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The dynamo effect/L'effet dynamo

The solar dynamo

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Abstract

We shortly review the basic observational facts concerning the solar dynamo. Then, a brief overview of our current understanding of the large-scale evolution of the magnetic field of the Sun is proposed, showing, in particular, some successes and difficulties of the mean-field models. We illustrate the complications of this problem with recent work on stellar dynamos. We also compare the solar situation to that of the core of the Earth as well as those of laboratory and numerical experiments. **To cite this article:** *M. Rieutord, C. R. Physique 9 (2008).*

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Résumé

La dynamo solaire. Nous passons brièvement en revue les principaux faits observationnels concernant la dynamo solaire. Nous exposons ensuite notre compréhension actuelle de la génération du champ magnétique solaire aux grandes échelles en montrant en particulier les succès obtenus par les théories de champ moyen, mais aussi les difficultés qui surgissent quand tous les aspects du problème sont pris en compte. Nous illustrons ces complications par les récentes observations de l'activité magnétique d'étoiles semblables au soleil, lesquelles montrent la grande sensibilité de certaines caractéristiques des dynamos stellaires aux paramètres des étoiles. Enfin, nous comparons la situation solaire à celle du noyau terrestre, des expériences de laboratoire et des expériences numériques actuellement possibles. **Pour citer cet article :** *M. Rieutord, C. R. Physique 9 (2008).*

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1. Introduction

The understanding of the solar dynamo and, more generally, that of stellar dynamos, is one of the grand challenges of astrophysics nowadays. Furthermore, a full knowledge of the solar dynamo would open interesting applications, like the prediction of the solar magnetic activity. This is a key parameter of the “space weather”, which is important for any space mission. Besides, such models would also be a useful input for the determination of the solar irradiance, which has an obvious influence on the Earth's climate.

The question is, however, so closely related to that of a theory of turbulence, that it is likely that we have to solve the latter to understand the former. Despite this daunting perspective, astrophysicists have tried to decipher the solar dynamo and, surprisingly, found some simple and robust rules that must be met by the models. These results leave the

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impression that we are not far from a self-consistent view of solar magnetism, but the devil is in the details: any close inspection of a given model stumbles over either an inconsistency or an ad hoc assumption.

Notwithstanding these difficulties, the solar dynamo (and more generally astrophysical dynamos) is a field of intense research as testified by the numerous reviews that have recently appeared (e.g. Ossendrijver [1], Charbonneau [2], Brandenburg and Subramanian [3], Proctor [4], Tobias and Weiss [5]), and which mention at least six hundred papers altogether!

Although I have not worked recently on the subject (my contribution was in the first direct numerical simulation of a dynamo based on compressible convection, (see Brandenburg et al. [6]), I nevertheless accepted the challenge of writing this short review with the hope that it will give an outside and synthetic view of the subject that may be useful to those of us who are not directly involved in this complicated, but fascinating, topic.

The followings will first present the most striking landmarks of observational constraints, followed by a summary of the current understanding of the general features of the solar dynamo. I then bring other stars on the stage in order to point out their potential merits for our understanding of the problem. Then, the geodynamo, numerical and experimental dynamos are compared to the solar situation, before some perspectives conclude this contribution.

2. A (very) short review of observational facts

The solar dynamo has its most obvious expression through the variations of the sunspot number and the famous butterfly diagram, which we show in Fig. 1. This diagram shows, as a function of time and latitude the number of sunspots. It gives the evidence of the dynamo wave which propagates from the mid-latitudes to the equator.

A detailed examination shows that the oscillation is not strictly periodic; cycle duration varies between 7 and 14 years with a mean value of 11 years. The diagram also shows a clear modulation of the cycles at a time scale of a hundred years (this is the so-called Gleissberg cycle with a 88 yrs period). Another modulation of ~ 200 yrs appears in the cosmogenic indicators (^{14}C , ^{10}Be) of the Earth atmosphere, which are sensitive to the Sun's magnetic field (e.g. Beer et al. [7]).

Observations of sunspots have also shown that they usually appear in pairs where the leading spot is of opposite polarity to the trailing one. Moreover, the polarities of leading and trailing spots reverse from one hemisphere to the other. They also reverse from one cycle to the next.

From these observations, known as Hale's sunspot polarity law, it has been inferred that the emergence of a sunspot pair is the signature of the rise of a magnetic flux tube coming from the toroidal magnetic field inside the convection zone of the Sun. The reversal of the polarities between two consecutive cycles indicates that the true period of the solar dynamo oscillation is 22 years in the mean. Furthermore, the fact that the leading sunspot is always closer to the equator than the trailing one (Joy's law), can be nicely interpreted in the model of a buoyantly rising flux tube (D'Silva and Choudhuri [8]).

Finally, longitudinally averaged magnetograms clearly show the large-scale dipolar magnetic field, which reverses during the sunspot maximum (see Dikpati et al. [9]).

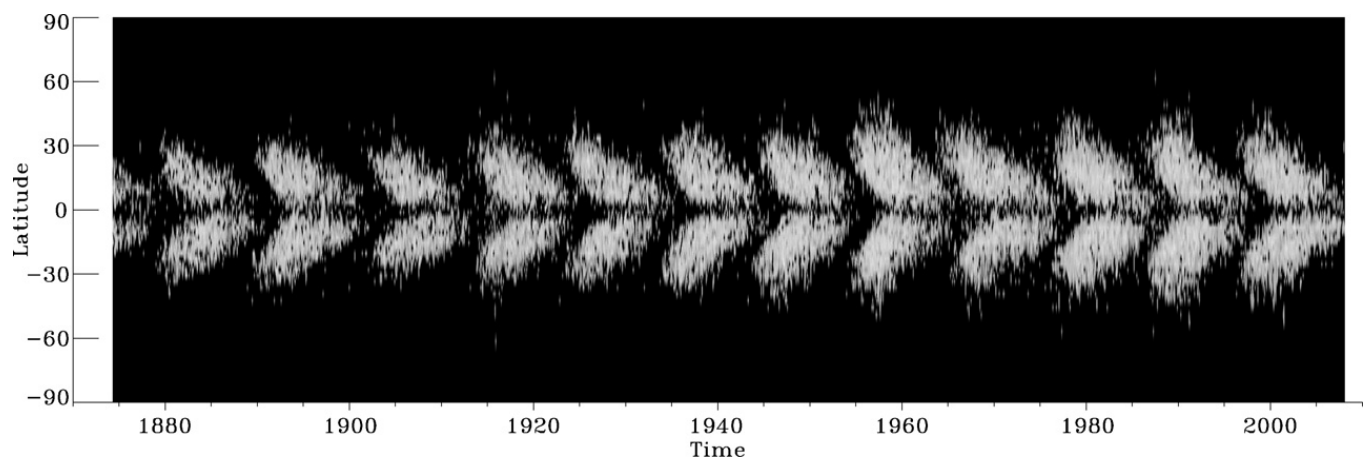


Fig. 1. The butterfly diagram during 140 years (data from the Greenwich Sunspot Database). This is the number of sunspots (the colour in log scale) as a function of the latitude of emergence and time.

3. Current understanding

3.1. Physical conditions of the solar dynamo

The structure of the Sun is well known (to some extent, of course!). The most stringent data come from helioseismology with the profiles of the sound velocity. Standard solar models typically tune three parameters (the initial mass fraction of helium and metals, and the ratio of the convective mixing-length to the pressure scale height) to make a model as close as possible to the Sun that we see now, i.e. after 4.6 Gyrs of evolution (e.g. Basu et al. [10]). Current precision is around a percent. Hence, the physical conditions under which the solar dynamo operates are rather well-known. Conductivity and viscosity are those of a plasma whose mass fraction of hydrogen is 0.71, helium 0.27 and metals 0.02. In numbers, the solar plasma is almost as if of pure hydrogen (the mean charge of ions is $\langle Z \rangle \sim 1.1$). Magnetic diffusivity η and dynamical viscosity μ are dominated by electron–ion collisions and plasma physics, in the fluid régime, says (Spitzer [11]):

$$\begin{cases} \eta = \frac{1}{\mu_0 \sigma} = \frac{1}{2\mu_0} \left(\frac{\pi}{2} \right)^{3/2} \left(\frac{e}{4\pi\epsilon_0} \right)^2 \frac{Z}{\gamma(Z)} \frac{\sqrt{m_e} \ln \Lambda}{(k_b T)^{3/2}} \\ \mu = 0.406 \left(\frac{4\pi\epsilon_0}{e^2} \right)^2 \frac{\sqrt{m_i} (k_b T)^{5/2}}{Z^4 \ln \Lambda} \end{cases} \quad (1)$$

where m_e and m_i are the masses of electrons and ions, respectively; k_b is Boltzmann constant and T the temperature; ϵ_0, μ_0 are the usual electromagnetic constants, e the electric charge and Z the charge number of ions; $\gamma(Z)$ is a non-dimensional function with $\gamma(1) \simeq 0.58$. $\ln \Lambda$ is the usual Coulomb logarithm with $\Lambda = \frac{8\pi(\epsilon_0 k_b T)^{3/2}}{Ze^3 \sqrt{n_e}}$, where n_e is the number density of electrons (from Delcroix and Bers [12]). The heat transport is dominated by photon diffusion, therefore we use the heat radiative conductivity $\chi_{\text{rad}} = 16\sigma T^3/3\kappa\rho$ where κ is the mean Rosseland opacity and σ Stefan constant. From these expressions we can estimate the kinetic and magnetic Prandtl numbers as well as the kinetic and magnetic Reynolds numbers. For the latter, a velocity and a length scale are also necessary. We take them from the standard mixing-length model of solar convection.

To summarise the values, we plot these numbers as a function of the fractional radius of the Sun (see Fig. 2). It turns out that the solar dynamo works at small Prandtl numbers and (very) high Reynolds numbers. It is now useful to compare them with the same numbers for the Earth's core and the VKS experiment (Monchaux et al. [13], Berhanu et al. [14]).

Table 1 shows that the solar dynamo shares with the VKS laboratory experiment and the geodynamo a low magnetic Prandtl number and a high kinetic Reynolds number, while the main difference comes from the magnetic Reynolds number. This latter number is quite similar in VKS and geodynamo, but much lower than the Sun's. This means that the Sun (and stars) harbours a very large number of magnetic scales that we cannot find in the geodynamo and present laboratory experiments.

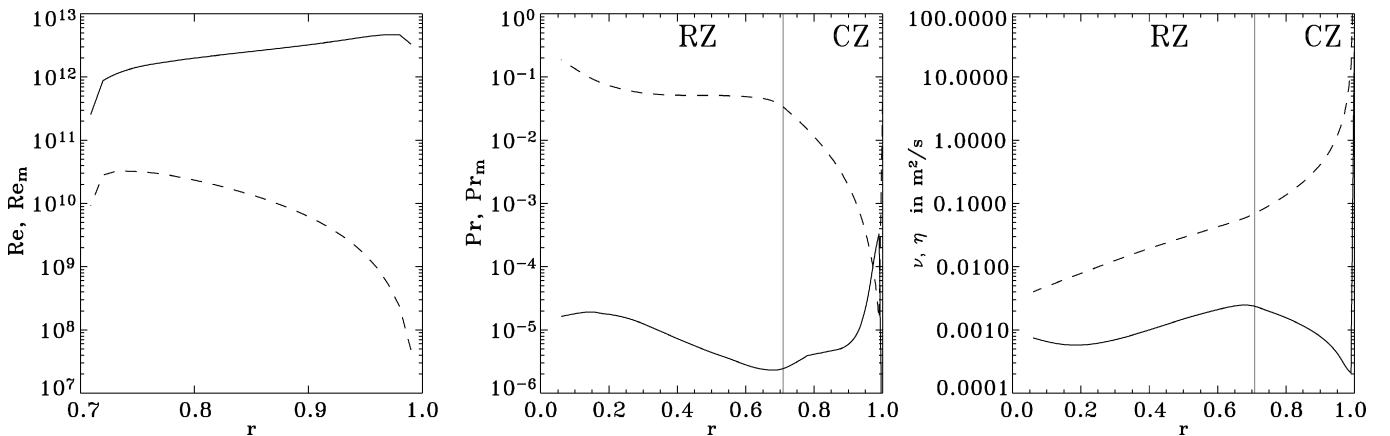


Fig. 2. Left: Kinetic (solid line) and magnetic (dashed line) Reynolds numbers in the solar convection zone as a function of the fractional radius. Middle: The kinetic (solid line) and magnetic (dashed line) Prandtl numbers in the whole Sun. Right: The kinematic viscosity (solid line) and magnetic diffusivity (dashed line). The vertical line separates the radiative (RZ) and convective zones (CZ).

Table 1
Values of the numbers in the core of the Earth and in VKS

Numbers	Earth's core	VKS
\mathcal{P}_m	5×10^{-7}	10^{-5}
Re	7×10^8	5×10^6
Re_m	360	49

3.2. The mean-field approach

The foregoing discussion has shown that the solar dynamo is operating at very high Reynolds numbers, and for this reason a large number of dynamic and magnetic scales are expected to contribute to it. However, our quick review of the observations shows that the solar dynamo has energy in large spatial scales, which evolve slowly compared to those of the turbulent convection; at first glance, their behaviour is rather simple and should be amenable to a simple theory

Thus mean-field approaches have been favoured and have had some successes although they still stumble on inconsistencies or ad hoc assumptions.

The mean magnetic field, usually taken as the longitudinally-averaged field $\langle \vec{B} \rangle$ has two components: the toroidal and the poloidal ones. It evolves according to

$$\frac{\partial \langle \vec{B} \rangle}{\partial t} = \vec{\nabla} \times (\langle \vec{v} \rangle \wedge \langle \vec{B} \rangle + \langle \vec{v} \wedge \vec{b} \rangle - \eta \vec{\nabla} \times \langle \vec{B} \rangle) \quad (2)$$

where we used the Reynolds decomposition of turbulent fields ($\vec{B} = \langle \vec{B} \rangle + \vec{b}$, etc.). In this evolution equation, difficulties come from the mean electromotive force $E_i = \langle \vec{v} \wedge \vec{b} \rangle_i$, which needs a closure of the turbulence to be related to other mean-fields. Since the pioneering work of Steenbeck et al. [15], this is usually done through an expansion over the derivatives of the mean-field, namely

$$E_i = \alpha_{ij} \langle B_j \rangle + \beta_{ijk} \partial_j \langle B_k \rangle + \dots \quad (3)$$

In this expression the third order pseudo-tensor β is related to the turbulent diffusion of the magnetic field, while the second order pseudo-tensor α provides the famous alpha-effect at the heart of mean-field dynamos. Notwithstanding the conditions of use of expansion (3) (for instance the separation of large and small scales), this expression of the mean electromotive force \vec{E} has proved to be a very useful paradigm for deciphering the solar dynamo (a more detailed introduction may be found in Moffatt [16]).

Early work on the solutions of (2) using (3) have discovered the now classical $\alpha - \Omega$ and α^2 dynamos. In these works, like Roberts [17], the α , β tensors follow simple prescriptions and the large-scale velocity field is just a differential rotation (or zero). The mean-fields appear as a solution of a linear eigenvalue problem that depends on the so-called dynamo numbers $C_\alpha = \frac{\alpha R}{\beta}$ and $C_\Omega = \frac{\Delta \Omega R^2}{\beta}$. The instability turns out to be usually oscillatory in the $\alpha - \Omega$ case and steady in the α^2 case (this is the case when no differential rotation is present and where the α -effect regenerates the toroidal and poloidal field). As shown in Roberts [17], the $\alpha - \Omega$ case is reminiscent of the solar dynamo: the toroidal and the dipolar fields oscillate periodically, showing an equatorward propagation of the activity belts in properly tuned cases.

3.3. The difficulties of the linear régime . . .

In the kinematic régime where the non-linear feedback of the Lorentz force is neglected, the α , β tensors can be estimated from the underlying turbulence. Setting τ as the typical turnover time scale, it turns out that

$$\alpha \sim -\frac{\tau}{3} \langle \vec{v} \cdot \vec{\nabla} \times \vec{v} \rangle, \quad \text{and} \quad \beta \sim \frac{\tau}{3} \langle v^2 \rangle \quad (4)$$

showing in particular that α is related to the local kinetic helicity, and of opposite sign to it (Moffatt [16], Charbonneau [2]).

A first difficulty already comes at this linear level. As noticed by Parker [18], downward motions ($V_z < 0$) concentrate background vorticity, which is positive in the northern hemisphere ($(\vec{\nabla} \times \vec{v})_z > 0$), hence the α -effect is positive

in the northern hemisphere. $\alpha - \Omega$ mean-field models have shown however that this implies a negative radial gradient of Ω , i.e. $\partial_r \Omega < 0$, for the dynamo wave to propagate, as observed, to the equator.

A popular model for the solar dynamo argues that, for the build-up of the toroidal field, one needs the interface between the radiative and convective zones, namely the tachocline, where strong fields can be stored and eventually emerge as Ω -shaped flux ropes (e.g. Tobias and Weiss [5]). However, within this region or just above, helioseismology shows that $\partial_r \Omega > 0$ in the latitude band $-30^\circ < \lambda < +30^\circ$, thus implying a poleward propagation of the dynamo wave. In this respect, helioseismology would suggest to take into account a layer at $r \sim 0.95 R_\odot$ where $\partial_r \Omega < 0$, but the small thickness of this layer, the latitude extension of activity belts and the storage of the toroidal field raise new problems (see Brandenburg [19] and Brandenburg and Käpylä [20]).

Other models attribute an important role to the meridional circulation like the Babcock–Leighton one (but see Charbonneau [2] for a complete discussion).

3.4. ... and the non-linear régime

Once the dynamo instability is obtained for a given set-up, one faces the question of how this instability saturates and at which level. As the alpha tensor is the key parameter of the instability, a natural way of incorporating the feedback of the Lorentz force in the mean-field model is to devise an alpha-quenching mechanism.

Various scenarios have been proposed for the quenching mechanism but the resulting form of the alpha-coefficient is always something like

$$\alpha = \frac{\alpha_0}{1 + \langle \vec{B} \rangle^2 / B_{\text{eq}}^2}$$

where $B_{\text{eq}}^2 = \mu_0 \langle \rho v^2 \rangle$ is the equipartition field. Such a recipe just expresses the fact that a strong enough mean-field suppresses small-scale flows responsible of the alpha effect. In the case of the Sun, the Ω -effect (the differential rotation) is also important and could also support some feed-back. This possibility is usually not considered since the toroidal field, i.e. the main component of the solar magnetic field, cannot reduce the shear that makes it growing. Moreover, observations indicate a very slight modulation of the differential rotation with the magnetic cycle (these are the torsional oscillations, e.g. Ulrich and Boyden [21]).

Thus non-linear effects should concentrate in the α coefficients. However, some authors (like Vainshtein and Cattaneo [22]) argued that the quenching should rather be like

$$\alpha = \frac{\alpha_0}{1 + Re_m \langle \vec{B} \rangle^2 / B_{\text{eq}}^2}$$

which was actually observed in a numerical experiment by Cattaneo and Hughes [23]. Such a quenching, called *catastrophic alpha quenching* after Blackman and Brandenburg [24], is extremely strong in the solar case where $Re_m \gtrsim 10^8$.

The understanding of this catastrophic quenching has motivated much work (cf. Brandenburg and Subramanian [3]). It turns out that it is the consequence of the conservation of magnetic helicity $\int_{(V)} \vec{A} \cdot \vec{B} dV$, where \vec{A} is the vector potential. As shown for instance in Moffatt [16], such a quantity is conserved in ideal MHD, when either no flux is allowed through the boundaries, or these are periodic. The role of the conservation of magnetic helicity has thus been intensively studied as a possible source of alpha quenching.

First studies, as reviewed by Pouquet [25], have demonstrated that the inverse cascade of the magnetic helicity and the non-linear feedback of the Lorentz force come into the alpha effect through the current helicity $\langle \vec{j} \cdot \vec{b} \rangle$, so that the kinematic expression of alpha in (4) should be written

$$\alpha = -\frac{1}{3} \tau \left(\langle \vec{v} \cdot \vec{\nabla} \times \vec{v} \rangle - \left\langle \frac{\vec{b} \cdot \vec{\nabla} \times \vec{b}}{\mu_0 \rho} \right\rangle \right)$$

when non-linear feedback is important. Following this line of research, Brandenburg and collaborators have devised a dynamical equation for α (e.g. equation 9.15 in Brandenburg and Subramanian [3]) which reproduces the algebraic catastrophic quenching observed by Cattaneo and Hughes [23].

Together with the catastrophic α -quenching appeared the catastrophic β -quenching, thus a catastrophic reduction of the turbulent diffusion (e.g. Vainshtein and Cattaneo [22]). Such a quenching, even if it favours the amplification of

the magnetic field, has also catastrophic consequences in the models since the turbulent diffusion controls the period of the dynamo oscillation (e.g. Roberts [26]): a strong reduction would imply unrealistic long time scales. Hence, here too the solar dynamo does not support the catastrophic quenching.

The foregoing discussion shows that the role of the various helicities in the solar dynamo needs to be well studied; α is likely not a simple function (e.g. Courvoisier et al. [27], Sur et al. [28]). It is urgent to understand how stellar dynamos escape to these catastrophic quenchings: what is the role of inhomogeneities, of open boundaries, which allow magnetic helicity losses via coronal mass ejections, etc.

4. The Sun and other stars

The solar dynamo is however only a particular, well detailed, case of the general stellar dynamo. One may thus wonder what other stars tell us about the generation of magnetic fields. As may be guessed, the data on stellar magnetic fields are much less numerous than those of the Sun. Broadly speaking, stellar magnetic fields are detected in three ways: directly, through the polarisation of many spectral lines, indirectly through the chromospheric emission (UV and X rays) or photometrically by the transit of spots on the stellar discs. In the best cases, where the magnetic field is detected directly through spectropolarimetry, and the star is rotating, Doppler imaging can be used to reconstruct the magnetic field on the large fraction of the stellar surface (Donati et al. [29]). This technique is, however, very recent and the time basis of such data is still very short compared to that available for the Sun. With the indirect indicators that we mentioned, data are less detailed but stars have been observed on a longer time scale.

Typically, stellar observations tell us about the intensity of the magnetic field, the magnetic flux, the time variations of the field. These magnetic quantities may then be related to global parameters of the stars, namely: (i) to the depth of the convection zone, which conditions the possible existence of a tachocline and controls also the vigour of the convection; these are direct consequences of the mass and the metallicity of the star; (ii) to the rotation rate; and (iii) to the differential rotation.

A promising way of research is the investigation of solar twins, i.e. stars which are very similar to the Sun except for one parameter. The work of Petit et al. [30] applies this idea to the rotation rate. An interesting result of this study is to show that the star HD146233 (18 Sco), which is almost not distinguishable from the Sun except for its rotation period (22.7 days instead of 24.7), has a magnetic cycle period of 7 years. This result indicates that the magnetic field oscillation is likely a sensitive observable of the dynamo. As it is related (in mean-field models) to the turbulent diffusion, we see that some details of the turbulent convection are quite influential (actually, the intrinsic variability of the Sun's magnetic cycle also points in this direction). On the other hand, the case of HD190771 (a young sister of the Sun, which has the same mass, but is likely only 2×10^9 yrs old, and rotates in 8.8 days) displays a strong toroidal field as well as a strong differential rotation, underlining the well-known link between these quantities (see Petit et al. [30]).

It is clear that, when a large data base of the magnetic activity of stars is complete, very interesting constraints will be given to stellar dynamos. Presently, stars do not bring very stringent cases, but this is just a question of time.

5. Hints from other dynamos

5.1. The geodynamo

In view of the difficulties faced at modelling the solar dynamo, it may be useful to compare it to the geodynamo where a lot of data are also available. Unfortunately the geodynamo operates in a rather different régime. Indeed:

- As shown by Table 1, the magnetic Reynolds number is much lower, meaning much fewer magnetic scales;
- From the drift of the Earth magnetic dipole, velocities of 10 km/year are thought to be typical of the fluid core; compared to the rotational velocity this means a very small Rossby number ($\sim 10^{-6}$), much smaller than the Sun's;
- The fluid of the core, liquid iron, is much less compressible than the Sun's plasma;
- If, as often admitted, the turbulence is driven by a solutal convection forced by the light elements released by the crystallisation of the inner core, the associated Prandtl number is likely much larger than unity;

- The boundary conditions are different: solid boundaries limit the liquid core of the Earth; the magnetic diffusivity is continuous at the inner core boundary, while it has a sharp variation at the bottom of the solar convection zone;
- The shell is thicker: the ratio of inner radius to outer radius is 0.35 for the core and 0.7 for the Sun.

Besides all these differences, two similarities remain: the flow is turbulent (high kinetic Reynolds number), and in both cases the flow occurs in a spherical shell.

The main differences are of course the range of the magnetic Reynolds number, and even more, that of the Rossby number (in the Sun $Ro \gtrsim 0.1$). This suggests that the Coriolis force plays a dominant role and that the geodynamo is in a magnetostrophic state where the Lorentz force balances the Coriolis force (see the contribution of Dormy and Le Mouél in this volume [31]).

5.2. The laboratory experiments

The development of the sodium technology has allowed the design of laboratory experiments showing the dynamo effect with a fluid flow. If the first successful experiments used rather constrained flows (see the contributions of Gailitis et al. [32] and Müller et al. [33] in this volume), the recent success of the VKS experiments opens new windows on natural dynamos.

The numbers shown in Table 1 show that the VKS dynamo operates in a range of parameters close to that of the Earth's core, namely low \mathcal{P}_m and low Re_m . The results of Berhanu et al. [14] are quite appealing for the geodynamo. As far the solar dynamo is concerned, the connection is not so direct as the Re_m is very large there.

We have seen in Section 3 that the most promising way of studying the solar dynamo is through the mean-field theories. This means that we need to determine the mean-field coefficients describing the effects of the small scales (to make short). The recent work of Brandenburg and collaborators has shown that these coefficients may be obtained from extra-equations determining their temporal evolution (in a homogeneous model). Thus, these studies are following the tracks of studies of pure hydrodynamic turbulence. Indeed, quite some time ago now, Launder and Spalding [34] proposed mean-field equations for non-homogeneous HD turbulence, where the mean-field coefficients are evaluated through the computation of the turbulent kinetic energy (K) and turbulent dissipation (ε), yielding the famous K - ε model (see also Mohammadi and Pironneau [35]). Such a model has been fruitful because non-dimensional coefficients used in modelling third order correlations have been calibrated by experiments.

Therefore, an interesting use of experiments for stellar dynamos, will be in the determination of the parameters controlling the alpha-effect or the turbulent diffusion, or any quantity that may be used in the modelling of the small scales. Compared to pure HD turbulence, MHD turbulence includes an inverse cascade of magnetic helicity. Things may thus be more difficult. Not surprisingly, we suggest the building of bigger experiments to achieve higher Re_m , allowing a scan of a greater range of parameters with a dynamo and including more magnetic scales.

5.3. The numerical experiments

Before concluding this short review, let us do a little tour of the numerical simulations of the solar and stellar dynamos. These numerical experiments concentrate in two kinds of work: those around the ASH code, developed in Boulder Colorado and initiated by P. Gilman and G. Glatzmaier at the beginning of the 1980s (Gilman and Glatzmaier [36]), and those, more recent, of W. Dobler and collaborators developed with the Pencil Code.¹ The first code addressed mainly the solar case, extending only recently to other types of stars (see Brun and Toomre [37], Brun et al. [38]), while the second focused on general stellar dynamo with noteworthy results on fully convective stars (e.g. Dobler et al. [39]). Such simulations should be termed as Large-Eddy Simulations (LES) as they never reach the real values of the parameters. Rather, small-scales are represented by enhanced diffusion coefficients. Despite the drastic simplifications that are used, these simulations are able to catch some observed features of the stellar dynamos (for instance the steady dipolar field of a completely convective, rapidly rotating star, e.g. Dobler [40]). But, again the devil is in the details: recently, Browning [41], using the ASH code for the same fully convective stars, finds that the differential rotation in these models is solar like (fast equator, slow pole), while Dobler et al. [39] find it anti-solar!

¹ <http://www.nordita.org/software/pencil-code>.

Regardless of these difficulties, such global simulations are likely to be a key to decipher stellar dynamos. However, they are presently lacking good subgrid-scale models, which make the comparison of their results to real stars rather delicate.

Another way of progress is to make Direct Numerical Simulations of a small chunk of fluid. Like experiments, these simulations teach us about the behaviour of mean-field coefficients or of the subgrid scales (e.g. Rieutord et al. [42], Courvoisier et al. [27], Brandenburg and Käpylä [20], Sur et al. [28]). Such a line of research is complementary of laboratory experiments as numerical experiments can explore other ranges of parameters and give a detailed output on all the fields.

6. Conclusions/outlook

The solar dynamo operates at very large kinetic and magnetic Reynolds numbers and therefore both the magnetic and velocity fields contain a large number of interacting scales. Despite much progress in the recent years, no satisfactory self-consistent model has yet emerged. Mean-field approaches have allowed us to understand many sides of the problem, but the successes are always partial. At the heart of the solar dynamo is the interaction of small and large scales and the inverse cascade of the magnetic helicity. An additional difficulty of the solar case is the strong variations of the turbulence from the top to the bottom and from the pole to the equator. The situation is far from homogeneous and in the end models will have to be cast into a sophisticated LES. But for this aim to be reached, numerical experiments, laboratory experiments and observations of other stars are essential guides.

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