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# THE ESTER PROJECT

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**Abstract.** The ESTER project aims at building a stellar evolution code in two dimensions of space for the study of effects of rotation. The numerical scheme is based on spectral methods with a spherical harmonic decomposition in the horizontal direction and a Chebyshev polynomial expansion in the vertical direction. Coordinates adapted to the centrifugally distorted shape are mapped to spherical coordinates. First tests on rotating polytropes are presented.

## 1 Introduction

Rotating stars are more and more focusing the attention of stellar physicists, especially because of the recent progress of observational technics like interferometry or high precision photometry. The best illustrative example is certainly the nearby star Altair whose rotation, diameter, etc. have been determined by interferometry (Domiciano de Souza et al. 2005, Peterson et al. 2005) and which has also been identified as an oscillating $\delta$-Scuti star (Buzasi et al. 2005).

There is therefore much need to develop stellar evolution codes which deal with rotation and all the associated hydrodynamical effects in an accurate way. The challenge is to understand:

- The structure of a rapidly rotating star, the flows in radiative regions and their effects on surface abundances,
- the mechanism of angular momentum loss,
- the effects of rotation on stellar oscillations,
- the relation between rotation and magnetic activity.

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Hence to be able to understand the effects of rotation at all evolutionary time scales in the life of a star.

Work on two-dimensional stellar models started with James (1964) with the computation of the structure of rotating polytropes. More physics has then been incorporated into the models but since 25 years 2D-models make no real progress because the physics is still essentially one dimensional: a barotropic approximation is used, and only Poisson's equation is solved in two dimensions. Most recent works are those of Roxburgh (2004) and Jackson et al. (2005). All these models neglect the dynamics and none have reached the 'evolution stage'.

The project ESTER (Evolution STEllaire en Rotation) takes up the challenge of computing the structure of a rapidly rotating star with a self-consistent description of the hydrodynamics.

## 2 A first step: the bipolytrope

As a first step we considered a simple model of a star, namely the bipolytrope. In this model the radiative and convective zones are describred by polytropic equations. Typically, we use a $n = 3/2$-polytrope for the convective region and a $n = 3$-polytrope for the radiative one.

In order to derive the set of equations to be solved, it is useful to recall some properties of a one dimensional composite polytrope (see Chandrasekhar 1939).

The continuity of pressure (mechanical equilibrium) together with the continuity of temperature (thermal equilibrium) imply the continuity of density. From the expression of the "enthalpy" $h = (n+1)P/\rho$, we see that this quantity is not continuous at the interface. From the equation of hydrostatic,

$$(n+1)P/\rho + \phi - \frac{1}{2}\Omega^2 s^2 = Cte$$

the constants are not necessarily the same in each domain (see below). We also note that $\phi$, the gravitational potential, is not constant at the interface but $\phi_{\rm eff} = \phi - \frac{1}{2}\Omega^2 s^2$ is constant and discontinuous. In the first (central) domain we write $\rho = H^{n_1}$ with

$$H = 1 - (\phi - \phi_0) + \frac{1}{2}\Omega^2 s^2$$

$\phi_0$ is the central value of $\phi$, $s = r\sin\theta$ and the density is scaled by its central value. In the second domain, the enthalpy has the same form, but the constant is not unity:

$$H = C - (\phi - \phi_0) + \frac{1}{2}\Omega^2 s^2 = C - \phi_{\rm eff} = (n_2+1)K_2\rho^{\gamma_2 - 1}$$

Since $P = K\rho^\gamma$, pressure continuity implies $K_2 = K_1\rho_1^{\gamma_1-\gamma_2}$ so that

$$C = \phi_{\rm eff}(1) + (n_2+1)K_1\rho_1^{\gamma_1 - 1}$$

Scaling potentials with $h_0 = (n_1+1)K_1\rho_0^{\gamma_1-1}$, we find

$$\rho = \rho_1 \left(1 + \left(\frac{n_1+1}{n_2+1}\right) H_1^{-1} \left(\phi_{\rm eff}(1) - \phi_{\rm eff}\right)\right)^{n_2} \tag{2.1}$$

The index 1 represents the surface where the polytropic index changes. Setting

$$H_< = 1 - (\phi - \phi_0) + \frac{1}{2}\Omega^2 s^2, \quad H_> = 1 + \left(\frac{n_1+1}{n_2+1}\right) H_1^{-1} \left(\phi_{\rm eff}(1) - \phi_{\rm eff}\right)$$

the equations to be solved are:

$$\Delta\phi = \Lambda H_<^{n_1}, \qquad \Lambda = \frac{4\pi G\rho_0 R^2}{h_0} \tag{2.2}$$

in the inner domain and

$$\Delta\phi = \Lambda\rho_1 H_>^{n_2} \tag{2.3}$$

in the outer domain; $\rho_1$ is the density at the interface scaled by the central density. We note that $\phi_{\rm eff}(1) = 1 - \rho_1^{1/n_1} = 1 - H_1$ so that

$$H_> = 1 + \left(\frac{n_1+1}{n_2+1}\right) H_1^{-1} \left(1 - H_1 - \Lambda(\Phi - \Phi_0) + \frac{1}{2}\Omega s^2\right)$$

where we set $\phi = \Lambda\Phi$. Parameters of the configuration are $n_1, n_2$ and $\rho_1$. $\Lambda$ must be such that the density vanishes at the equator. From the expression of $H_>$, we find that

$$\Lambda = \frac{H_1}{\Phi_{\rm eq} - \Phi_0}\left(\frac{n_2 - n_1}{n_1+1}\right) + \frac{1 + \frac{1}{2}\Omega^2}{\Phi_{\rm eq} - \Phi_0} = \frac{\Omega^2}{2(\Phi_{\rm eq} - \Phi_{\rm pole})}$$

Note that in a non-rotating polytrope with $n_1 = n_2$, the radius of the polytrope is $\xi_1 = \sqrt{\Lambda}$ with classical notations.

## 3 Results

To solve (2.2) and (2.3) we use the method developed by Bonazzola et al. (1998). Briefly, the computational domain is divided into two domains: the inner one contains the star, the outer one the vacuum. The outer boundary is a sphere where boundary conditions on the potential can be applied. Poisson's equation is solved iterativly following Bonazzola et al. (1998).

The result is shown in figure 1, where the density distribution and the gravitational potential have been plotted. The dashed line on the graph shows the surface of the star. The configuration which has been computed mimics a young solar type star with an outer convective envelope represented by a n=3/2-polytrope and a radiative core by a n=3 polytrope. The density at the interface is at $\rho_1 = 0.677$. In figure 2 we plotted the spectra of $\Phi$ in the vertical and horizontal directions. Note the fast convergences. In the domain with matter the vertical convergence is not so fast because of the jump in polytropic index. Spectral convergence can be recovered if separating the two parts of the star into two separate domains.

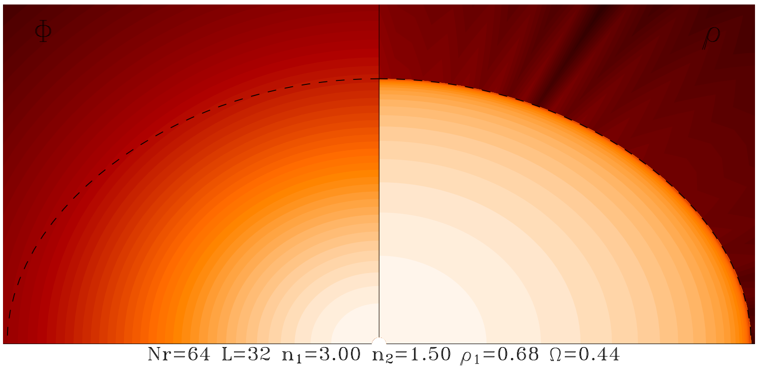


**Fig. 1.** A bipolytrope in rapid rotation (core n=3, envelope n=3/2). Left is gravitational potential; right is density. Ratio of equatorial to polar radius is 1.27 .

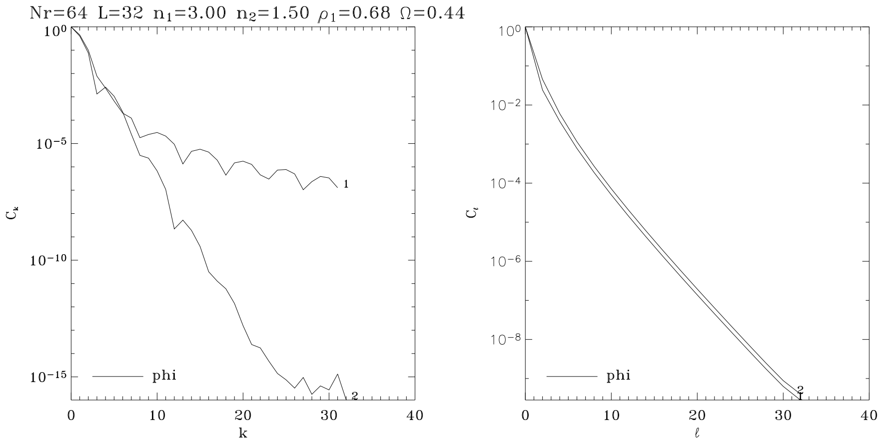


**Fig. 2.** Spectra in Chebyshev polynomial (left) and in spherical harmonics (right) and in inner (1) and outer (2) domains. Note the fast convergence except in the vertical direction in the star because of polytropic index jump.

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