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EKMAN PUMPING AND TIDAL DISSIPATION IN CLOSE BINARIES: A REFUTATION OF TASSOUL'S MECHANISM

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ABSTRACT

We show that the existence of an Ekman boundary layer does not enhance the tidal dissipation in a close binary star because the tides do not exert a stress on the stellar surface. The synchronization timescale is of order $(\varepsilon_T)^{-2} t_{\rm adj}$, where $t_{\rm adj}$ is the (global) viscous damping time and ε_T is the tidal deformation caused by the companion (Darwin 1879; Zahn 1966; Scharlemann 1982; Rieutord & Bonazzola 1987). We thus refute the claim made by Tassoul (1987), who thought to have found a very efficient mechanism for the synchronization and circularization of binary systems. We analyze the paper by Tassoul & Tassoul (1992b) and prove that the alleged magnitude of their Ekman pumping is due to an improper treatment of the surface boundary conditions. Their mechanism would have dramatic, yet unverified consequences, as illustrated by two examples of tidal interaction: between Io and Jupiter, and in the newly discovered planetary system 51 Peg.
Subject headings: binaries: close — hydrodynamics — stars: interiors — stars: rotation

1. INTRODUCTION

Both the rotation and the orbital eccentricity of the early-type binaries are reasonably well explained by invoking the radiative damping of the dynamical tide (Zahn 1975, 1977). It is true that in some binaries the synchronization with the orbital motion seems to proceed faster than predicted (Rajamohan & Venkatakrishnan 1981; Giuricin et al. 1984). But the synchronization rate drawn from the tidal theory concerns the star as the whole, whereas the surface layers are spun down much faster than the deep interior because they experience a higher torque per unit mass. This was pointed out by Zahn (1984), and soon after Goldreich & Nicholson (1989) completed the picture by describing how the synchronization is achieved gradually from the surface toward the interior.

Thus there is no need for another mechanism to interpret the observations, although the theory clearly calls for further developments. For instance, the calculation of the surface rotation speed would require an adequate treatment of all physical processes that transport angular momentum inside the star (turbulence, waves, meridian circulation, perhaps even magnetic stresses), an ambitious program which has not been tackled so far.

But the present article has a different purpose. It is to convince the reader that the so-called hydrodynamical mechanism proposed by Tassoul (1987) to resolve the apparent discrepancies mentioned above does not operate with the announced efficiency and that it plays a negligible role in the tidal interaction. An earlier attempt by Rieutord (1992, hereafter Paper I) to refute this claim apparently did not reach its goal: Tassoul persists in his error, and, regrettably, efforts are spent here and there to confront his predictions with the observations.

In § 2 we discuss again the role of the Ekman layer in a rotating star and prove that the viscous dissipation which occurs within this layer is negligible to leading order as a result of the stress-free boundary conditions. In § 3 we point out the major flaws of Tassoul's analysis. For the reader who is not much acquainted with hydrodynamics, we take in § 4 a well documented example, namely the tides induced by Io on Jupiter, to demonstrate that Tassoul's mechanism would have dramatic, yet unobserved consequences. We illustrate this also with 51 Peg and its recently discovered planet.

2. THE FLUID FLOW IN A TIDALLY DISTORTED STAR

The theory of the tides in a viscous star is hardly a new subject (see Darwin 1879; Zahn 1966; Scharlemann 1982), but only in recent years has the role of the Ekman layer been examined more closely. Such layers appear near the boundaries of a rotating fluid, and since they are generally very thin, they enhance the viscous dissipation and can therefore increase drastically the rate at which the flow evolves from given initial conditions. This is why Tassoul (1987) invoked this mechanism to speed up the tidal evolution in close binary stars, but at the same time Rieutord & Bonazzola (1987) showed that the contribution of the Ekman layer is negligibly small because the fluid star experiences stress-free boundary conditions, unlike what occurs in laboratory experiments.

To explain how they reached their result, we shall take the same approach as Rieutord in Paper I, which is somewhat easier to follow for the nonspecialist than the original demonstration. It will prove convenient to consider a reference frame rotating with the angular velocity Ω of the tidal potential and to write the governing equations in nondimensional form. To achieve this we choose as the length scale the mean radius R of the star and $(2\Omega)^{-1}$ as the timescale. Then the dimensionless equations of the flow are

$$\begin{cases} \dfrac{\partial}{\partial \tau} \boldsymbol{u} + \boldsymbol{e}_z \times \boldsymbol{u} = -\nabla p + E \, \Delta \boldsymbol{u} \, , \\ \operatorname{div} \boldsymbol{u} = 0 \, , \end{cases} \tag{1}$$

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where $E = \nu/2\Omega R^2$ is the Ekman number, with ν being the kinematic viscosity of the fluid. The scalar function p contains the pressure and all the potentials acting on the fluid. For simplicity, we have assumed that the star is incompressible (the case considered also by Tassoul), that the velocity $\boldsymbol{u}$ is small enough to allow the linearization, that the viscosity is constant, that the orbit is circular, and that the orbital velocity does not vary in time.

This differential system needs to be completed by the boundary conditions applied on the free surface of the star. (Strictly speaking, the surface of the star is modified by the flow itself; however, since we are working at linear approximation, we may neglect this extra deformation and assume that the surface of the star does not depart from its equilibrium position.) The boundary conditions on the velocity are thus

$$\begin{cases} \boldsymbol{u} \cdot \boldsymbol{n} = 0 \,, \\ \boldsymbol{n} \times ([\sigma]\boldsymbol{n}) = \boldsymbol{0} \,, \end{cases} \tag{2}$$

where $\boldsymbol{n}$ is the outer normal to the surface of the star and $[\sigma]$ is the viscous stress tensor. These boundary conditions express that the velocity field is tangent to the surface and that no tangential stress—in particular no torque—is applied on the surface of the star.

2.1. *The Solution*

To construct the solution of equations (1) and (2) we shall proceed by steps, and we first solve the inviscid steady state problem, the solution of which is the so-called geostrophic flow. This flow $\boldsymbol{u}_0$ verifies the following system of equations:

$$\begin{cases} \boldsymbol{e}_z \times \boldsymbol{u}_0 = -\boldsymbol{\nabla} P_0 \,, \\ \operatorname{div} \boldsymbol{u}_0 = 0 \,, \\ \boldsymbol{u}_0 \cdot \boldsymbol{n} = 0 \quad \text{on the surface } P_0 = 0 \,, \end{cases} \tag{3}$$

which has the analytic solution (Greenspan 1969)

$$\boldsymbol{u}_0 = \left[\frac{\partial}{\partial h} P_0(h)\right] \boldsymbol{n}_b \times \boldsymbol{n}_t \,. \tag{4}$$

Here the following notations have been introduced: the surface of the star is represented by two equations: $z = f(x, y)$ for the part above the equatorial symmetry plane and $z = -g(x, y)$ for the part below. From these equations one derives the vectors $\boldsymbol{n}_t = \boldsymbol{\nabla}[z - f(x, y)]$ and $\boldsymbol{n}_b = -\boldsymbol{\nabla}[z + g(x, y)]$, which are respectively normal to the top and the bottom surfaces, and $h = f + g$, which is the total height comprised between these two surfaces.

Equation (4) shows that $\boldsymbol{u}_0$ is a two-dimensional flow, in agreement with the Taylor-Proudman theorem.

Let us stress that this result applies also to a tidally distorted star, the surface of which may be approximated by an oblate triaxial ellipsoid elongated in the direction of the companion (the x-axis). Its equation is

$$P_0(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0 \,, \tag{5}$$

and thus the normal vectors are given by

$$\boldsymbol{n}_b = -\boldsymbol{\nabla}(z + h) \,, \quad \boldsymbol{n}_t = \boldsymbol{\nabla}(z - h) \,,$$

where

$$h = \frac{c}{b} \sqrt{b^2 - x^2(1 - \varepsilon)^2 - y^2} \,,$$

with the elongation $\varepsilon = (a - b)/a$. Hence, the geostrophic velocity field is given by

$$\boldsymbol{u}_0 = \frac{c^2}{b^2} \frac{1}{h} \frac{dP_0}{dh} \left| \begin{matrix} y \\ -x(1 - \varepsilon)^2 \end{matrix} \right. , \tag{6}$$

which illustrates how this flow departs from uniform rotation.

2.2. *The Boundary Layer*

The geostrophic flow $\boldsymbol{u}_0$ is such that $\boldsymbol{u}_0 \cdot \boldsymbol{n} = 0$, but in general it does not verify the stress-free condition of equation (2). Therefore, we must add to it a boundary layer flow $\boldsymbol{b}_1$, the components of which lie in the tangential plane and the amplitude of which varies much faster in the direction (ξ_1) normal to the surface than in the tangential directions (ξ_2 and ξ_3). With these prerequisites $\boldsymbol{b}_1$ obeys a rather simple equation, describing the famous Ekman spiral:

$$\frac{\partial^2}{\partial \zeta^2} (\boldsymbol{n} \times \boldsymbol{b}_1 + i\boldsymbol{b}_1) = i(\boldsymbol{n} \cdot \boldsymbol{e}_z)(\boldsymbol{n} \times \boldsymbol{b}_1 + i\boldsymbol{b}_1) \,, \tag{7}$$

where ζ is the stretched normal coordinate pointing inward:

$$\zeta = (1 - \xi_1)/\sqrt{E} \,,$$

and $\xi_1 = 1$ delineates the surface of the star. The thickness δ of the Ekman layer is thus $\delta/R = \mathcal{O}(E^{1/2})$.

Equation (7) results from a combination of the projections of the momentum equations in a plane tangent to the surface and where only the depth dependence of the solution has been retained. Its solution is

$$\boldsymbol{n} \times \boldsymbol{b}_1 + i\boldsymbol{b}_1 = \boldsymbol{C} \exp\left[-(i\boldsymbol{n} \cdot \boldsymbol{e}_z)^{1/2}\zeta\right] \,, \tag{8}$$

where $\boldsymbol{C}$ is a complex vector, which will be chosen such that it cancels the tangential components of the viscous stress:

$$\begin{cases} \sigma_{12}(\boldsymbol{u}_0) + \sigma_{12}(\boldsymbol{b}_1) = 0 \\ \sigma_{13}(\boldsymbol{u}_0) + \sigma_{13}(\boldsymbol{b}_1) = 0 \end{cases} \quad \text{at } \xi_1 = 1 \,. \tag{9}$$

To lowest order, where we can neglect the curvature terms, these conditions translate into

$$\begin{cases} \dfrac{\partial u_{0,3}}{\partial \xi_1} + \dfrac{\partial b_{1,3}}{\partial \xi_1} = 0 \\ \dfrac{\partial b_{1,2}}{\partial \xi_1} = 0 \end{cases} \quad \text{at } \xi_1 = 1 \,. \tag{10}$$

For the tangential stress to vanish, $\boldsymbol{b}_1$ must be $\mathcal{O}(E^{1/2})$, which ensures that $\partial b_{1,3}/\partial \xi_1$ be of the order of $\partial u_{0,3}/\partial \xi_1$. Let us make this even more explicit; we combine the two equations of equation (10) into a single one so that we have

$$\frac{\partial}{\partial \xi_1} (b_{1,2} + ib_{1,3}) = -i \frac{\partial u_{0,3}}{\partial \xi_1} \quad \text{at} \quad \xi_1 = 1 \,.$$

Using now equation (8), we find the constant $\boldsymbol{C}$:

$$C_3 = -\frac{i\sqrt{E}}{\sqrt{i\boldsymbol{n} \cdot \boldsymbol{e}_z}} \left(\frac{\partial u_{0,3}}{\partial \xi_1}\right)_{\xi_1 = 1} \,, \quad C_2 = iC_3 \,,$$

which again shows that $\boldsymbol{b}_1$ is $E^{1/2}$ smaller than $\boldsymbol{u}_0$.

This small boundary layer flow $\boldsymbol{b}_1$ satisfies the (steady) momentum equation and the boundary conditions, but it does not conserve mass; i.e., in general $\operatorname{div} \boldsymbol{b}_1 \neq 0$. This is why one has to add to this flow yet another one, which we call $\boldsymbol{b}_2$ since it exists only in the boundary layer. It has just one component, which is normal to the surface:

$$\boldsymbol{b}_2 = B_2(\xi_2, \xi_3) \exp\left[-(i\boldsymbol{n} \cdot \boldsymbol{e}_z)^{1/2}\zeta\right]\boldsymbol{n} .$$

Its amplitude is determined by the continuity equation taken at the surface

$$\operatorname{div}(\boldsymbol{b}_1 + \boldsymbol{b}_2) \equiv \frac{\partial b_{2,1}}{\partial \xi_1} + \frac{\partial b_{1,2}}{\partial \xi_2} + \frac{\partial b_{1,3}}{\partial \xi_3} = 0 ,$$

which yields

$$B_2(\xi_2, \xi_3) = \sqrt{E}\, \frac{\operatorname{div} \boldsymbol{b}_1}{\sqrt{i\boldsymbol{n} \cdot \boldsymbol{e}_z}} ,$$

showing that this new component is $\mathcal{O}(E^{1/2})$ smaller than $\boldsymbol{b}_1$; it is therefore $\mathcal{O}(E)$.

Finally, since $\boldsymbol{b}_2 \cdot n$ does not vanish at the surface, one has to add an interior flow $\boldsymbol{u}_2$, which verifies the inviscid equations and the boundary condition $(\boldsymbol{u}_2 + \boldsymbol{b}_2) \cdot \boldsymbol{n} = 0$ and has therefore a component normal to the surface. This flow $\boldsymbol{u}_2$ is usually called the Ekman circulation; we see here that, like $\boldsymbol{b}_2$, it is $\mathcal{O}(E)$.

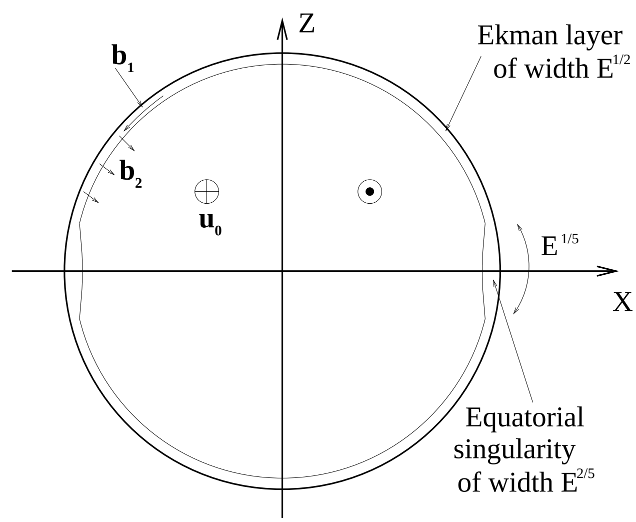

FIG. 1.—Schematic view of Ekman layers and the equatorial singularity; sizes of boundary layers have been much exaggerated. Note also that the main component of $\boldsymbol{b}_1$ is parallel to $\boldsymbol{u}_0$ and is not represented.

2.3. *Rigid Boundary Conditions*

In classical textbooks, such as Greenspan (1969), one considers usually the case of rigid boundary conditions, which are the most relevant to laboratory experiments. These require the vanishing of the velocity, and therefore the boundary layer flow is of the same order as $\boldsymbol{u}_0$ since

$$\boldsymbol{u}_0 + \boldsymbol{b}_0 = \boldsymbol{0}$$

on the boundary. This flow $\boldsymbol{b}_0$ (note the index 0) may be calculated in the same way as $\boldsymbol{b}_1$ above; similarly, it does not in general satisfy div $\boldsymbol{b}_0 = 0$ and needs to be corrected by a normal flow (the Ekman pumping), which is $E^{1/2}$ smaller than $\boldsymbol{b}_0$. In turn, that normal boundary flow induces in the bulk of the fluid an Ekman circulation $\boldsymbol{u}_1$ that is now $\mathcal{O}(E^{1/2})$.

This illustrates the importance of the boundary conditions in determining the magnitude of the different components of the flow: the Ekman circulation is $E^{1/2}$ weaker with stress-free boundary conditions than with rigid ones, and for this reason it plays a negligible role in the viscous dissipation, as we shall see.

2.4. *The Equatorial Singularity*

The boundary layer analysis outlined above breaks down close to the equator where the Coriolis force becomes horizontal, i.e., where $\boldsymbol{n} \cdot \boldsymbol{e}_z \to 0$. There is indeed a singularity in the asymptotic development in powers of $E^{1/2}$, and it arises because the scaling of the boundary layer changes in the region close to the equator. It has been shown (Roberts & Stewartson 1963; Stewartson 1966) that for latitudes less than $\mathcal{O}(E^{1/5})$ the width of the boundary layer is $\mathcal{O}(E^{2/5})$ instead of $\mathcal{O}(E^{1/2})$. Therefore, the boundary layer in the equatorial region appears to be "infinitely" thick compared to the rest of the Ekman layer, although it remains vanishingly small when confronted with the size of the star. Figure 1 gives an illustration of the situation.

However, this region of the flow does not change the overall dynamics of a spin-up/down process since the mass flux into the interior is still $\mathcal{O}(E)$. Indeed, as the width of the layer is now $\mathcal{O}(E^{2/5})$, $\boldsymbol{b}_2$ in this region is $\mathcal{O}(E^{4/5})$, but the latitudinal extension of the layer is only $\mathcal{O}(E^{1/5})$; therefore, the total flux generated in the interior is $\mathcal{O}(E^{1/5} \times E^{4/5}) = \mathcal{O}(E)$. There is thus no reason to expect that the equatorial bulge of the Ekman layer transfers angular momentum more efficiently than the rest of the layer.

The same conclusion may be reached by comparing the viscous dissipation in the equatorial singularity: $\mathcal{O}(E \times E^{1/5} \times E^{2/5}) = \mathcal{O}(E^{8/5})$, with that in the Ekman layer: $\mathcal{O}(E \times E^{1/2}) = \mathcal{O}(E^{3/2})$, or in the bulk of the star: $\mathcal{O}(E)$.

2.5. *Evolution Timescales of the Geostrophic Flow*

So far we have treated the problem as if it were steady in time. This was possible because at lowest order in $E^{1/2}$ the time derivative can be neglected in the momentum equation, compared to the other terms. But the quasi-steady solutions derived above will now serve to determine their evolution in time. The problem may be formulated as follows: given some initial flow which departs from the corotation with the tidal potential, i.e., $\boldsymbol{u} \neq 0$, on which timescale will this motion be damped out through viscous dissipation?

There are several ways of answering this question. A detailed demonstration has been given in Paper I (§ 2.3), but since it is a little technical, we choose here the more intuitive approach, based on the energy argument, which was also sketched in Paper I (§ 2.5).

Let us first assume that the star is not submitted to a tidal torque. The equation governing the evolution of the kinetic energy of the flow is

$$\frac{\partial}{\partial \tau}\left(\int_v \frac{1}{2} u^2\, dV\right) = -E \int_v \mathscr{D}\, dV , \qquad (11)$$

where $\mathscr{D}$ is defined by

$$\mathscr{D} = \sum_{i,j} s_{ij}^2(\boldsymbol{u}) \text{ with } s_{ij}(\boldsymbol{u}) = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} ;$$

the components s_{ij} of the shear stress are linear functions involving the first derivatives of the velocity. The time the star needs to reach uniform rotation, which we shall call the adjustment time, is thus given by

$$\frac{1}{\tau_{\rm adj}} = E\,\frac{\int_v \mathscr{D}\,dV}{\int_v \frac{1}{2}u^2\,dV}\,.$$

To estimate this time, it suffices to identify the main contributions to the kinetic energy and the viscous dissipation, respectively. The case of the kinetic energy is easy to settle since $\boldsymbol{u}_0$ dominates the flow, and thus

$$\int_v \tfrac{1}{2}u^2\,dV = \mathscr{O}(1)$$

with our nondimensionalization. For the dissipation we need to evaluate $s_{ij}(\boldsymbol{u}_0 + \boldsymbol{b}_1 + \boldsymbol{b}_2 + \boldsymbol{u}_2)$. Referring back to § 2.2, we have

$$s_{ij}(\boldsymbol{u}_0) = \mathscr{O}(1)\,, \quad s_{ij}(\boldsymbol{b}_1) = \mathscr{O}(1)\,, \quad s_{ij}(\boldsymbol{b}_2) = \mathscr{O}(E^{1/2})\,, \ldots$$

and therefore $\int_v \mathscr{D}\,dV$ is $\mathscr{O}(1)$, implying that the kinetic energy decays on the (overall) viscous timescale:

$$\tau_{\rm adj} \approx E^{-1} \quad \text{or} \quad t_{\rm adj} \approx \frac{R^2}{\nu}\,.$$

This is also the rate at which a binary component smooths out its internal rotation. However, the tidal potential does not allow the final state to be of uniform rotation since the flow must be tangent to the distorted surface. After that adjustment phase, the flow is thus of the form

$$\boldsymbol{u} = (\omega_s - \omega_o)\boldsymbol{e}_z \times \boldsymbol{r} + \varepsilon \boldsymbol{u}_T\,, \tag{12}$$

where ω_s and ω_o $(=1)$ are, respectively, the (mean) angular velocity of the star and that of the orbital motion. The magnitude of the extra term $\varepsilon \boldsymbol{u}_T$, which represents the tide, is proportional to the lack of synchronism and to the tidal deformation:

$$\varepsilon = (\omega_s - \omega_o)\varepsilon_T = (\omega_s - \omega_o)\,\frac{M'}{M}\left(\frac{R}{a}\right)^3\,,$$

M'/M being the mass ratio (companion/star), R the radius of the considered star, and a the semimajor axis of the orbit.

Let us now ask on which timescale the star evolves toward synchronism, i.e., to $\omega_s = \omega_o$. At present, the kinetic energy that is dissipated through viscous friction is drawn both from the rotation and from the orbital motion; neglecting the rotation of the companion, the total mechanical energy of the system is

$$\mathscr{E} = \tfrac{1}{2}I\omega_s^2 - \tfrac{1}{2}\mu\omega_o^2 a^2\,,$$

where I is the moment of inertia of the star and μ is the reduced mass of the system. Neglecting a possible mass loss, the system will evolve while conserving its total angular momentum:

$$J = I\omega_s + \mu\omega_o a\,.$$

We differentiate these two equations

$$d\mathscr{E} = I\omega_s\,d\omega_s - \tfrac{1}{3}\mu(GM_T)^{2/3}\omega_o^{-1/3}\,d\omega_o\,,$$
$$dJ = I\,d\omega_s - \tfrac{1}{3}\mu(GM_T)^{2/3}\omega_o^{-4/3}\,d\omega_o = 0\,,$$

and combine them, to establish the following relation between the star's deceleration (or acceleration) and the energy decrease of the system:

$$\frac{d\mathscr{E}}{d\tau} = I(\omega_s - \omega_o)\,\frac{d\omega_s}{d\tau}\,.$$

It remains to equate this to the viscous dissipation rate. With the velocity field given by equation (12), it is easy to check that

$$s_{ij}(\boldsymbol{u}_0) = \varepsilon s_{ij}(\boldsymbol{u}_{\mathrm{T},0}) = \mathscr{O}(\varepsilon)\,, \quad s_{ij}(\boldsymbol{b}_{\mathrm{T},1}) = \mathscr{O}(\varepsilon)\,, \ldots,$$

and therefore that

$$E\int_v \mathscr{D}\,dV = E\mathscr{O}(\varepsilon^2) = E(\omega_s - \omega_o)^2\mathscr{O}[(\varepsilon_T)^2]\,.$$

Hence, the synchronization timescale is given by

$$\frac{1}{\tau_{\rm sync}} = -\frac{1}{(\omega_s - \omega_o)}\,\frac{d\omega_s}{d\tau} \approx E(\varepsilon_T)^2\,,$$

or, equivalently, in natural units,

$$\frac{1}{t_{\rm sync}} = -\frac{1}{(\Omega_s - \Omega_o)}\,\frac{d\Omega_s}{dt} \approx \frac{\nu}{R^2}\,(\varepsilon_T)^2\,, \tag{13}$$

which is the classical result also found by Rieutord & Bonazzola (1987).[2]

3. COMMENTS ON TASSOUL'S SO-CALLED HYDRODYNAMICAL MECHANISM

In an article published the same year, Tassoul (1987) examines also the role of the Ekman layer, but he claims that the synchronization time is of order $\tau_{\rm sync} \approx E^{-1/2}\varepsilon_T^{-1}$ in nondimensional units, much shorter than the actual time given above. In a series of papers, his hydrodynamical mechanism, as he calls it, is described in more detail and is applied to various cases, including stars possessing a convective envelope (Tassoul & Tassoul 1990, 1992a, 1992b, Tassoul 1995).[3]

Tassoul's error, as the reader may have guessed, lies in his improper treatment of the Ekman flow. In the first paper, he does not even mention the stress-free character of the boundary conditions and states simply that the Ekman circulation is of order $|\boldsymbol{u}| \approx \varepsilon E^{1/2}$ (in our notation), as it would be with rigid boundaries. One may wonder whether he fully appreciates then the properties of the Ekman layer: he describes it as diffusing "throughout the bulk of the radiative envelope," whereas its thickness depends only on the viscosity and on the rotation rate.

In a subsequent paper, Tassoul & Tassoul (1990) elaborate on the eigenfunctions of the viscous decay. Following Rieutord (1987) and Rieutord & Bonazzola (1987), they expand these functions into spherical harmonics, and for reasons which will be analyzed below, they overestimate again the strength of the Ekman pumping, and hence of the tidal dissipation.

[2] For the complete expression, see Scharlemann (1982) or Zahn (1989).

[3] In a footnote in his latest paper, Tassoul (1995) declares that "[Rieutord's] analysis is severely flawed" and that "the currents defined in his eq. (13) do not satisfy the causality principle" since "they do not vanish in the limiting case $\varepsilon_T \rightarrow 0$." Apparently he fails to see that some solid rotation will then still persist in the reference frame rotating with the companion, unless the star was initially synchronized.

It is only in Tassoul & Tassoul (1992b, hereafter TT92) that the flow within the Ekman layer gets due attention. This paper is a reply to Rieutord's criticism of Tassoul's mechanism, and until its equation (16) the results fully agree with those of Paper I. TT92 obtain also the boundary layer flow $\boldsymbol{b}_1$ ($B_{1\eta}$ in their notation) of $\mathcal{O}(\varepsilon E^{1/2})$, which, as we have seen above in § 2, induces an Ekman circulation $\boldsymbol{u}_2$ of $\mathcal{O}(\varepsilon E)$. Surprisingly, they state that this flow $\boldsymbol{b}_1$ is of "paramount" importance, but they do not include it in their expansion (eq. [17]). Instead, they introduce an *interior* flow $\boldsymbol{u}_1$, allegedly also of $\mathcal{O}(\varepsilon E^{1/2})$, which they hold responsible for their fast tidal synchronization.

According to TT92, this extra flow originates in the equatorial singularity where the vertical component of the Coriolis force vanishes. In reality, as we have seen above in § 2.4, with stress-free boundaries this singularity induces a circulation that is only $\mathcal{O}(E)$ of the geostrophic flow, hence $\mathcal{O}(\varepsilon E)$.

The Tassouls' interior flow $\boldsymbol{u}_1$ is just an artifact of the expansion procedure they use to circumvent the singularity, which they describe in some detail in their Appendix B. Such expansions in spherical harmonics have indeed been employed in the theory of rotating fluids by Rieutord (1987, 1991), but they must be handled with care.

First, a result expanded in series of spherical harmonics is valid only if the convergence of the series can be assessed. The solutions given by TT92 contain at most two terms ($l = 1$, $l = 3$ or $l = 2$, $l = 4$) of the infinite series, which is far from sufficient since in the asymptotic case ($E \to 0$) the spherical harmonics are coupled by $\mathcal{O}(1)$ coefficients and the convergence of the series is only algebraic.

But there is a more serious reason why TT92 do not retrieve the correct scaling: some terms are missing in their expansions of the boundary conditions in spherical harmonics, and they are $\mathcal{O}(\varepsilon)$; in particular, the stress is ill-developed. It is then as if a spurious stress of $\mathcal{O}(\varepsilon)$ were applied on the flow, in which case it is quite natural that a circulation $\boldsymbol{u}_1$ arises, of $\mathcal{O}(\varepsilon E^{1/2})$. We give in the Appendix the correct form of the boundary conditions projected on the spherical harmonics and point out the terms missing in the Tassouls' derivation.

Finally, let us comment about Tassoul's decision to increase his synchronization time by an arbitrary factor 10, "to make allowance for small but finite departure from synchronism." It is true that the classical treatment of the spin-up/down problem assumes that the Rossby number $v/2\Omega R$ is small enough to permit linearization. But the nonlinear effects have been analyzed thoroughly by several authors (Wedemeyer 1964; Greenspan & Weinbaum 1965; Benton 1973; Weidman 1976); the latter summarizes the results, which apply to the case of rigid boundaries, where the characteristic timescale $t_{\rm adj}$ (in our notation) depends on the thickness of the Ekman layer, as it does in Tassoul's prescription. His first conclusion is that $t_{\rm adj}$ must be evaluated with the initial rotation rate Ω_i when the fluid is spun down, and with the final rotation rate Ω_f when it is spun up. If we transpose these rules to the synchronization of close binaries, we see that the thickness of the Ekman layer is determined by the star's rotation rate Ω_s when it is larger than the orbital rate Ω_o. The second conclusion of these studies is that, except for the extreme case where the boundaries are at rest, the spin-down follows closely the predictions of linear theory. Contrary to Tassoul's assertion, there is no justification whatsoever in Weidman's paper (which he quotes in Tassoul 1987) to lengthen his synchronization time; instead, that time should be shortened by a factor $(\Omega_o/\Omega_s)^{1/2}$ when the star rotates faster than the orbital motion, as it occurs most often.

4. TWO ILLUSTRATIVE EXAMPLES: IO's TIDES ON JUPITER AND 51 PEG

Some readers may find the developments above a trifle too technical, and they would rather prefer to be convinced by a suitable example. An excellent test case is provided by the tidal interaction of Jupiter and its closest satellite Io. Except for its innermost core, Jupiter is a fluid planet, and the theory of viscous dissipation applies.

Most parameters of the problem may be found in Allen (1973): Jupiter's rotation period $2\pi/\Omega$ is 9 hr 50.5 minutes, its radius measures $R = 71{,}300$ km, the mass ratio is 26,500, and the orbit is circular with a radius d of 422,000 km. The subsurface layers of Jupiter are convective, and their turbulent viscosity ν may be inferred from the convective flux, using the mixing length recipe: it is about $2 10^4$ m^2 s^{-1} (Guillot 1994).

The tidal deformation of Jupiter is thus

$$\varepsilon_T = \frac{M_{\rm Io}}{M_{\rm Jup}} \left(\frac{R}{d}\right)^3 = 1.855 \times 10^{-7} ,$$

and the Ekman number is given by

$$E = \frac{\nu}{2\Omega R^2} = 1.11 \times 10^{-8} .$$

The Ekman layer has a thickness of about 7 km, less than a pressure scale height (≈ 20 km), and to first approximation the turbulent viscosity may be considered as constant within the layer.

According to Tassoul's prescription, the rotation of Jupiter would be synchronized with the orbital motion of Io in a time of the order of

$$t_{\rm sync} = \frac{1}{2\Omega} E^{-1/2} \varepsilon_T^{-1} = 4.6 \times 10^6 \text{ yr} ,$$

which is clearly incompatible with the fast rotation we observe. A tidal interaction of that strength would have transferred long ago the proper angular momentum of Jupiter to its satellites, and these would orbit today at much greater distances from the planet.

The correct expression (eq. [13]) yields

$$t_{\rm sync} = \frac{1}{2\Omega} E^{-1} \varepsilon_T^{-2} = 2.3 \times 10^{17} \text{ yr} ,$$

which explains why Jupiter is still a rapid rotator.

We could multiply such examples, which would all show that Tassoul's prescription largely overestimates the tidal damping. Let us consider just another case, which has received much attention recently: 51 Peg with its planet of Jovian mass (Mayor & Queloz 1995). The star is a slow rotator of solar type, with a period of about 30 days, which proves that it has been spun down by the same mechanism as the Sun (mass loss plus magnetic activity). But according to Tassoul, it should be perfectly synchronized with the orbital motion whose period is 4.23 days, since the tidal synchronization time he would predict is about 1.6×10^4 yr, many orders of magnitude shorter than the spin-down time ($\approx 10^9$ yr at that age). The synchronization time drawn from the correct expression (eq. [13]) is 3×10^{11} yr, which again explains much better what is observed. (These figures have been worked out with $\varepsilon_T = 7.5 \times 10^{-6}$, $\nu = 10^9$ m^2 s^{-1}, and thus $E = 5.9 \times 10^{-5}$.)

5. CONCLUSIONS

The purpose of this paper was to show again that the Ekman pumping occurring in close binary stars does not enhance the tidal dissipation because no stress is applied by the tides on the stellar surface. The synchronization timescale is of order $(\varepsilon_T)^{-2} t_{\rm adj}$, where ε_T is the tidal deformation caused by the companion and $t_{\rm adj} = R^2/\nu$ is the viscous adjustment time, R being the radius of the considered star and ν the (turbulent) viscosity (Darwin 1879; Zahn 1966; Scharlemann 1982; Rieutord & Bonazzola 1987).

We hope to have convinced the reader that the so-called "hydrodynamical mechanism" invoked by Tassoul for the synchronization and circularization of binary stars is based on an improper treatment of the star's boundary conditions, a mechanism which was unfortunately endorsed by an incorrect expansion in spherical harmonics.

For the benefit of the reader who does not feel at ease with fluid dynamics, we have also shown that if Tassoul's mechanism were operating, Jupiter would not be the fast rotator we know, but would have transferred most of its angular momentum to the satellites (in particular, to Io). We also quoted 51 Peg, the slow rotation of which demonstrates again how much the tidal dissipation is overestimated by this mechanism. And in both cases the predictions of the tidal theory outlined in § 2 agree well with the observations.

APPENDIX

In this appendix we expand the stress-free boundary conditions to be applied onto an ellipsoidal surface to first order of the elongation, and we project them on the relevant spherical functions.

Let us write the surface of the ellipsoid as

$$r = 1 + \varepsilon f(\theta, \varphi);$$

the function $f(\theta, \varphi)$ represents the departure from the sphere. This function may be expanded into spherical harmonics $Y_l^m(\theta, \phi)$ and in the case of tidal deformation the main components are

$$f(\theta, \varphi) = Y_2^2 + Y_2^{-2} = \sqrt{\frac{15}{8\pi}} \sin^2\theta \cos 2\varphi \, . \tag{A1}$$

Now we develop the boundary conditions with respect to the sphere; we find

$$\begin{cases} u_r + \varepsilon\left[f\left(\dfrac{\partial u_r}{\partial r}\right) + n_\theta u_\theta + n_\varphi u_\varphi\right] = 0 \\ \sigma_{r\theta} + \varepsilon\left[f\left(\dfrac{\partial \sigma_{r\theta}}{\partial r}\right) + (\sigma_{\theta\theta} - \sigma_{rr})n_\theta + \sigma_{\theta\varphi} n_\varphi\right] = 0 \\ \sigma_{r\varphi} + \varepsilon\left[f\left(\dfrac{\partial \sigma_{r\varphi}}{\partial r}\right) + (\sigma_{\varphi\varphi} - \sigma_{rr})n_\varphi + \sigma_{\theta\varphi} n_\theta\right] = 0 \end{cases} \tag{A2}$$

taken at $r = 1$; $n_\theta = -\partial f/\partial\theta$ and $n_\varphi = -(1/\sin\theta)(\partial f/\partial\varphi)$, and terms of $\mathcal{O}(\varepsilon^2)$ have been dismissed.

Then these equations need to be projected on the basis of spherical harmonics. The expression of the viscous stress tensor $[\sigma]$ has been given in Rieutord (1987). For the sake of clarity we recall that

$$\boldsymbol{u} = \sum_{l=0}^{+\infty} \sum_{m=-l}^{+l} u_m^l \boldsymbol{R}_l^m + v_m^l \boldsymbol{S}_l^m + w_m^l \boldsymbol{T}_l^m \, ,$$

where

$$\boldsymbol{R}_l^m = Y_l^m \boldsymbol{e}_r \, , \qquad \boldsymbol{S}_l^m = \boldsymbol{\nabla} Y_l^m \, , \qquad \boldsymbol{T}_l^m = \boldsymbol{\nabla} \times \boldsymbol{R}_l^m \, ,$$

the Y_l^m are normalized spherical harmonics, and the gradients are taken on the unit sphere. In addition, we write

$$\begin{pmatrix} \sigma_{r\theta} \\ \sigma_{r\varphi} \end{pmatrix} = s_m^l \boldsymbol{S}_l^m + t_m^l \boldsymbol{T}_l^m \, ,$$

so that equation (A2) now reads

$$\begin{cases} u_L^M + \varepsilon\left(\dfrac{\partial u_m^l}{\partial r} F_{l,L}^{m,M} + v_m^l G_{l,L}^{m,M} + w_m^l H_{l,L}^{m,M}\right) = 0 \, , \\ l(l+1)s_M^L + \varepsilon\left(-3\dfrac{\partial u_m^l}{\partial r} B_{l,L}^{m,M} + v_m^l D_{l,L}^{m,M} + w_m^l E_{l,L}^{m,M} + \dfrac{\partial s_m^l}{\partial r} P_{l,L}^{m,M} + \dfrac{\partial t_m^l}{\partial r} H_{l,L}^{m,M}\right) = 0 \, , \\ l(l+1)t_M^L + \varepsilon\left(3\dfrac{\partial u_m^l}{\partial r} H_{l,L}^{m,M} - v_m^l D_{l,L}^{m,M} + w_m^l D_{l,L}^{m,M} - \dfrac{\partial s_m^l}{\partial r} H_{l,L}^{m,M} + \dfrac{\partial t_m^l}{\partial r} P_{l,L}^{m,M}\right) = 0 \, . \end{cases} \tag{A3}$$

We have introduced the following coupling integrals:

$$B_{l,L}^{m,M} = \int_{4\pi} Y_l^m (n_\theta N_{\theta,L}^M + n_\varphi N_{\phi,L}^M) d\Omega \,, \tag{A4}$$

$$D_{l,L}^{m,M} = \int_{4\pi} [(X_l^m n_\theta + Z_l^m n_\varphi) N_{\theta,L}^M + (Z_l^m n_\theta - X_l^m n_\varphi) N_{\phi,L}^M] d\Omega \,, \tag{A5}$$

$$E_{l,L}^{m,M} = \int_{4\pi} [(Z_l^m n_\theta + X_l^m n_\varphi) N_{\theta,L}^M - (X_l^m n_\theta + Z_l^m n_\varphi) N_{\phi,L}^M] d\Omega \,, \tag{A6}$$

$$F_{l,L}^{m,M} = \int_{4\pi} f(\theta, \varphi) Y_l^m Y_L^M \, d\Omega \,, \tag{A7}$$

$$G_{l,L}^{m,M} = B_{L,l}^{M,m} \,, \tag{A8}$$

$$H_{l,L}^{m,M} = \int_{4\pi} (n_\theta N_{\varphi,l}^m - n_\varphi N_{\theta,l}^m) Y_L^M \, d\Omega \,, \tag{A9}$$

$$P_{l,L}^{m,M} = \int_{4\pi} f(\theta, \varphi)(n_\theta N_{\theta,L}^M + n_\varphi N_{\phi,L}^M) d\Omega \,, \tag{A10}$$

with

$$N_{\theta,l}^m = \frac{\partial Y_l^m}{\partial \theta} \quad \text{and} \quad N_{\phi,l}^m = \frac{1}{\sin\theta} \frac{\partial Y_l^m}{\partial \varphi} \,,$$

$$X_l^m(\theta,\phi) = 2 \frac{\partial^2 Y_l^m}{\partial \theta^2} + l(l+1) Y_l^m \,,$$

$$Z_l^m(\theta, \phi) = 2 \frac{\partial}{\partial \theta} \left(\frac{1}{\sin\theta} \frac{\partial Y_l^m}{\partial \phi} \right) .$$

If we use equation (A1) to define the surface, the coupling integrals are nonzero only if

$$l = L \pm 1 \quad \text{for} \quad C, H, E \,; \tag{A11}$$

$$l = L \pm 0 \pm 2 \quad \text{for} \quad B, D, F, G, P \,. \tag{A12}$$

Four of these integrals are irreducible (D, E, F, H), two of which (F, H) are known as Elsasser and Adams integrals.

We may now write down the boundary conditions developed on spherical harmonics up to order $l = 3$, as was done (incorrectly) by Tassoul & Tassoul (1992b) in their Appendix B:

$$\begin{aligned}
u_0^2 + \varepsilon[\partial_r u_{\pm 2}^2, v_{\pm 2}^2, w_{\pm 2}^3] &= 0 \,, \\
u_{\pm 2}^2 + \varepsilon[\partial_r u_0^2, v_0^2, w_0^1, w_0^3] &= 0 \,, \\
s_0^2 + \varepsilon[\partial_r u_{\pm 2}^2, v_{\pm 2}^2, w_{\pm 2}^3, \partial_r s_{\pm 2}^2, \partial_r t_{\pm 2}^3] &= 0 \,, \\
s_{\pm 2}^2 + \varepsilon[\partial_r u_0^2, v_0^2, w_0^1, w_0^3, \partial_r s_0^2, \partial_r t_0^1, \partial_r t_0^3] &= 0 \,, \\
t_0^1 + \varepsilon[\partial_r u_{\pm 2}^2, v_{\pm 2}^2, w_{\pm 2}^3, \partial_r s_{\pm 2}^2, \partial_r t_{\pm 2}^3] &= 0 \,, \\
t_0^3 + \varepsilon[\partial_r u_{\pm 2}^2, v_{\pm 2}^2, w_{\pm 2}^3, \partial_r s_{\pm 2}^2, \partial_r t_{\pm 2}^3] &= 0 \,, \\
t_{\pm 2}^3 + \varepsilon[\partial_r u_0^2, v_0^2, w_0^1, w_0^3, \partial_r s_0^2, \partial_r t_0^1, \partial_r t_0^3] &= 0 \,.
\end{aligned} \tag{A13}$$

We have listed all the terms that obey the selection rules (eqs. [A11] and [A12]); for shortness sake, numerical constants and coupling integrals have been omitted. Let us now compare these boundary conditions to the one given by TT92. In the first place, their number is not the same: TT92 have nine conditions, while here there are 10. Their missing condition is equation (A13), which shows that their expansion is not complete to order $l = 3$.

However, their analysis has another flaw that is even worse. The reader may note that all the radial derivatives of the stress are missing in TT92, i.e., there is no equivalent of the terms $\partial_r t_0^1$, $\partial_r s_0^2$, ... These derivatives arise from the development of the stress around the sphere of radius 1:

$$\sigma(1 + \varepsilon) = \sigma(1) + \varepsilon \frac{\partial \sigma}{\partial r} + \mathcal{O}(\varepsilon^2) \,,$$

and they play an important role in the boundary layer flows, where they compete with the other terms. Their omission is equivalent to applying an $\mathcal{O}(\varepsilon)$ stress on the true surface, which is why TT92 obtain the wrong amplitude for the Ekman circulation.

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