ORBIT CONTROL OF MAX,
A NEW CONCEPT OF GAMMA RAY TELESCOPE

V. Martinot[*], A. Boutonnet[*] , P. Brousse[*], H. Sainct[*] and P. Von Ballmoos[+]

[*]ALCATEL SPACE, 26 avenue J.-F. Champollion, B.P. 1187
31037 Toulouse Cedex, France.

[+]Centre d’Etude Spatiale des Rayonements, 9, av. du Colonel-Roche,
31028 Toulouse Cedex 4, France

3rd International Workshop on Satellite Constellations and Formation Flying

Pisa, Italy, 24-26 February 2003

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V. Martinot*[1], A. Boutonnet*[2] , P. Brousse*[3], H. Sainct*[4] and P. Von Ballmoos[5]

*Alcatel Space, 26, av. J.-F. Champollion, 31037 Toulouse Cedex, France
[1] e-mail: vincent.martinot@space.alcatel.fr
[2] e-mail: arnaud.boutonnet@space.alcatel.fr
[3] e-mail: pascal.brousse@space.alcatel.fr
[4] e-mail: herve.sainct@space.alcatel.fr
[5]Centre d'Etude Spatiale des Rayonements 9, av. du Colonel-Roche, 31028 Toulouse Cedex 4, France - e-mail: peter.von-ballmoos@cesr.fr

ABSTRACT

This paper presents the Max mission and the orbit control strategy of the formation proposed to implement it. Max is actually a new concept of gamma ray telescope, to study supernovae of type Ia by measuring intensities, shifts and shapes of their nuclear gamma-ray lines. It is radically different from traditional gamma-ray telescopes: for the first time in this energy range, gamma-rays are focused from the large collecting area of a crystal diffraction lens onto a very small detector volume. As a consequence, the background noise is extremely low, making possible unprecedented sensitivities.

The mission concept is to use a master-slave tandem on an Earth-bound orbit where the distance between both satellites is roughly equal to 100 m. The main challenge of the control is to keep constant not only this distance but also the line of sight of the telescope. It is customary to qualify this geometry with the term "Non-Keplerian Formation Flying" since the satellites orbit must be continuously controlled.

Mission analysis is conducted for two types of operational orbits, namely a low-Earth orbit and a Geostationary Transfer Orbit to take advantage of possible shared launches. It is shown that a way to reduce the constraints on the propulsion subsystem is to favour the latter option but at the cost of a more complex control of the formation in terms of operations and guidance laws.

1. INTRODUCTION

A few years ago, formation flying seemed a promising approach but still to be demonstrated through the implementation of at least demonstration missions. Since then, a few missions have definitely proven that the paradigm change foreseen in formation flying was a reality, one of the most recent examples being the Grace mission which contributes to the knowledge of the Earth potential to a level never reached before. All the missions so far belong to the category of free (elliptic) motion formation flying according to the classification introduced in [RD1].

However, the Max mission the scientific objectives of which are first recalled requires a non-Keplerian formation flying to keep inertial the line of sight between the two satellites of the formation. After presenting the rationale for the Max operational orbit selection, the guidance laws for two candidate operational orbits – namely a Low-Earth orbit and a Geostationary Transfer orbit - are derived and estimations of the related propellant budget are presented.

This allows to balance the advantages and drawbacks of both possibilities and conclude on a preliminary feasibility of the proposed mission with the current technological status.

2. THE MAX MISSION

2.1 Scientific objectives

The primary scientific objective of MAX is the study of supernovae of type Ia (SN Ia) by measuring intensities, shifts and shapes of their nuclear gamma-ray lines. When finally understood and calibrated, these profoundly radioactive events will be determining in measuring the size, shape, and age of the Universe. Despite the intensive study of SN Ia in optical and infrared wavelengths, neither the nature of the exploding object nor the explosion mechanism are satisfactorily known. High resolution gamma-ray spectroscopy provides a key route to answer these questions by studying the conditions in which the thermonuclear explosion starts and propagates. Complementary observations using the optical monitor on MAX will provide the link that allows a comprehensive understanding of the phenomenon.

Having a large sample is imperative in exploring the different types of SN Ia. A sensitivity of 10^{-6} photons cm^{-2} s^{-1} to broadened gamma-ray lines allows observations of supernovae out to distances of 50–100 Mpc. Within this distance there will always be a type Ia SN in the phase of gamma-ray line emission, starting shortly after explosion, and lasting for several months.

The sensitive gamma-ray spectroscopy performed by MAX addresses a wide range of fundamental astrophysical questions such as the life cycles of matter and the behavior of matter under extreme conditions: Observing the radio-activities from individual classical novae and core collapse supernovae will significantly improve our understanding of explosive nucleo-synthesis. Sensitive gamma-ray measurements also hold out the prospect of observing SNe in optically obscured regions and resolving problems in understanding SN rates. Gamma-ray line spectroscopy is expected to clarify the nature of galactic micro-quasars (e+e- annihilation radiation from the jets), neutrons stars and pulsars, X-ray Binaries, AGN, solar flares and, last but not least, gamma-ray afterglow from gamma-burst counterparts. Accomplishing all these objectives will require significant improvement in sensitivity over current and planned missions.

2.2 Concept description

Since the wavelength of nuclear gamma-ray photons is two to three orders of magnitude shorter than the distance between atoms in solids, astrophysicists have been used to accept that it is “impossible to reflect or refract gamma-rays”. Present telescope concepts for nuclear astrophysics make use of geometrical optics (shadow-casting in modulating aperture systems) or quantum optics (kinetics of Compton scattering). Because the collecting area of such systems is identical to the detector area, nuclear astrophysics has in fact come to an impasse where “bigger is not necessarily better”: with the background noise being roughly proportional to the volume of a detector, a larger photon collection area is synonymous with higher instrumental background - consequently, the signal to noise ratio does not improve with larger collectors.

A possible way out of this impasse consists of taking advantage of the phase information of the photons. The concept of MAX is radically different from traditional gamma-ray telescopes: for the first time in this energy range, γ-rays are focused from the large collecting area of a crystal diffraction lens onto a very small detector volume. As a consequence, the background noise is extremely low, making possible unprecedented sensitivities. MAX constitutes a breakthrough for the study of compact sources by combining narrow line sensitivities of a few 10^{-7} ph s^{-1} cm^{-2} with high energy resolution ($E/\Delta E \approx 500$) and very good positioning. The concept of diffracting a narrow energy band is ideally matched to the narrow lines in the domain of nuclear transitions.

To implement this concept of diffraction, it is proposed to use two satellites flying in formation composed of:

- ✓ a stabilized spacecraft with a crystal diffraction telescope equipped with a gamma-ray lens to focus the gamma-rays,
- ✓ a small spacecraft equipped with a small array of germanium detectors.

2.3 Mission requirements

The following mission requirements are identified:

- ✓ [MR1]: The observation time in a fixed inertial direction shall typically last for 2 weeks.

  Justification: the flux is expected to be of the order of 10^{-6} to 10^{-7} ph s^{-1} cm^{-2}, which combined with a reasonable size for the collecting area of the order of 1 m^2, requires at least 10^6 seconds.
  Remark: The observation period can be interrupted periodically as long as the total time is reached.

- [MR2]: All the directions in the sky shall be accessible.
  Justification: there is no privileged locations for the appearance of SN Ia.

- [MR3]: Observations in Van Allen regions shall be avoided.
  Justification: Apart from the negative effects of the particles trapped in the Van Allen regions on the satellite electronics, they also have a disturbing effect on the measurements of the gamma rays seen by the detector in an indirect way. Indeed, the interactions between the trapped particles and the detector eventually lead to radioactive emissions of gamma rays that perturb the measurements of the gamma rays coming from SN Ia.

- [MR4]: The distance between the collector and the detector shall be of 112 metres.
  Justification: The diffraction lens mounted on one of the satellite has a focal length of 112 metres.

3. PRELIMINARY MISSION ANALYSIS

3.1 Selection of the orbit

Concerning the operational orbit, the following types can be proposed: Low-Earth Orbits (LEO), Highly Eccentric Orbit (HEO), escape hyperbola or trailing orbit (TO), Lagrangian points (LP). It is difficult at this stage of the project to definitely choose the operational orbit since numerous aspects – not only related to the mission, but also to the design of the satellites – need to be considered for the final selection of the operational orbit. Some elements of the trade-offs are however presented in Table 1.

Orbits	Characteristics	Advantages	Drawbacks
LEO / MEO	z : from 400 to 1500 km / 15000 to 25000 km i : from 0 to 90° $GG = 2.10^{-6}\ s^{-2}$ to $4.5\ 10^{-8}\ s^{-2}$	✓ Low Delta V required to reach the orbit ✓ Communication (Earth proximity) ✓ Launch cost (Small low-cost launcher compatible)	✓ High perturbations in LEO (atmospheric drag at low altitudes, gravity gradient, Earth potential variation, thermal variations, eclipses) ✓ Defiling orbits with short visibility number of ground stations / availability
HEO	Ex: GTO (12 h period) $GG = 2.5\ 10^{-9}\ s^{-2}$ at apogee	✓ Low perturbations at apogee ✓ Adapted for launch by small Russian launchers	✓ Radiation levels ✓ Number of ground stations ✓ Attitude control for formation flying complex near perigee ✓ High delta V needed for insertion on orbit: will induce the use of an important kick stage
TO	E.g. S/C slowly drifts from the Earth on a circumsolar orbit at 1 A.U. from the Sun	✓ Low level of perturbation (only solar radiation pressure) ✓ Station-keeping can be avoided (TBC)	✓ Launch cost ✓ High delta V for insertion ✓ Communications (Distance to Earth) with decreasing budget ✓ Number of ground stations / availability
LP	Along the Sun-Earth directions, 1.5 millions away from the Earth	✓ Low perturbations ✓ no problems for keeping an inertial control during formation flying	✓ Launcher cost ✓ Unstable orbit but small station keeping need ✓ Communications (Distance to Earth) ✓ Number of ground stations / availability

Table 1 : Candidate Orbits for Max

When accounting for the possibility of sharing the launch cost with another mission, the two most attractive types of orbit candidate for further study are:

- ✓ a LEO at an altitude of 1500 km. This altitude is selected:
  - ✓ to reduce the level of geopotential differential effect between the satellite to counteract,
  - ✓ to limit the drag acting on the satellite (although only the differential effect needs strictly speaking to be corrected),
  - ✓ to be under the first Van Allen region.
- ✓ a GTO orbit.

3.2 Theoretical formation flying guidance laws

The natural evolution of the differential motion between both satellites of the formation does not lead to a constant direction of the line of sight and therefore requires specific guidance laws. In a first approach, only the central term of the Earth gravitational field is considered.

Let $\vec{r}_1$ be the radius vector of the detector and $\vec{\rho}$, the differential position vector between the detector and the diffraction lens (or equivalently the two satellite of the formation). Assuming that the motion is purely Keplerian and that the distance between the two satellites is small compared to the distance to the centre of the Earth (a valid assumption in agreement with [MR4]), a limited development at first order in the ratio $\|\vec{\rho}\|/\|\vec{r}_1\|$, yields :

$$\ddot{\vec{\rho}} = \frac{\mu}{r_1^3}\left[-\vec{\rho} + 3\left(\frac{\vec{r}_1}{r_1}, \vec{\rho}\right)\frac{\vec{r}_1}{r_1}\right] + \vec{P} \qquad \text{Eq 1}$$

Where:

- ✓ $\vec{P}$ represents the perturbing differential accelerations between both satellites,
- ✓ (,) is the scalar product between two vectors.

To achieve a constant inertial line of observation, the differential acceleration shall be null, which gives a direct expression for the thrust acceleration to annihilate the relative motion (since no other differential acceleration is considered), an approach similar to the one used in [RD2], [RD3] and [RD4]:

$$\vec{\gamma} = -\frac{\mu}{r_1^3}\left[-\vec{\rho} + 3\left(\frac{\vec{r}_1}{r_1}, \vec{\rho}\right)\frac{\vec{r}_1}{r_1}\right] \qquad \text{Eq 2}$$

NB:

- ✓ Assuming a pure Keplerian motion and an inertial frame associated with the orbital plane of the detector, the distance to the centre of the Earth r_1 is expressed by the following relationship:

$$r_1 \overset{\rightharpoonup}{=} \|\vec{r}_1\| \left|\begin{matrix}\cos(v)\\ \sin(v)\\ 0\end{matrix}\right. \text{ et } r_1 = \frac{a(1-e^2)}{1+e\cos(v)} \text{ where v is the true anomaly.}$$

- ✓ The acceleration can therefore be expressed analytically since it depends only on the shape of the orbit and the true anomaly

In the Hill reference frame, the differential radius vector of the diffraction lens with respect to the collector follows a cone normal to the orbital plane and its extremity, a circle. The half aperture angle of the cone becomes null when the observation direction is normal to the orbital plane, and 90 degrees when the observation direction is contained in the orbital plane. Note that in case of free elliptic motion, a circle can also be achieved but only when the axe of the cone is inclined at 30 degrees with respect to the orbital plane. Therefore, it does not correspond to the current situation where the line of sight is kept constant.

4. MISSION ANALYSIS RESULTS

4.1 Hypotheses and definitions

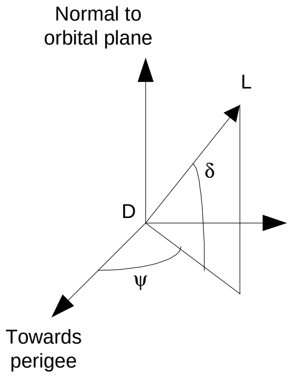

Figure 4-1 : Observation direction

A few definitions and hypotheses are necessary:

- ✓ The satellite on which the diffraction lens is mounted is the one performing the thrust to avoid ejecting contaminating material in the vicinity of the detector. Its mass is of the order of 200 kg. The satellite with the detector has only attitude control capability to orient the detector normal to the observation direction.

- ✓ The observation direction is indicated by two angles, the elevation δ with respect to the orbital plane of the detector and the azimuth angle ψ in the orbital plane of the detector counted positively from the perigee vector direction, as shown on Figure 4-1 where D stands for the Detector and L, for the lens.

4.2 Low-Earth Circular Orbit

4.2.1 Guidance laws

- ✓ Case of observation direction in the orbital plane

  The variation of the thrust level along the orbit is presented on Figure 4-2 for various in-plane observation directions. One can see that the guidance law is only shifted in phase when the in-plane observation directions varies, the average thrust level over one orbit being the same. The direction of the thrust is always contained in the orbital plane and rotates at twice the orbital pulsation.

  For a circular orbit at 1500 km, the maximum thrust level is of the order of 0.035 N for a 200-kg satellite (inter-satellite distance of 112 m).

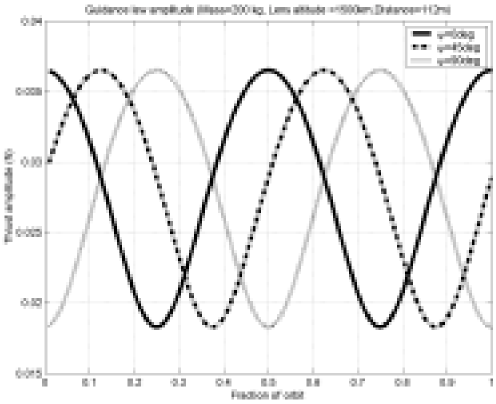

Figure 4-2 : Guidance law amplitude (in-plane observation)

- ✓ Case of observation direction out of the orbital plane

  The variation of the thrust level along the orbit is presented on Figure 4-3 for various out-of plane observation directions. One can see that the amplitude of the guidance law is all the higher as the elevation is small with respect to the orbital plane. The worse case is reached when the elevation is null i.e. when the observation direction is in the orbital plane. The amplitude of the thrust is periodic like in the in-plane observation direction with a period equal to half the orbital period.

  The sizing case is therefore the observation in the orbital plane. From now on, only the in-plane case will be considered.

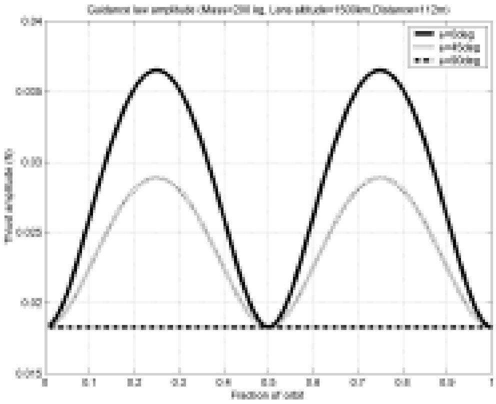

Figure 4-3 : Guidance law amplitude (out of plane observation)

NB: The ESA project Xeus [RD5] use guidance laws similar to the ones presented here.

4.2.2 Sensitivity to altitude and inter-satellite distance

Although an altitude of 1500 km is selected as baseline for Max in case the operational orbit is circular, other altitudes could be envisaged if a launch opportunity at higher altitude is strongly identified. Therefore, it seems interesting to show the sensitivity of the thrust level to the altitude of the operational orbit and to the distance between the satellites (the inter-satellite distance in Max is however pretty much fixed).

This is performed on Figure 4-4 where the operational orbit is taken either as a circular orbit at 1500 km (baseline altitude), at 25000 km or at 50000 km One can note that the current Max inter-satellite distance requires a thrust level compatible with existing low thrust propulsion systems on the whole range of altitudes.

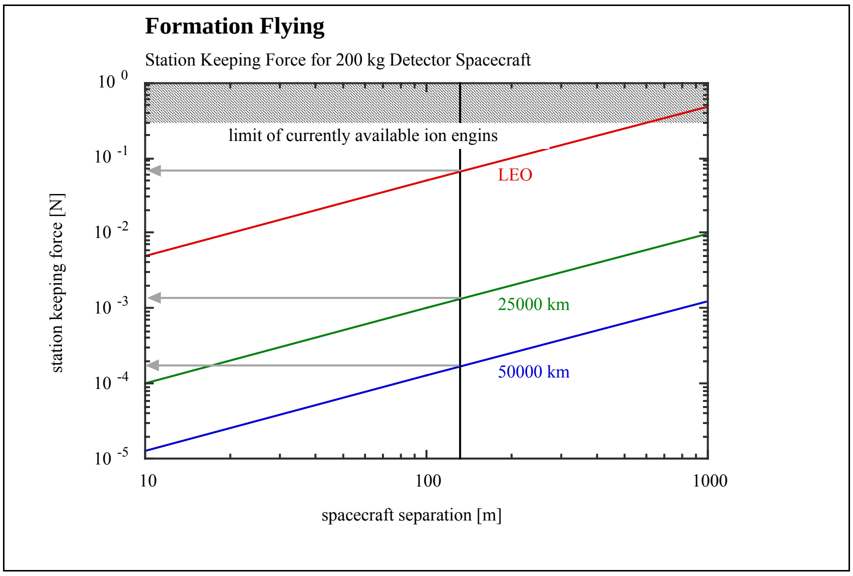

Figure 4-4 : Max thrust level for various altitudes and inter-satellite distances

Assuming a mean acceleration per orbit γ_m , one can deduce an estimation of the ergol consumption by the rocket equation:

$$\Delta m = m_0 \left(1 - e^{-\frac{\gamma_m \Delta t}{g_0 Isp}} \right) \qquad \text{Eq.3}$$

Which combined with a specific impulse of typically 3000s, yields the estimation of the required mass of Xenon presented on Figure 4-5.

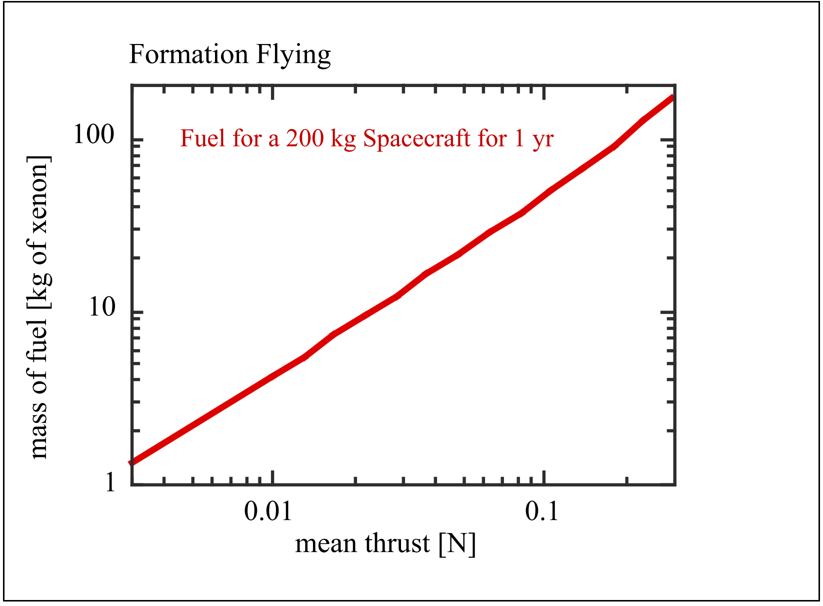

Figure 4-5 : Propellant mass per year for various mean thrust level (200 kg satellite)

4.3 Geostationary Transfer Orbit

4.3.1 Guidance laws

The evolution of the thrust magnitude along the orbit is presented on Figure 4-6 for different in-orbit observation angles ψ. It can be seen that the thrust level is high around perigee as expected to counter-act the higher differential gravity gradient between the satellites at this point of lower altitude.

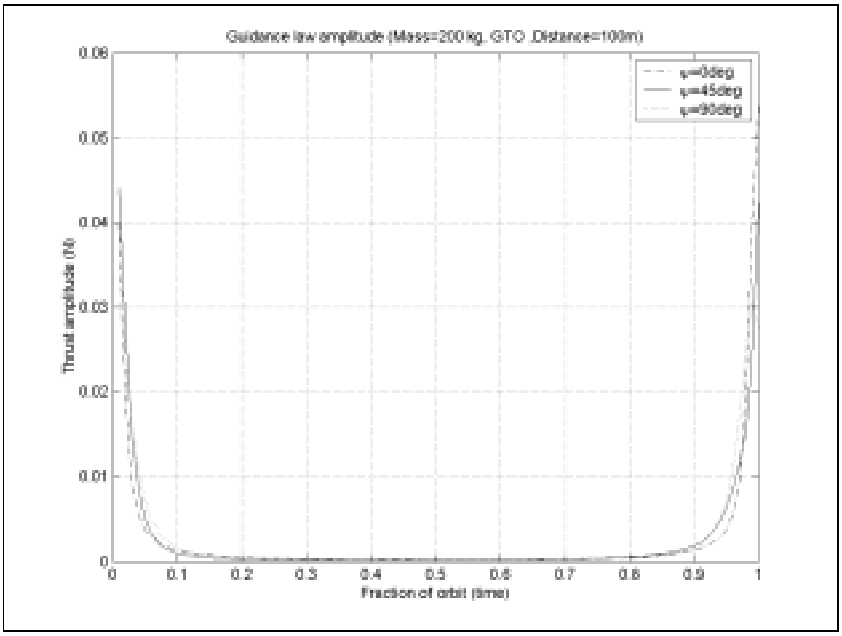

Figure 4-6 : Evolution of the thrust magnitude for different in-orbit observation angles

With such a variation of the thrust between the perigee and the rest of the orbit, it is difficult to distinguish what are the most favourable in-orbit directions. If we concentrate on the level of thrust at perigee, it can be shown that the thrust level is minimal when the in-orbit observation direction is perpendicular to the line of apsides, and Maximal when aligned with the lines of apsides.

However, when looking at the average thrust level along the orbit and its dependence with respect to the in-orbit observation ψ angle, one can note that it becomes independent from ψ (the perigee is therefore not a good indicator of the average thrust level along the orbit).

4.3.2 Sensitivity to apogee duty-cycle duration

Since thrusting to keep constant the observation direction is at high level around perigee and the time spent at perigee represents only a small portion of the orbital period, it is natural to think of performing the measurements only on a portion of the orbit around the apogee to reduce the overall consumption and the requirement on the propulsion system in terms of Maximum thrust level.

NB: From a mission point of view, this means that the observation time per orbit is reduced, which increases the total time required to collect sufficient gamma rays (see [MR1]). The increase in total time is however not simply inversely proportional since the emission of gamma rays by a SN Ia decreases in intensity with time. The impact on the measurement has therefore to be precisely studied.

This remarks being said, let us introduce the apogee duty cycle τ defined as the ratio of the time during which the thrust is activated over the orbit period. By definition, the apogee duty cycle τ belongs to the interval [0 1] where 1 corresponds to thrusting (and performing the measurement) all along the orbit (previous case) and 0 to no thrusting at all (and no measurements, which has no practical meaning).

The introduction of an apogee duty-cycle leads to stopping and starting the thrust at specific points of the orbit without consideration of the achieved orbital energy at these points. This aspects has however to be considered for example at the end of the thrust arc to ensure that no secular term appears in the evolution of the relative position between the satellites. A dedicated manoeuvre is therefore implemented at the end of the thrust arcs to ensure that this condition on equal orbital energy is met and at the beginning of the thrust arcs to ensure that the initial relative position is the required one (separation by the inter-satellite distance and satellites correctly aligned in the observation direction). When accounting for that phenomenon, the gain is no more infinite for a null duty cycle since there is anyhow to account for the contribution of two manoeuvres - one at the beginning of the thrust arc and one at the end – whatever the duty cycle.

The ratio of the ergol expenditure along the orbit for a duty cycle of one divided by the ergol expenditure along the orbit for a given duty cycle is plotted along the apogee duty cycle τ on Figure 4-7 (when accounting for the start and end manoeuvres).

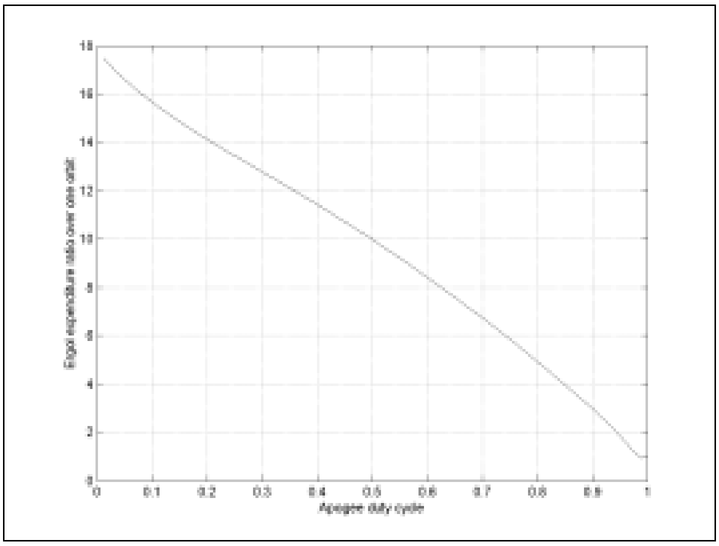

Figure 4-7 : Ergol expenditure ratio over one orbit versus apogee duty cycle

The variation is roughly linear, which is an unexpected result for a situation where the relationship between time and true anomaly is no more linear (contrary to circular orbits), but this is only due to the introduction of start and end manoeuvres.

But it must be pointed out that the previous ratio does not correspond to the real ergol expenditure ratio for one observation: let us suppose that an observation must be done during one orbit duration. If the duty cycle is equal to 0.5, the observation must be done during 2 orbits meaning that the real ergol expenditure ratio is twice less than the previous ratio. In other words the real ergol expenditure ratio is equal to the previous ratio divided by the duty cycle:

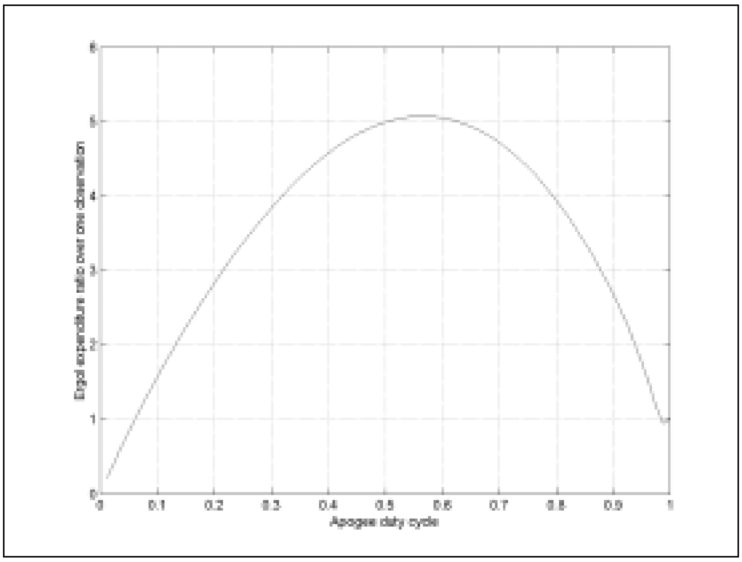

Figure 4-8 : ergol expenditure ratio per observation time

The results presented in Figure 4-8 are obtained by considering just the semi-major axis changes that need to be performed at the beginning (respectively at the end) of the observation periods to acquire the line of sight (respectively to bring back the satellites in a stable relative motion). It does not pay attention to the additional possible contributions due to the changes of the other orbital elements and therefore should be taken only as first approximation.

5. CONCLUSION

The Max mission aiming at studying supernovae of type Ia (SN Ia) proposes a paradigm change in the way of measuring the intensities, shifts and shapes of their nuclear gamma-ray lines. The related concept is based on two satellites – one embarking the diffraction lens, the other, the detector - flying in close formation and looking in the same inertial direction. The guidance laws to fulfil this constraint were derived and appear to be compatible with existing low-thrust propulsion technologies. Two candidate operational orbits were further studied: a Low-Earth Orbit and a Geostationary Transfer Orbit, the latter allowing to reduce the propellant needed to perform the mission. At this stage, no conclusion is however drawn on the relative interest between both types of orbits since further studies are needed to address finer phenomena appearing when thrusting (or equivalently observing) on a reduced portion of the elliptic orbit.

6. REFERENCES

[1] A. Boutonnet, B. Escudier, and V. Martinot, “Direct approach for spacecraft formation flying positioning in quasicircular low Earth orbit”, Paper 02-4638, AIAA ASC, Monterey, California

[2] S. R. Vadali and S. S. Vaddi, “Large angle kinematics for the control of satellite relative motion”, AIAA 2002-4742, Astrodynamics Specialist Conference, Aug. 2002, Monterey, California

[3] S. Kemble and B. Swinburne, “Design of satellite formations in low Earth orbit and deep space”, IAC-02-A.2.03, Houston, Oct. 2002

[4] H. Yamakawa and al. , “Formation flight using low propulsion”, 2nd International Symposium on low thrust trajectories, Toulouse, Jun. 2002

[5] About XEUS: http://sci.esa.int/content/doc/de/2526.htm