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STABILITY OF A COMPRESSIBLE FLUID LAYER IN A MAGNETIC FIELD: A SIMPLE MODEL FOR SUPERGRANULATION

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Abstract. We investigate the onset of steady Rayleigh-Bénard magnetoconvection in a polytropic layer of index $m = 1$ enclosed between two thermally insulating plates. In a non-magnetic Boussinesq context, such boundary conditions, also called fixed flux conditions, are known to favour an infinite horizontal scale as the first unstable mode. We show that the situation can be very different when compressibility and magnetic field combine, giving birth to a finite horizontal scale in most cases. We explore different regimes and demonstrate in particular that the transition to a non-zero wave number occurs for a Chandrasekhar number $Q = 394.3$ in the Boussinesq limit, while compressibility tends to reduce this critical value. We discuss these results in the context of large scale stellar convection such as supergranulation where fixed flux conditions are physically relevant and for which the effects of magnetic fields and stratification cannot be bypassed.

1 Introduction

In models of large scale convection at the solar surface, the choice of thermal boundary conditions proves to be an important point (Depassier *et al.*, 1981). *Thermal flux conditions* (i.e. imposing a constant heat flux on both boundaries) are relevant at the scale of supergranulation (30 Mm), since most of the solar flux is transported by granules (1 Mm). Owing to the scale separation between granules and larger structures, it is also reasonable to assume that granules act as diffusing structures and to model them by using large eddy viscosities/diffusivities. These properties suggest a simple model of the photosphere, viewed from the large scale point of view : a highly stratified plane fluid layer (a $m = 1$ polytrope) permeated by a constant vertical magnetic field. We investigate the onset of steady convection in such a layer, when thermal flux boundary conditions apply (stress-free

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conditions are chosen for the velocity field), in order to determine the influence of stratification and magnetic field on the preferred scales of motion.

Unlike the usual case of fixed temperature conditions, the critical wave number is zero when flux conditions apply, as proved by Hurle *et al.* (1966). The corresponding critical Rayleigh number is 120 in the non-magnetic stress-free case. However, Murphy *et al.* (1977) briefly reported that a transition to finite critical horizontal wavelength is possible in the Boussinesq (incompressible) approximation when a sufficiently strong magnetic field is imposed, while stratification is known to favour smaller horizontal scales compared to the Boussinesq case (Gough *et al.*, 1976). It is therefore expected that the critical scale at the onset of convection highly depends on both magnetic field and stratification.

2 Main results

2.1 Notations and physical regimes

In the following, a is the horizontal wave number of the convection cells and R is the Rayleigh number taken in the middle of the layer. Q, the Chandrasekhar number (Chandrasekhar, 1961), is proportional to the square of the magnetic field. The temperature is z_o at the top of the layer and $z_o + 1$ at the bottom (Depassier *et al.*, 1981). To fix ideas small z_o describe highly stratified layers, whereas large z_o represent nearly incompressible fluids. The $z_o \to +\infty$ limit is the Boussinesq limit.

2.2 Boussinesq study

We briefly report Boussinesq results showing the transition to finite horizontal wavelength at marginal stability (Fig. 1). In Rincon and Rieutord (2003), it is shown that the transition occurs when the Chandrasekhar number is greater than 394.3, and that the critical Rayleigh number for $a = 0$ is given by

$$\mathrm{R}_c(a=0,Q) = \frac{12\,Q^{5/2}}{Q^{3/2} + 24\,\tanh\left(\frac{\sqrt{Q}}{2}\right) - 12\sqrt{Q}} \qquad (2.1)$$

In the large Q limit, $\mathrm{R}_c \to 12Q$, similar to the $\pi^2 Q$ law of Chandrasekhar (Chandrasekhar, 1961) in the case of fixed temperature boundary conditions (this asymptotic behaviour is illustrated on the full line curve of Fig. 2b).

2.3 Combined influence of stratification and magnetic field

When compressibility is taken into account without any magnetic field, no transition such as the one outlined before occurs. Stratification alone is unable to change the horizontal geometrical properties of the system. Mixing stratification and magnetic fields is more interesting. When stratification increases (lower z_o), the required magnetic field for the transition to occur is reduced from 394.3 in the

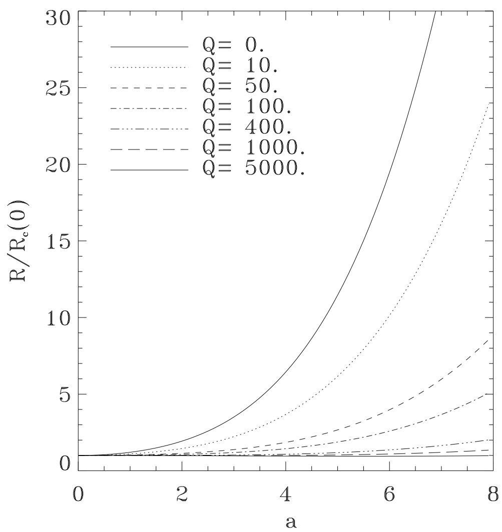


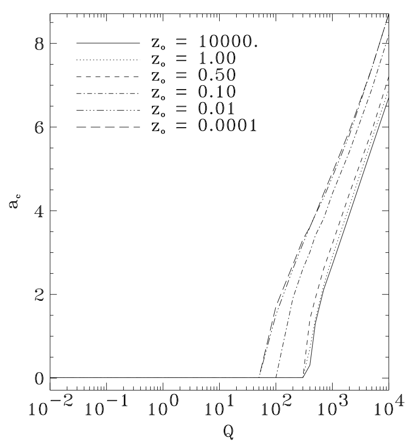
Fig. 1. Marginal stability curves and the transition to non-zero wave number a in a Boussinesq layer, as a function of the Chandrasekhar number Q. The critical wave number, associated with the minimum of the curve, shifts from zero to non-zero when Q exceeds 394.3.

Boussinesq limit to 61 in the case of an infinite number of density scale heights. Fig. 2a and Fig. 2b illustrates that the range of unstable Rayleigh and wave numbers is limited to a narrow band between infinite and zero stratification. In all cases compressibility and magnetic field combine to raise the threshold of instability.

3 Application to supergranulation in the upper solar convection zone

We conclude this study with an estimation of the required magnetic field amplitude in order to create supergranules. Assuming a photosphere with a hundred density scale heights ($z_o = 0.01$), a depth of 5 Mm with bottom density $\rho_b = 10^{-3}\,\mathrm{kg.m}^{-3}$, and a turbulent viscosity $\nu_T = 10^8\,\mathrm{m}^2.\mathrm{s}^{-1}$, $a \sim 1$, corresponding to supergranulation, is critical when $Q \sim 100$, following Fig. 2a. The associated magnetic field is 100 G, in good agreement with recent intra-network fields estimations (Lin, 1995).

Even though our model is obviously oversimplified and ignores the detailed properties of the upper convection zone flows and magnetic structures, it outlines the importance of the relations between the solar magnetic field and the velocity patterns at the scale of supergranulation. An observational investigation of the evolution of the typical horizontal scale of supergranulation over a solar dynamo cycle could give some insight on the physical processes that give birth to supergranulation.



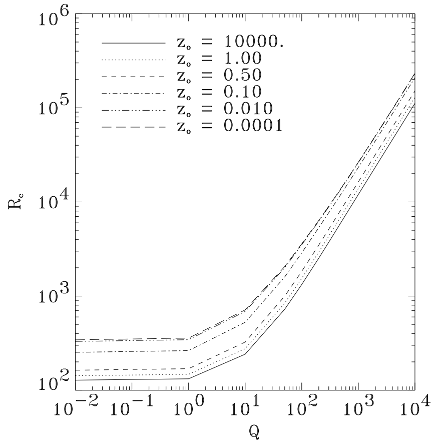


Fig. 2. a) Influence of stratification and magnetic field on the transition to non-zero wave number. b) Behaviour of the critical Rayleigh number when the magnetic field increases, for various stratifications.

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