Db	Sg	Bh	Hs	5   M	t									
				58	59	60	61	62	63	64	65	66	(	67	68	69	70	71
			(	Ce	Pr   I	<b>N</b> d []	Pm	Sm	Eu	Gd	Tb	Dy	7   H	Io	Er	Tm	Yb	Lu
				90	91	92	93	94	95	96	97	98	9	99	100	101	102	103
				Γh	Pa	$\mathbf{U}$	Np	Pu	Am	Cm	Bk	Cf	:   F	Es  ]	Fm	Md	No	Lr
#### Positron-atom binding

Stochastic Variational calculations by Mitroy and Ryzhikh (1997 on) Configuration Interaction by Mitroy and Bromley (2002 on) Configuration Interaction + many-body by Dzuba et al. (1999-2000)

1																		1	2
$ \mathbf{H} $																		H	He
3	4												5	6	5	7	8	9	10
Li	Be														2	Ν		F	Ne
11	12												13	1	4	15	16	17	18
Na	Mg												A	l   S	i	Р			Ar
19	20	21	22	23	24	25	26	27	2	8	29	30	31	3	2	33	34	35	36
K	Ca	Sc	Ti			$ \mathbf{M} $	n  Fe	e   Co	0   N	Ji	Cu	Zr	I Ga	a   G	re	As	Se	e   Br	Kr
37	38	39	40	41	42	43	44	45	6 4	6	47	48	49	5	0	51	52	53	54
Rb	Sr	Y	Zr	Nb	M	5  Te	$c \mid R^{*}$	u   R1	h   P	d	Ag	Cc	l In	1   S	n	Sb	Te		Xe
55	56	57	72	73	74	75	76	77	7 7	8	79	80	81	8	2	83	84	85	86
Cs	Ba	La	Hf	Ta	W		e   O	s   Iı	:   P	<b>°</b> t	Au	Hg	T	l   P	b	Bi	Pc	)   At	Rn
87	88	89	104	105	106	10'	7 10	8 10	9 11	0	111	112		11	4		116	,	118
Fr	Ra	Ac	Rf	Db	Sg		h H	s   M	[t										
				-									_						
				58	59	60	61	62	63	6	54	65	66	67	6	68	69	70	71
			(	Ce	Pr	Nd	Pm	Sm	Eu	G	d '	Гb	Dy	Ho	E	Er   1	Гm	Yb	Lu
				90	91	92	93	94	95	9	)6	97	98	99	1	00	101	102	103

Pu Am Cm Bk

Th

Pa

IJ

Cf | Es

Fm Md No

Lr

#### Positron-atom binding

Stochastic Variational calculations by Mitroy and Ryzhikh (1997 on) Configuration Interaction by Mitroy and Bromley (2002 on) Configuration Interaction + many-body by Dzuba et al. (1999-2000)

<sup>1</sup> H	Mitroy et al., J. Phys. B <b>35</b> , R81 (2002)													$\mathbf{H}^{1}$	<sup>2</sup> He		
<sup>3</sup> Li	<sup>4</sup> Be												<sup>6</sup> C	$\overset{7}{\mathbf{N}}$	<sup>8</sup> O	9 F	Ne
<sup>11</sup> Na	$\mathbf{Mg}^{12}$		$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$												<sup>17</sup> Cl	Ar	
19 K	Ca	<sup>21</sup> Sc	<sup>22</sup> Ti	23 V	Cr	Mn <sup>25</sup>	Fe <sup>26</sup>	<sup>27</sup> Co	<sup>28</sup> Ni	<sup>29</sup> Cu	$\frac{30}{Zn}$	Ga <sup>31</sup>	Ge	$\mathbf{As}^{33}$	<sup>34</sup> Se	<sup>35</sup> Br	<sup>36</sup> Kr
<sup>37</sup> <b>Rb</b>	<sup>38</sup> Sr	39 Y	<sup>40</sup> Zr	$\mathbf{N}^{41}$	$\mathbf{M}^{42}$	43 Tc	A4 Ru	<sup>45</sup> Rh	$\mathbf{P}^{46}$	$\stackrel{47}{\operatorname{Ag}}$	<sup>48</sup> Cd	<sup>49</sup> In	<sup>50</sup> Sn	<sup>51</sup> Sb	Te	53 I	Xe
55 Cs	<sup>56</sup> Ba	57 La	<sup>72</sup> Hf	<sup>73</sup> Ta	$\mathbf{W}^{74}$	<sup>75</sup> Re	76 Os	77 Ir	<sup>78</sup> Pt	<sup>79</sup> Au	<sup>80</sup> Hg	<sup>81</sup> Tl	<sup>82</sup> Pb	<sup>83</sup> Bi	Po	<sup>85</sup> At	<sup>86</sup> Rn
Fr	Ra	Ac	<sup>104</sup> <b>R</b> f	105 Db	106 Sg	<sup>107</sup> Bh	<sup>108</sup> Hs	<sup>109</sup> Mt	110	111	112		114		116		118
	•	•	•	•		•				•						•	

58	59	60	61	62	63	64	65	66	67	68	69	70	71
Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu
90	91	92	93	94	95	96	97	98	99	100	101	102	103
Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr

J. Mitroy, M.W. J. Bromley and G. Ryzhikh, J. Phys. B 32, 2203 (1999)



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# More candidates for binding

 $\begin{array}{l} \mbox{Conditions for binding} \\ \alpha_d > 40 \ \mbox{a.u.} \\ I < 10 \ \mbox{eV} \end{array}$ 

## More candidates for binding



V.A. Dzuba, V.V. Flambaum and G. F. Gribakin, Detecting positron-atom bound states through resonant annihilation, Phys. Rev. Lett. **105**, 203401 (2010).

## More candidates for binding



V.A. Dzuba, V.V. Flambaum and G. F. Gribakin, Detecting positron-atom bound states through resonant annihilation, Phys. Rev. Lett. **105**, 203401 (2010).

For nonrelativitistic positrons in electronic environment



For nonrelativitistic positrons in electronic environment



Annihilation rate in the positron bound state

$$\Gamma^{a} = \pi r_{0}^{2} c \int \sum_{i=1}^{Z} \delta(\mathbf{r}_{i} - \mathbf{r}) |\Psi_{0}(\mathbf{r}_{1}, \dots, \mathbf{r}_{Z}, \mathbf{r})|^{2} d\mathbf{r}_{1} \dots d\mathbf{r}_{Z} d\mathbf{r}$$

For nonrelativitistic positrons in electronic environment

$$\begin{array}{ll} \mbox{Probability of} \\ \mbox{annihilation per} &= \pi r_0^2 c \times \\ \mbox{unit time} & \mbox{QED} \end{array} \begin{bmatrix} \mbox{Density of the} \\ \mbox{electrons at the} \\ \mbox{positron} \end{bmatrix}$$

Annihilation rate in the positron bound state

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Annihilation cross section for the positron with momentum  $\mathbf{k}$  on a target

$$\sigma_a = \pi r_0^2 \frac{c}{v} \int \sum_{i=1}^Z \delta(\mathbf{r}_i - \mathbf{r}) |\Psi_{\mathbf{k}}(\mathbf{r}_1, \dots, \mathbf{r}_Z, \mathbf{r})|^2 d\mathbf{r}_1 \dots d\mathbf{r}_Z d\mathbf{r}$$

For nonrelativitistic positrons in electronic environment



Annihilation rate in the positron bound state

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$$\sigma_a = \pi r_0^2 \frac{c}{v} Z_{\text{eff}}$$

For nonrelativitistic positrons in electronic environment



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Estimate in atomic units:

 $r_0 = e^2/mc^2 \sim 10^{-4}, \ \sigma_a \sim 10^{-6} Z_{\text{eff}}$ 

 $\sigma_a = \pi r_0^2 \frac{c}{d} Z_{\text{eff}}$ 

 $c \approx 137$ 

# $Z_{\rm eff}$ for atoms and small molecules

Atom	$Z_{\rm eff}^{*}$	
н	8.5	calc.
He	3.94 **	
Ne	<b>5.99</b> **	
Ar	26.8 **	
Kr	<b>65.7</b> **	
Xe	<b>40</b> I ***	

\* Values at room T, k~0.05 au
\*\* Coleman, Charlton, Kileen, Griffith, Heyland, Wright (1975-82) (UCL)
\*\*\* Iwata et al. (1995) (UCSD)

Molecule	Zeff	Ζ
H <sub>2</sub>	15	2
N <sub>2</sub>	31	14
<b>O</b> <sub>2</sub>	37	16
СО	39	14
CO <sub>2</sub>	55	20
N <sub>2</sub> O	78	20
SF <sub>6</sub>	86	70
H <sub>2</sub> O	319	18
NO <sub>2</sub>	1090	23
NH <sub>3</sub>	1600	10

#### Positron annihilation: many-body theory

$$Z_{\text{eff}} = \int \sum_{i=1}^{Z} \delta(\mathbf{r}_i - \mathbf{r}) |\Psi_{\mathbf{k}}(\mathbf{r}_1, \dots, \mathbf{r}_Z, \mathbf{r})|^2 d\mathbf{r}_1 \dots d\mathbf{r}_Z d\mathbf{r}_Z$$



#### Positron annihilation: many-body theory

$$Z_{\text{eff}} = \int \sum_{i=1}^{Z} \delta(\mathbf{r}_i - \mathbf{r}) |\Psi_{\mathbf{k}}(\mathbf{r}_1, \dots, \mathbf{r}_Z, \mathbf{r})|^2 d\mathbf{r}_1 \dots d\mathbf{r}_Z d\mathbf{r}_Z$$



$$Z_{\text{eff}}^{(0)} = \sum_{n} \int |\psi_n(\mathbf{r})|^2 |\varphi_{\varepsilon}(\mathbf{r})|^2 d\mathbf{r}$$

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$$Z_{\text{eff}}^{(0)} = \sum_{n} \int |\psi_n(\mathbf{r})|^2 |\varphi_{\varepsilon}(\mathbf{r})|^2 d\mathbf{r}$$

$$Z_{\text{eff}}^{(1)} = \sum_{\nu,\,\mu,\,n} \frac{\langle \varepsilon n | \delta | \mu \nu \rangle \langle \nu \mu | V | n \varepsilon \rangle}{\varepsilon - \varepsilon_{\nu} - \varepsilon_{\mu} + \varepsilon_{n}}$$





$$\Psi_{\mathbf{k}}(\mathbf{r}_1,\ldots,\mathbf{r}_Z,\mathbf{r})\simeq\Phi_0(\mathbf{r}_1,\ldots,\mathbf{r}_Z)\left[e^{i\mathbf{k}\cdot\mathbf{r}}+frac{e^{ikr}}{r}
ight]$$



$$\Psi_{\mathbf{k}}(\mathbf{r}_1,\ldots,\mathbf{r}_Z,\mathbf{r}) \simeq \Phi_0(\mathbf{r}_1,\ldots,\mathbf{r}_Z) \begin{bmatrix} e^{i\mathbf{k}\cdot\mathbf{r}} + f \frac{e^{ikr}}{r} \end{bmatrix} \quad \begin{array}{l} \text{Scattering} \\ \text{amplitude} \end{array}$$



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#### Annihilation on noble gases Krypton



#### Annihilation on noble gases Krypton



#### Thermally averaged $Z_{eff}$ at room T

Atom	He	Ne	Ar	Kr	Xe
Oth order, static	0.69	0.97	0.74	0.68	0.61
0th order, Dyson	1.34	2.30	6.86	14.6	71
total, Dyson	3.78	5.53	26.5	66.4	402
Exp. (gas)	3.94	5.99	26.8	65.7	320
Exp. (trap, UCSD)	-	-	33.8	90.I	40 I

#### Positron annihilation in molecules

Martin Deutsch (1917-2002)

- Discovery of Ps (1951)
- Measurement of lifetime of Ps
- Single-triplet Ps energy splitting Rapid annihilation of positrons in Freon-12, CCl<sub>2</sub>F<sub>2</sub>



The Times 22 October 2002

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Paul & Saint-Pierre (1963):  $Z_{eff} \sim 10^4$  for C<sub>4</sub>H<sub>10</sub> and CCl<sub>4</sub> Tao (1965):  $Z_{eff} \sim 1600$  for Cl<sub>2</sub> (UNSW) Heyland *et al.* (1982), Wright *et al.* (1985): C<sub>3</sub>H<sub>8</sub>, ...

### Positron annihilation in molecules

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- Discovery of Ps (1951)
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For room-temperature positrons  $Z_{\rm eff} = \frac{F}{\kappa^2 + k^2} \lesssim 10^3$ 



The Times 22 October 2002

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Heyland et al. (1982), Wright et al. (1985):  $C_3H_8$ , ...

## $Z_{\rm eff}$ for alkanes and substitutes

Molecule	Zeff	Ζ
$H_2$	15	2
N <sub>2</sub>	31	14
<b>O</b> <sub>2</sub>	37	16
CO	39	14
CO <sub>2</sub>	55	20
N <sub>2</sub> O	78	20
SF <sub>6</sub>	86	70
H <sub>2</sub> O	319	18
NO <sub>2</sub>	1090	23
NH <sub>3</sub>	1600	10

# $Z_{\rm eff}$ for alkanes and substitutes

Molecule	X = H	X = F	X = CI	X = Br
CX <sub>4</sub>	142	54.4	9 530	39 800
C <sub>2</sub> X <sub>6</sub>	660	152	68 600	-
C <sub>3</sub> X <sub>8</sub>	3 500	317	-	-
C <sub>4</sub> X <sub>10</sub>	11 300	-	-	-
C <sub>5</sub> X <sub>12</sub>	37 800	-	-	-
C <sub>6</sub> X <sub>14</sub>	120 000	630	-	-
C7X16	242 000	-	-	-
C <sub>8</sub> X <sub>18</sub>	585 000	I 064	-	-
C <sub>9</sub> X <sub>20</sub>	643 000	-	-	-
C10X22	728 000	-	-	-
C <sub>12</sub> X <sub>26</sub>	I 780 000	-	-	-
C16X34	2 230 000	-	-	-

Iwata, Greaves, Murphy, Tinkle and Surko, PRA 51, 473 (1995)








# Chemical sensitivity of $Z_{eff}$



# Weakly bound positron states

$$\begin{split} \Psi_0(\mathbf{r}_1,\ldots,\mathbf{r}_Z,\mathbf{r}) \simeq \underbrace{\Phi_0(\mathbf{r}_1,\ldots,\mathbf{r}_Z)}_{\text{Atomic/molecular}} \underbrace{A \frac{e^{-\kappa r}}{r}}_{\text{Positron}} & \varepsilon_0 = -\frac{\kappa^2}{2} \\ \\ \varepsilon_0 = -\frac{\kappa^2}{2} \\ \varepsilon_0 = -\frac{\kappa^2}{2} \\ \end{array}$$
Normalisation:  $A \simeq \sqrt{\kappa/2\pi}$ 

# Weakly bound positron states

$$\Psi_0(\mathbf{r}_1,\ldots,\mathbf{r}_Z,\mathbf{r})\simeq \underbrace{\Phi_0(\mathbf{r}_1,\ldots,\mathbf{r}_Z)}_{\mathbf{A}}A\frac{e^{-\kappa r}}{r}$$

$$\varepsilon_0 = -\frac{\kappa^2}{2}$$

Atomic/molecular ground state

wavefunction

Positron

Normalisation:  $A \simeq \sqrt{\kappa/2\pi}$ 

#### Annihilation rate

$$\Gamma^{a} = \pi r_{0}^{2} c \rho_{ep}$$

$$\rho_{ep} = \frac{F}{2\pi} \kappa$$

$$\kappa = \sqrt{2|\varepsilon_{0}|}$$

$$F\sim 1$$
 in atomic units









#### Vibrational Feshbach resonance



Breit-Wigner theory

$$\sigma_a = \frac{\pi}{k^2} \sum_{\nu} \frac{g_{\nu} \Gamma_{\nu}^a \Gamma_{\nu}^e}{(\varepsilon - \varepsilon_{\nu})^2 + \frac{1}{4} \Gamma_{\nu}^2}$$



#### Vibrational Feshbach resonance





$$\sigma_{a} = \frac{\pi}{k^{2}} \sum_{\nu} \frac{g_{\nu} \Gamma_{\nu}^{a} \Gamma_{\nu}^{e}}{(\varepsilon - \varepsilon_{\nu})^{2} + \frac{1}{4} \Gamma_{\nu}^{2}}$$

$$e^{+} + M(\nu_{i}) \longrightarrow Me^{+}(\nu) \longrightarrow M^{+} + 2\gamma$$
Vibrational Feshbach  
resonance















Averaging over positron energy spread  $\bar{\sigma}_{a} = \frac{2\pi^{2}}{k^{2}} \frac{\Gamma^{a}\Gamma^{e}(\varepsilon)}{\Gamma(\varepsilon)} \rho(E)$ 



$$Z_{\text{eff}} = \frac{2\pi^2 \rho_{ep}}{k} \frac{\Gamma^e(\varepsilon)}{\Gamma(\varepsilon)} \rho(E)$$
$$Z_{\text{eff}} = \pi F \sqrt{\frac{|\varepsilon_0|}{\varepsilon}} \frac{\rho(\varepsilon + |\varepsilon_0| + E_{\nu_i})}{N(\varepsilon + E_{\nu_i})}$$
$$N(E) = \int_0^E \rho(E') dE'$$

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Molecule	$Z_{\rm eff}$	ε <sub>0</sub>   (meV)
C <sub>3</sub> X <sub>8</sub>	3 500	22
C <sub>4</sub> X <sub>10</sub>	11 300	42
C <sub>5</sub> X <sub>12</sub>	37 800	65
C <sub>6</sub> X <sub>14</sub>	120 000	90
C7X16	242 000	103
C <sub>8</sub> X <sub>18</sub>	585 000	122

$$Z_{\text{eff}} = \frac{2\pi^{2}\rho_{ep}}{k} \frac{\Gamma^{e}(\varepsilon)}{\Gamma(\varepsilon)} \rho(E)$$

$$Z_{\text{eff}} = \pi F \sqrt{\frac{|\varepsilon_{0}|}{\varepsilon}} \frac{\rho(\varepsilon + |\varepsilon_{0}| + E_{\nu_{i}})}{N(\varepsilon + E_{\nu_{i}})}$$

$$N(E) = \int_{0}^{E} \rho(E')dE'$$

$$\frac{\text{Molecule}}{C_{3}X_{8}} \frac{Z_{\text{eff}}}{3500} \frac{22}{22}$$

$$\frac{C_{4}X_{10}}{C_{5}X_{12}} \frac{11}{37} \frac{300}{42}$$

$$\frac{42}{C_{5}X_{14}} \frac{120}{2000} \frac{90}{90}$$

$$\frac{C_{7}X_{16}}{C_{8}X_{18}} \frac{585}{000} \frac{122}{12}$$

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# Measuring $Z_{eff}$ as a function of positron energy

S. J. Gilbert, L. D. Barnes, J. P. Sullivan and C. M. Surko, Phys. Rev. Lett. 88, 043201 (2002)



By reducing the depth of the trap, the positrons are "pushed" over the edge to form a beam

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Positron parallel energy distribution is approximately Gaussian with FWHM ~ 25 meV

S. J. Gilbert et al., Appl. Phys. Lett. 70, 1944 (1997)

# Dependence of $Z_{eff}$ on positron energy for alkanes



Barnes et al. PRA 67, 032706 (2003)

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C-H vibrational mode  $\omega pprox 0.37~{
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For infrared-active vibrations this coupling is mediated by the dipole potential

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Calculation of the elastic (capture) width for vth resonance

$$\begin{split} \Gamma_{\nu}^{e} &= 2\pi \int |A_{\nu\mathbf{k}}|^{2} \delta(k^{2}/2 - \omega_{\nu} - \varepsilon_{0}) \frac{d^{3}k}{(2\pi)^{3}} \\ A_{\nu\mathbf{k}} &= \int \varphi_{0}(\mathbf{r}) \Phi_{\nu}^{*}(\mathbf{R}) \frac{\hat{\mathbf{d}} \cdot \mathbf{r}}{r^{3}} e^{i\mathbf{k}\cdot\mathbf{r}} \Phi_{0}(\mathbf{R}) \, d\mathbf{r} d\mathbf{R} \\ &= \frac{4\pi i}{3} \frac{\mathbf{d}_{\nu} \cdot \mathbf{k}}{\sqrt{2\pi\kappa}} \, _{2}F_{1}\left(\frac{1}{2}, 1; \frac{5}{2}; -\frac{k^{2}}{\kappa^{2}}\right) \end{split}$$

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# Fundamentals and elastic widths in $CH_3CI$

TABLE I. Characteristics of the vibrational modes of $CH_3Cl$ .					
Mode	Symmetry	$g_{\nu}$	$\omega_{\nu}^{a}$ (meV)	$d_{\nu}$ (a.u.)	$\omega_{\nu}d_{\nu}^2$ (a.u.)
$\nu_1$	$a_1$	1	363	0.0191	$4.87 \times 10^{-6}$
$\nu_2$	$a_1$	1	168	0.0176	$1.91 \times 10^{-6}$
$\nu_3$	$a_1$	1	91	0.0442	$6.52 \times 10^{-6}$
$ u_4$	e	2	373	0.0099	$1.34 \times 10^{-6}$
$ u_5$	е	2	180	0.0162	$1.74 \times 10^{-6}$
$\nu_6$	е	2	126	0.0111	$5.66 \times 10^{-7}$

<sup>a</sup>Mode energies ω<sub>ν</sub> and dipole amplitudes d<sub>ν</sub> from Ref. [29].
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Annihilation width vs elastic width:  $|\varepsilon_0| = 25 \text{ meV}$  $\kappa = 0.043 \text{ a.u.}$   $\Gamma^{(a)} = 5.5 \times 10^{-9} \text{ a.u.} \ll \Gamma^{(e)}$ 

# Putting it all together: total $Z_{\text{eff}}$ $Z_{\text{eff}} = \bar{Z}_{\text{eff}}^{(\text{res})} + \bar{Z}_{\text{eff}}^{(\text{dir})}$ $Z_{\text{eff}}^{(\text{res})} = \frac{\pi}{k} \rho_{ep} \sum_{\nu} \frac{g_{\nu} \Gamma_{\nu}^{e}}{(\varepsilon - E_{\nu} - \varepsilon_{0})^{2} + \Gamma_{\nu}^{2}/4} \qquad Z_{\text{eff}}^{(\text{dir})} = \frac{F}{\kappa^{2} + k^{2}}$

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Averaging over the positron energy distribution in the beam

$$f_{\epsilon}(\varepsilon_{\perp}, \varepsilon_{z}) = \frac{1}{k_{B}T_{\perp}\sqrt{2\pi\sigma^{2}}} \exp\left[-\frac{\varepsilon_{\perp}}{k_{B}T_{\perp}} - \frac{(\varepsilon_{z} - \epsilon)^{2}}{2\sigma^{2}}\right]$$
$$\sigma = \delta_{z}/\sqrt{8\ln 2}, \text{ where } \delta_{z} = \text{FWHM}$$
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$$\bar{Z}_{\text{eff}}^{(\text{res})}(\epsilon) = 2\pi^2 \rho_{ep} \sum_{\nu} \frac{g_{\nu} \Gamma_{\nu}^e}{k_{\nu} \Gamma_{\nu}} \Delta(\epsilon - \varepsilon_{\nu})$$

Function  $\Delta(E)$  describes the shape of a resonance as observed with the beam

$$\Delta(E) = \frac{1}{2k_B T_{\perp}} \exp\left[\frac{\sigma^2}{2(k_B T_{\perp})^2} + \frac{E}{k_B T_{\perp}}\right] \left\{1 + \Phi\left[-\frac{1}{\sqrt{2}}\left(\frac{E}{\sigma} + \frac{\sigma}{k_B T_{\perp}}\right)\right]\right\}$$

## Shapes of resonances observed with cold positron beam



#### Theory vs experiment for halomethanes

Halogen-substituted methane: CH<sub>3</sub>Hal all 6 vibrational modes infrared active



#### CH<sub>3</sub>F

[Barnes, Gilbert and Surko, PRA 67 032706 (2003)]

#### CH<sub>3</sub>Cl, CH<sub>3</sub>Br

[Barnes, Young and Surko, PRA 74 012706 (2006)]

#### Theory vs experiment for halomethanes





#### Theory vs experiment for halomethanes





#### Predictions for deuterated halomethanes

CD<sub>3</sub>Hal - same binding energy as CH<sub>3</sub>Hal, but different vibrational spectrum

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Young, Gribakin, Lee and Surko, Phys. Rev. A **77**, 060702 (2008)













Similarity between  $Z_{eff}$  and spectrum of modes Vibrational modes act as "doorways" into VFR G. F. Gribakin and P. M.W. Gill, Nucl. Instrum. Methods B **221**, 30 (2004)

$$Z_{\rm eff} = \frac{F\Gamma_{\rm spr}}{2\Gamma(\varepsilon)} \sqrt{\frac{\varepsilon_b}{\varepsilon}} \sum_n \frac{\Gamma_n^e}{(\varepsilon - \omega_n + \varepsilon_b)^2 + \frac{1}{4}\Gamma_{\rm spr}^2}$$

sum over the modes

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sum over the modes  
Fit using fixed ratios  
$$\frac{\Gamma_n^e}{n}/\Gamma$$
for C-H and low-  
energy modes
$$\prod_{n=1}^{\infty} \frac{\Gamma_n^e}{1 \times 10^4} \sqrt{\frac{\varepsilon_b}{\varepsilon}} \sqrt{\frac{\varepsilon_b}{\varepsilon$$

#### Zero-range potential model for positron binding

G. F. Gribakin and C. M. R. Lee, Nucl. Instrum. and Methods B 247, 31 (2006)

Experimental binding energies			
No. of C's	BE (meV)	2nd (meV)	
3	10	-	
4	35	-	
5	60	-	
6	80	-	
7	105	-	
8	115	-	
9	145	-	
12	205	0	
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#### Zero-range potential model for positron binding

G. F. Gribakin and C. M. R. Lee, Nucl. Instrum. and Methods B 247, 31 (2006)

Experimental binding energies			
No. of C's	BE (meV)	2nd (meV)	
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Zero-range wavefunction:  $\Psi(\mathbf{r}) = \sum_{i} A_{i} \frac{e^{-\kappa |\mathbf{r} - \mathbf{R}_{i}|}}{|\mathbf{r} - \mathbf{R}_{i}|}$ Boundary condition:  $\Psi|_{\mathbf{r} \to \mathbf{R}_{i}} \simeq \text{const} \times \left(\frac{1}{|\mathbf{r} - \mathbf{R}_{i}|} - \kappa_{0i}\right)$ 

#### Binding energies for alkanes: theory vs experiment



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#### Binding energies for alkanes: theory vs experiment



Theory predicts emergence of the 2nd positron bound state!

#### 2D density of the bound states



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#### Dependence of $Z_{eff}$ on positron energy for alkanes



C-H vibrational mode  $\omega pprox 0.37~{
m eV}$ 

Barnes et al. PRA 67, 032706 (2003)

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### Open questions

- Positron binding to open-shell atoms?
- Positron-atom electronic Feshbach resonances?
- Positron binding to nonpolar molecules?
- Positron annihilation rates in molecular bound states?
- Positron coupling to non-IR-active vibrations?
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# THANK YOU