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Does solar differential rotation arise from a large scale instability?

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Abstract. The suggestion by several authors that the solar differential rotation is caused by a large scale instability of the basic convective state is examined. We find that the proposed mean-field models are unstable to a Rayleigh-Bénard type instability, but argue that this cannot explain the differential rotation of the Sun, because such a flow would become nonaxisymmetric. We discuss the applicability of the mean-field equations to the problem.

Key words: convection – hydrodynamics – Sun: rotation

1. Introduction

Commonly proposed mechanisms for explaining the origin of the solar and stellar differential rotation in terms of mean-field theories include the effects of anisotropic viscosity (Biermann 1951; Kippenhahn 1963), non-diffusive parts of the Reynolds stress (i.e. the Λ-effect; see Rüdiger 1977), and a latitude-dependent heat transport (Weiss 1965; Durney & Roxburgh 1971). However, there have been several indications (Gierasch 1974; Schmidt 1982; Chan et al. 1987; Rüdiger 1989; Chan & Mayr 1991; Rüdiger & Tuominen 1991; Rüdiger & Spahn 1992) that the governing equations exhibit a global instability and it has been proposed that the solar differential rotation might be caused by this instability, the nature of which has not been fully elucidated.

One purpose of the present paper is to point out that this instability, as discussed in the literature (cf. Rüdiger 1989, Section 11.4), is simply a large scale convective Rayleigh-Bénard type instability. At first glance this may seem paradoxical, since the equations solved are mean-field equations where the effects of small scale flows (such as small scale convection) have been subsumed into mean-field transport coefficients. In general, one would expect the mean-field equations to be stable to small perturbations and not subject to instabilities, as has been argued by Orszag (1970) in a somewhat different context. That this may not generally be the case is seen in the mean-field dynamo problem (Krause & Rädler 1980), where the finite amplitude mean magnetic field $\langle \boldsymbol{B} \rangle$ is a result of an instability of the state $\langle \boldsymbol{B} \rangle = \boldsymbol{0}$. This mechanism is often invoked to explain the mean magnetic field in the Sun. Thus, an instability in the mean-field equations is *a priori* not novel or unphysical.

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2. The mean-field Rayleigh-Bénard problem

In this section we discuss the field equations that are commonly used to describe the evolution of the large scale hydrodynamical structure of the solar convection zone. The most crucial equation is the mean-field energy equation

$$\langle\rho\rangle\langle T\rangle\left(\frac{\partial}{\partial t}+\langle\boldsymbol{u}\rangle\cdot\boldsymbol{\nabla}\right)\langle s\rangle=\boldsymbol{\nabla}\cdot(\chi_t\langle\rho\rangle\langle T\rangle\boldsymbol{\nabla}\langle s\rangle), \qquad (1)$$

(Durney & Roxburgh 1971), where χ_t is the turbulent heat conductivity. This equation is rather similar to the original energy equation

$$\rho T\left(\frac{\partial}{\partial t}+\boldsymbol{u}\cdot\boldsymbol{\nabla}\right)s=\boldsymbol{\nabla}\cdot(\chi\rho c_p\boldsymbol{\nabla}T), \qquad (2)$$

(eg, Chandrasekhar 1961), where χ is the radiative diffusion coefficient. In Eq. (1) we have neglected terms involving correlations between velocity $\boldsymbol{u}$ and density ρ, which are expected to be small (cf. Rüdiger 1989, Chapter 8.2). Furthermore, we have here also neglected the radiative flux and the turbulent viscous heating (but see Sect. 6 of Brandenburg et al. 1992). The main difference between these two equations is that in (2) the diffusion is over temperature T, whilst in (1) diffusion is over the mean specific entropy $\langle s\rangle$. This property of Eq. (1) ensures that no large scale convective energy transport is possible in the adiabatic case $\langle s\rangle$ =constant.

Let us consider the case where the turbulent viscosity tensor is isotropic. The momentum equation for the mean velocity is then of a similar form as the original equation, with the viscosity being replaced by a turbulent viscosity ν_t:

$$\langle\rho\rangle\left(\frac{\partial}{\partial t}+\langle\boldsymbol{u}\rangle\cdot\boldsymbol{\nabla}\right)\langle\boldsymbol{u}\rangle=-\boldsymbol{\nabla}\langle p\rangle+\langle\rho\rangle\boldsymbol{g}-\boldsymbol{\nabla}\cdot(\langle\rho\rangle\mathcal{Q}), \quad (3)$$

where again, correlations between velocity and density have been neglected and in Cartesian coordinates we have

$$\mathcal{Q}_{ij}=-\nu_t(\langle u_{i,j}\rangle+\langle u_{j,i}\rangle-\tfrac{2}{3}\delta_{ij}\langle u_{k,k}\rangle). \quad (4)$$

In the limit of weak stratification the set of mean-field equations reduces to those used to describe Boussinesq convection (Chandrasekhar 1961). Thus, it is not surprising that the set of mean-field equations can also show a Rayleigh-Bénard type instability. The Rayleigh number now has to be defined using turbulent coefficients for viscosity and conductivity:

$$\mathrm{Ra}_t=-\frac{gd^4}{\nu_t\chi_t}\frac{1}{c_p}\frac{ds_0}{dr}, \quad (5)$$

where g is gravity, d the thickness of the layer, and s_0 is the entropy gradient of the quasi-hydrostatic reference solution. It is preferable to evaluate Ra_t in the middle of the layer, at a radius $r_{\mathrm{mid}}=\frac{1}{2}(r_0+R)$, where r_0 and R are the inner and outer radii of the shell. The Rayleigh numbers evaluated at the top or the bottom of the layer, $\mathrm{Ra}_t^{\mathrm{top}}$ or $\mathrm{Ra}_t^{\mathrm{bot}}$, depend more on the strength of the stratification than does $\mathrm{Ra}_t^{\mathrm{mid}}$; see Glatzmaier & Gilman (1981) and references therein.

Using the expression for the radial component of the convective flux

$$F_{\mathrm{conv}}=-\chi_t\langle\rho\rangle\langle T\rangle\, ds_0/dr, \quad (6)$$

the perfect gas equation $p=(1-1/\gamma)c_p\rho T$, and neglecting the radiative flux (i.e. the luminosity is $L=4\pi r^2F_{\mathrm{conv}}$), we may write $\mathrm{Ra}_t^{\mathrm{mid}}$ in the form

$$\mathrm{Ra}_t^{\mathrm{mid}}=(1-1/\gamma)\,\mathrm{Pr}_t^2Lgd^4/(4\pi r^2\nu_t^3p), \quad (7)$$

where $\mathrm{Pr}_t=\nu_t/\chi_t$ is a turbulent Prandtl number. Using solar parameters, $\gamma=5/3$, $L=3.9\,10^{33}\mathrm{g\,cm^2/s^3}$, $g_{\mathrm{mid}}=3.7\,10^4\mathrm{cm/s^2}$, $d=2\,10^{10}$cm, $r_{\mathrm{mid}}=6\,10^{10}$cm, we have

$$\mathrm{Ra}_t^{\mathrm{mid}}=2\,10^5\,\mathrm{Pr}_t^2\left(\frac{\nu_t}{10^{13}\mathrm{cm^2s^{-1}}}\right)^{-3}\left(\frac{p}{10^{12}\mathrm{g\,cm^{-1}s^{-2}}}\right)^{-1}. \quad (8)$$

For typical values from solar mixing length models we have $\mathrm{Pr}_t=0.33$, $\nu_t=5\,10^{12}\mathrm{cm^2/s}$, $p=8\,10^{12}\mathrm{g\,cm^{-1}s^{-2}}$, and thus $\mathrm{Ra}_t^{\mathrm{mid}}=2\,10^4$. Below we will show that this is somewhat above the critical value for the onset of convection. However, a turbulent viscosity twice as large would make the Rayleigh number subcritical. Thus, given the uncertainties in ν_t, too much weight should not be attached to the mixing-length value.

The set of mean-field equations has been solved for spherical shells in the presence of strong density stratification by Glatzmaier & Gilman (1981). They determined critical turbulent Rayleigh numbers both for axisymmetric and nonaxisymmetric modes using a constant temperature boundary condition on $r=r_0, R$. Glatzmaier & Gilman found critical values $\mathrm{Ra}_t^{\mathrm{crit}}$ around 600...800 in the absence of rotation and around 5000...10^4 for solar values of the turbulent Taylor number $\mathrm{Ta}_t=4\Omega_0^2R^4/\nu_t^2=10^{6\ldots7}$, where Ω_0 is the basic angular velocity. Similar values for the critical Rayleigh number have been found by Schmidt (1982) in the context of axisymmetric models for solar differential rotation. His boundary conditions were that the flux be constant at $r=r_0$ and converted into radiation ($F\sim\sigma T^4$) at $r=R$, i.e.

$$F_{\mathrm{conv}}=L/(4\pi r_0^2) \quad \text{on} \quad r=r_0, \quad (9)$$

$$F_{\mathrm{conv}}=\sigma\langle T\rangle^4 \quad \text{on} \quad r=R. \quad (10)$$

These boundary conditions will also be used in our models presented below.

3. Models presented in the literature

We now estimate the Rayleigh numbers for various mean-field models presented in the literature in order to see whether or not these numbers are larger than the critical value, in which case the solutions should be unstable with respect to the Rayleigh-Bénard type instability. A similar comparison has also been presented by Rüdiger & Spahn (1992) using, however, a local axisymmetric model.

Schmidt (1982) found a Rayleigh-Bénard type instability for $\chi_t=3.5\,10^{13}\mathrm{cm^2/s}$, $\mathrm{Pr}_t=0.1$, $d=1.6\,10^{10}$cm, and $(ds/dr)_{\mathrm{bot}}=10^{-16}c_p$/cm. He quoted as critical value $\mathrm{Ra}_t^{\mathrm{bot}}=2\,10^3$. The value of the Rayleigh number in the middle, $\mathrm{Ra}_t^{\mathrm{mid}}$, may be computed from $\mathrm{Ra}_t^{\mathrm{bot}}$ using the condition r^2F_{conv}=constant, where $F_{\mathrm{conv}}\propto p_0\partial s_0/\partial r$ and $g\propto r^{-2}$. This gives

$$\mathrm{Ra}_t^{\mathrm{mid}}=\mathrm{Ra}_{\mathrm{bot}}(r^4p)_{\mathrm{bot}}/(r^4p)_{\mathrm{mid}}. \quad (11)$$

For the model of Schmidt (1982) we have then $\mathrm{Ra}_t^{\mathrm{mid}}\approx 6700$. (Since in this model, g=constant was assumed, a more consistent estimate would be $\mathrm{Ra}_t^{\mathrm{mid}}=\mathrm{Ra}_t^{\mathrm{bot}}(r^2p)_{\mathrm{bot}}/(r^2p)_{\mathrm{mid}}\approx 10^4$ in this case.) This value is close to critical.

Gierasch (1974) used $\nu_t=2\,10^{12}\mathrm{cm^2/s}$, $\mathrm{Pr}_t=0.5$, $d=1.4\,10^{10}$cm, and $(ds/dr)_{\mathrm{bot}}=4\,10^{-15}c_p$/cm. This leads to $\mathrm{Ra}_t^{\mathrm{bot}}\approx 10^6$ and $\mathrm{Ra}_t^{\mathrm{mid}}\approx 4\,10^6$, which are clearly supercritical values. In fact, Gierasch noted that the region covered by his model is convectively unstable, but he associated this with the intrinsic background convection, rather than with an instability of the mean-field solution itself.

The parameters for the model considered by Chan & Mayr (1991) are $\nu_t=1.2\,10^{10}\mathrm{cm^2/s}$, $\mathrm{Pr}_t=0.3$, and $(ds/dr)_{\mathrm{mid}}=5\,10^{-5}c_p/H_p\approx 10^{-15}c_p$/cm, where H_p is the pressure scale height. This gives a rather large value of $\mathrm{Ra}_t^{\mathrm{mid}}\approx 10^{10}$, which is highly supercritical. The model of Chan et al. (1987) is very similar, but with $\nu_t\approx 10^{12}\mathrm{cm^2/s}$, which leads to $\mathrm{Ra}_t^{\mathrm{mid}}\approx 10^6$, that is also supercritical. Chan et al. identify their solution with an "axisymmetric resonant mode", which they relate to similar solutions obtained by Schmidt, but no connection is drawn with a convective instability of this solution.

The governing nondimensional parameters employed by Rüdiger (1989, Section 10.1) and Tuominen & Rüdiger (1989, hereafter TR89) are $f^* = (R/c_p)(ds_0/dr)_{\rm top}$, $g^* = gR/(\gamma\mathscr{R}T_{\rm top})$, and $T^* = \gamma\mathscr{R}T_{\rm top}R^2/\nu_t^2$, where $\mathscr{R}T = p/\rho$. These parameters are related to $\mathrm{Ra}_t^{\rm top}$ by

$$\mathrm{Ra}_t^{\rm top} = (d/R)^4 \mathrm{Pr}_t\, f^* g^* T^*. \tag{12}$$

As solar parameters they take $f^* = 200$, $g^* = 1500$, $T^* = 5\,10^7$, $\mathrm{Pr}_t = 0.33$, corresponding to $\mathrm{Ra}_t^{\rm top} = 4\,10^{10}$ (with $\gamma = 5/3$). $\mathrm{Ra}_t^{\rm mid}$ may be computed from $\mathrm{Ra}_t^{\rm top}$ analogous to Eq. (11) which gives $\mathrm{Ra}_t^{\rm mid} = 4\,10^5$, where we have used the value of $p_{\rm mid}/p_{\rm top}$ from the zero-order model of TR89. Again, the value of $\mathrm{Ra}_t^{\rm mid}$ is supercritical. However, Rüdiger (1989) characterizes the solution for these parameters as stable, but he mentioned the possibility of a "new instability" for parameters that correspond to $\mathrm{Ra}_t^{\rm mid} \approx 2\,10^5$. In the following section we point out that this is the value at the second bifurcation, which is physically irrelevant.

4. A detailed example

We now analyse in more detail the case considered in TR89. We first compute critical Rayleigh numbers of the linearised system of hydrodynamical mean-field equations by seeking the zeros of the determinant $\det(M)$ of the coefficient matrix M of these equations. The inner radius of the spherical shell is $r_0 = 0.7\,R$. In Fig. 1 we show critical values of $\mathrm{Ra}_t^{\rm mid}$ for different Taylor numbers and $f^* = 200$, $g^* = 1500$, $\mathrm{Pr}_t = 0.33$. For the sake of completeness and for later discussion, we have also included higher order critical Rayleigh numbers which, however, are not of physical importance. Note that for large values of Ta_t the critical Rayleigh numbers approach an asymptotic law which is somewhat steeper than the $\mathrm{Ra} \sim \mathrm{Ta}^{2/3}$ law, derived in the theory for the onset of convection in the presence of rotation (Chandrasekhar 1961; Bisshopp & Niiler 1965; Busse 1970, 1973). This discrepancy will be discussed below.

The location of the Sun, indicated by an asterisk in Fig. 1, is in the unstable region just beyond the second bifurcation. The line marking the secondary bifurcation is strikingly similar to the form and the position of the stability curve shown by Rüdiger (1989, Figure 11.8), but there the regime where the Sun is located is indicated as stable. Since here the critical lines are obtained from the zeros of a determinant $\det(M)$ one cannot in principle tell which side of the critical line corresponds to stability. Below we will show that the lowest line in Fig. 1 indeed corresponds to the first bifurcation and that below this line the models are stable. In view of our present results we expect that the critical line shown by Rüdiger corresponds actually to a secondary bifurcation. From the qualitative and quantitative agreement between the critical line plotted by Rüdiger and the lowest dotted curve of Fig. 1 we may conclude that we have identified the instability quoted by Rüdiger.

In Fig. 1 only the lowest (latitude-dependent) modes in the spherical harmonic expansion of velocity components (see Eqs. 16-18 in TR89) were involved ($n = 2$ in the figure). This has generally been sufficient for models of the solar differential rotation. When using the system of TR89 in the present context, for an investigation of a Rayleigh-Bénard type instability, it is necessary to check the importance of the higher modes in the expansion. As shown in Table 1 the first unstable mode is of increasingly higher order when the Taylor number is increased. The slope in this case remains slightly smaller than 2/3; see Fig. 2. It should be noted, however, that our model has a quite strong density stratification of 15 pressure scale heights, which is as strong as in the mixing length convection zone models for the Sun (eg, Spruit 1974).

The models presented in Brandenburg et al. (1992, hereafter BMT92) have a weaker stratification than used in TR89. The parameters used in the convectively stable "Model T5" of BMT92 are $\mathrm{Pr}_t = 0.1$, $\xi = (\gamma g^*)^{-1} = 0.01$, $\Gamma = g^*T^* = 3\,10^{11}$, and $\mathscr{L} = f^*T^*\rho_{\rm top}/(\overline{\rho}\gamma\mathrm{Pr}_t) = 10^6$, where $\overline{\rho} = \int \rho dV / \int dV$ is the average density of the convection zone. These parameters correspond to $g^* = 60$, $f^* = 8\,10^{-4}$, and $T^* = 5\,10^9$. In order to compare the models of TR89 and BMT92 we show in Fig. 3 critical Rayleigh numbers for these values of g^* and f^*, with T^* varying. The curve for TR89 again represents the first unstable

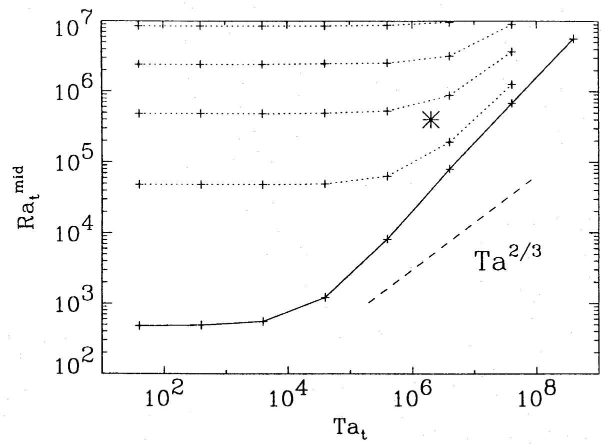

Fig. 1. The critical Rayleigh numbers for different values of the Taylor number for $f^* = 200$, $g^* = 1500$, and $\mathrm{Pr}_t = 0.33$, and varying values of T^*. The location of the Sun in this diagram is indicated by an asterisk. The slope for the asymptotic $\mathrm{Ra} \sim \mathrm{Ta}^{2/3}$ law is shown by a dashed line

Table 1. The first unstable mode and the critical values of $\mathrm{Ra}_t^{\rm mid}$ for the high stratification model of Fig. 2 (2nd and 3rd column) and for the low stratification model of Fig. 3 (4th and 5th column) as a function of Taylor number

Ta	n	$\mathrm{Ra}_t^{\rm mid}$	n	$\mathrm{Ra}_t^{\rm mid}$
$4\,10^1$	4	$4.9\,10^2$	2	$3.0\,10^2$
$4\,10^2$	4	$4.9\,10^2$	2	$3.1\,10^2$
$4\,10^3$	4	$5.3\,10^2$	2	$3.6\,10^2$
$4\,10^4$	6	$8.8\,10^2$	6	$6.8\,10^2$
$4\,10^5$	14	$2.3\,10^3$	12	$2.1\,10^3$
$4\,10^6$	28	$6.0\,10^3$	24	$7.3\,10^3$
$4\,10^7$	56	$1.5\,10^4$	44	$2.9\,10^4$

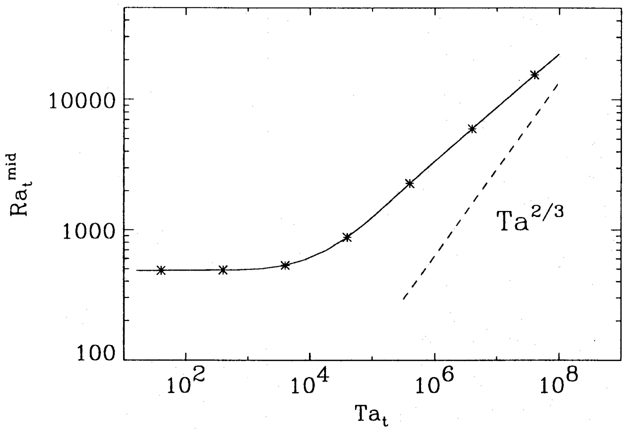

Fig. 2. The critical Rayleigh numbers for the most easily excited mode for different values of the Taylor number, for $f^* = 200$, $g^* = 1500$, and $Pr_t = 0.33$, and varying values of T^*. Table 1 gives the value of n for each value of the Taylor number. The slope for the asymptotic $Ra \sim Ta^{2/3}$ law is shown by a dashed line

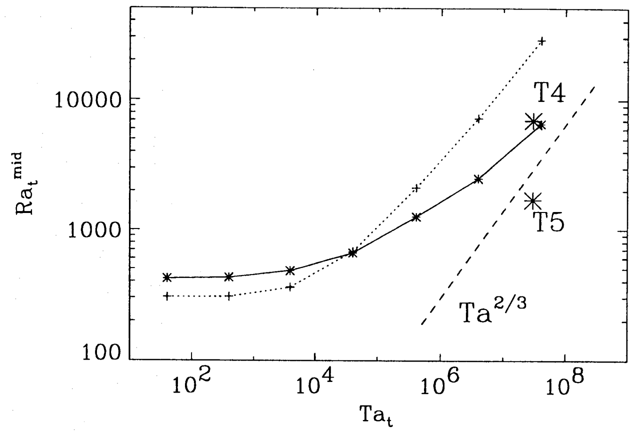

Fig. 3. The critical Rayleigh numbers as a function of the Taylor number for $f^* = 8\,10^{-4}$, $g^* = 60$, and $Pr_t = 0.1$. The solid line corresponds to data obtained with the code of BMT92, whilst the dotted curve shows the first unstable mode as determined by the code of TR89. The locations of the two models T4 and T5 of BMT92 are indicated by asterisks. The slope for the asymptotic $Ra \sim Ta^{2/3}$ law is shown by a dashed line

mode, whose latitudinal order n increases with Ta_t (see Table 1). Both curves now approach the 2/3 law, although somewhat more quickly for TR89. For small values of Ta_t the two curves are in rough agreement, but discrepancies occur for larger values of Ta_t. This could be a consequence of the fact that in the model of BMT92 the hydrostatic reference state changes as the Taylor number increases: the faster the shell rotates, the more oblate become surfaces of constant pressure and density. Another factor is that the stable model in BMT92 already has a rotationally induced meridional circulation. This is not the case in TR89. Nevertheless, it is clear that the instability found by the two different approaches has the same origin. The supercritical "Model T4" of BMT92 is for $Pr_t = 0.2$, but has otherwise the same parameters as "Model T5".

For illustration, we now discuss solutions computed for slightly supercritical Rayleigh numbers and different values of Ta_t, using the nonlinear code of BMT92. In all cases we used 41×41 mesh points in both the radial and latitudinal directions. In some cases we checked that results are similar when using lower and higher spatial resolutions. In Fig. 4 we show contours of Ω and stream lines of the poloidal flow for different values of Ta_t. Note that the contours of angular velocity are distinctly noncylindrical. For intermediate Taylor numbers, $Ta_t \lesssim 4\,10^4$, the contours are more "disk-shaped", i.e. perpendicular to the rotation axis. Such contours are similar to those observed in the Sun (e.g. Libbrecht 1988). For larger values of Ta_t the convection breaks up into smaller cells and both the stream lines and the Ω-contours develop more small scale structure. This increasingly small scale structure corresponds to the increasing latitudinal order of the most unstable mode found by the linear calculations discussed in Sect. 4. In the last panel for $Ta_t = 4\,10^7$, however, the mesh point solution appears to be not fully resolved. This also might help to explain the increasing discrepancy between the two solutions in Fig. 3 for large Taylor numbers.

In some cases we have included differential rotation generated by the Λ-effect (eg, Rüdiger 1989). The strength and sign of the radial gradient of the angular velocity is controlled by the parameter $V^{(0)}$ (for details see BMT92). For the model with $Ta_t = 4\,10^5$ and $V^{(0)} = -1$ we find that the onset of the convective instability occurs at slightly smaller value of Ra_t^{mid} ($\approx$ 1150 instead of $\approx$ 1250), whilst for $V^{(0)} = +1$ it occurs for larger values ($\approx$ 1350). Thus, a radially outwards increasing angular velocity ($V^{(0)} > 0$) has a stabilising effect, which is to be expected from the stability criterion for Couette flow (eg, Chandrasekhar 1961). In models with subcritical Rayleigh numbers and differential rotation generated by the Λ-effect the resulting contours of the angular velocity are nearly parallel to the rotation axis if $Ta_t \gtrsim 10^6$ (e.g. BMT92).

5. Turbulent convection with large scale flows

We now discuss the physical significance of the large scale Rayleigh-Bénard instability and the differential rotation generated by the resulting circulation. In the beginning of this discussion we refer to well-known experimental results for nonrotating laboratory convection in Cartesian and cylindrical geometry, but we anticipate that some aspects of the qualitative behaviour of convection at different Rayleigh numbers will carry over to spherical geometry. We then consider the possibility that the mean-field equations might actually become invalid before the mean-field Rayleigh-Bénard type instability occurs.

Consider first a fluid in the absence of any convection. As we increase the heat input from below, i.e. as we increase the (ordinary, nonturbulent) Rayleigh number, the system undergoes a sequence of bifurcations marking the onset of oscillatory convection, chaotic behaviour, soft and finally hard turbulence (see Heslot et al. 1987). Moreover, as discovered by Krishnamurti & Howard (1981), a large scale flow sets in at some Rayleigh

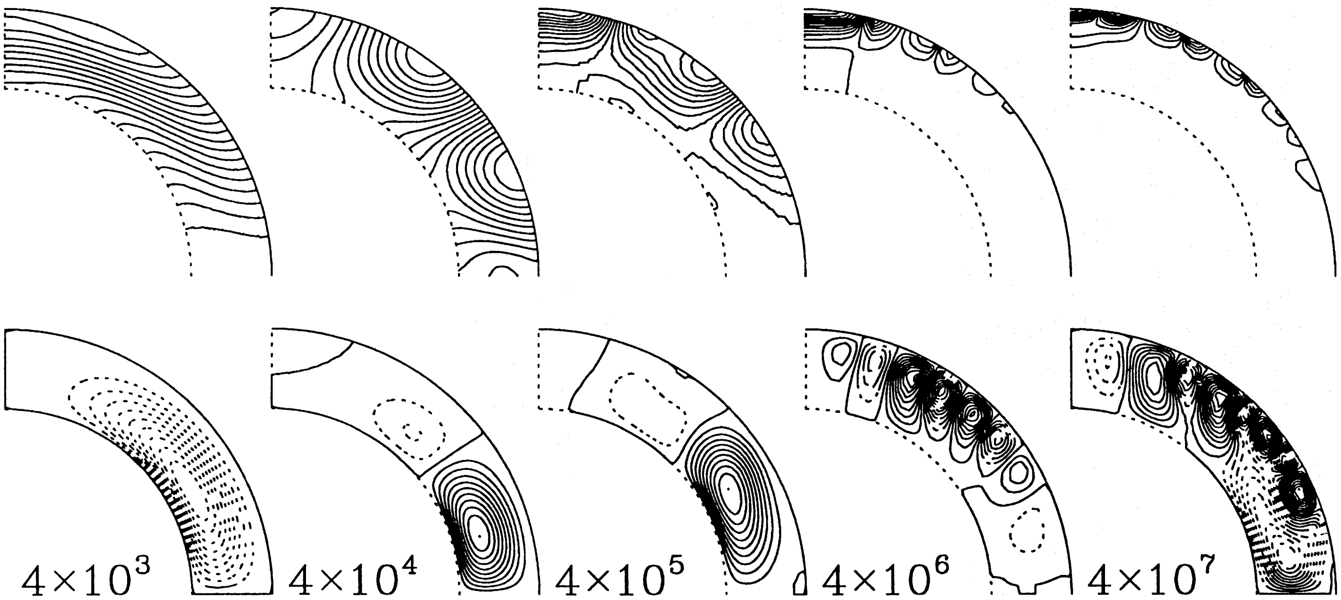

Fig. 4. Large scale convective flow. Contours of Ω (top) and stream lines of the poloidal flow (bottom) for the marginal or slightly supercritical case and different values of Ta_t ($4\,10^3 ... 4\,10^7$). The values for $\mathrm{Ra}_t^{\mathrm{mid}}$ used in the different cases are $\mathrm{Ra}_t^{\mathrm{mid}}$ = 480, 720, 1250, 2100, and 7000. The Ω contours are equidistantly spaced between the minimum and maximum values of Ω. For $\mathrm{Ta}_t \leq 4\,10^4$ the differential rotation is about 1%, for $\mathrm{Ta}_t = 4\,10^5$ about 3%, and for $\mathrm{Ta}_t \geq 4\,10^6$ around 100%

number around $2\,10^6$; and this is also observed in the experiments of Heslot et al. (1987). Recently, Libchaber et al. (1991, and private communication) found that at even larger Rayleigh numbers around 10^{13} this large scale flow can become time dependent, changing its direction continuously, indicating either reversals or a rotation of the large scale flow pattern about the vertical axis. These features of large scale flows, first laminar and later (at higher values of Ra) time dependent, seem to indicate empirically that the bifurcation sequence of ordinary Rayleigh-Bénard convection (from steady to oscillatory convection etc.) is similar and might qualitatively be described at the level of mean-field theory. From this point of view it seems that an explanation of the large scale flow in terms of a mean-field Rayleigh-Bénard type instability is *a priori* not unreasonable. An outline of such a bifurcation sequence is given in Table 2. We should, however, mention that other mechanisms to explain such large scale flows have also been proposed. For example, Howard & Krishnamurti (1986) interpret the large scale flow as being driven by a nonlinear interaction between already existing convective modes (and not through buoyancy, as discussed in the present paper). Massaguer et al. (1992) analyse the same bifurcation, but including also three-dimensional modes. Further interesting bifurcations may be expected before we reach the solar regime of Rayleigh numbers around $10^{20...30}$. For example Kraichnan (1962) expects a marked enhancement of the heat transport as a result of shear boundary-layer effects.

Table 2. Sketch of the bifurcation sequence for laboratory Rayleigh-Bénard convection. The critical values of Ra are taken from Heslot et al. (1987)

Ra	convection	large scale flow
$6\,10^3$	onset of convection	
$9\,10^4$	oscillatory convection	
$1.5\,10^5$	chaotic convection	
$3\,10^5$	soft turbulence	
$\approx 2\,10^6$		onset of large scale flow
$4\,10^7$	hard turbulence	
$\approx 10^{13}$		osc. large scale flow

The solar differential rotation is a clearly axisymmetric phenomenon and there is so far no evidence for significant nonaxisymmetric contributions to the rotation law. If a large scale instability, as discussed in the literature (see Sect. 3), were really responsible for driving the observed solar differential rotation, then we would have to require that the dominant mode be axisymmetric. According to the results of Glatzmaier & Gilman (1981) this will not be the case for modest and rapid rotation. For reasonable values of the Taylor number and in the presence of strong stratification, Glatzmaier & Gilman find that the most unstable modes have azimuthal orders of around $m = 10-20$. This would correspond to "banana cell" convection patterns, which have been observed in numerical simulations (eg, Gilman 1977) and in small scale space experiments (Hart et al. 1986a,b); but in spite of several attempts to detect these banana cells in the Sun no observational evidence has so far been found (eg, Simon & Weiss 1991). Thus, explaining solar differential rotation in terms of a Rayleigh-Bénard type instability leads to fundamental difficulties. We conclude that, unless there are mechanisms causing a preference for axisymmetric patterns in the highly nonlinear regime, for example a strong toroidal magnetic field (cf. Eltayeb 1972; Jones & Galloway 1988), such large scale convection cannot be the cause of the solar differential rotation.

6. Conclusions

Some attempts to explain the solar differential rotation as a result of a global instability have been based on mean-field models that exhibit a large scale convective Rayleigh-Bénard type instability. This instability drives a meridional circulation that in turn causes the angular velocity to be nonuniform. In the Sun this type of mechanism is unlikely to operate, because it is known that the meridional flow is, at least at the surface, very small.

The large scale convective instability perhaps seems somehow paradoxical, because the basic state is already convectively unstable, but the analogy with the occurrence of a large scale flow in laboratory convection suggests that a description in terms of mean-field theory is not inherently implausible. We would argue, however, that such a phenomenon is unlikely to occur in the Sun, because the resulting flow pattern is expected to be nonaxisymmetric, and even axisymmetric calculations show detailed latitudinal structures as the Taylor number is increased (see Fig. 4). We thus reject the possibility that the solar differential rotation is a result of such an instability. Instead, other explanations in terms of anisotropic viscosity, Λ-effect and nonuniform heat transfer remain valid candidates for driving the solar differential rotation.

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