The ESTER project: modelling fast rotating stars

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Why should we make 2D-models ?

To deal properly with rotation !

Rotation means

- non spherical stars
- baroclinic flows in radiative region
- anisotropic convection

We note that

- 1D rotating models are valid when $\Omega \rightarrow 0$
- A lot of physics is condensed inside adjustable (transport) coefficients
- 1D models are not usable in asteroseismology of rapid rotators
- New data from optical/IR interferometry require a 2D view...

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Interferometry : Achernar

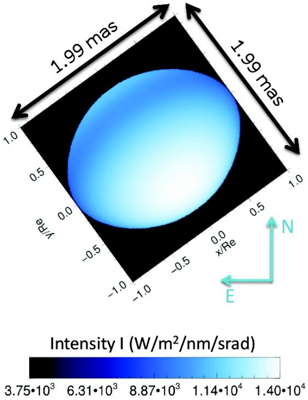

FIGURE – Achernar with VLTI (Domiciano de Souza et al. 2014, AA **569**)

Interferometry : Altair

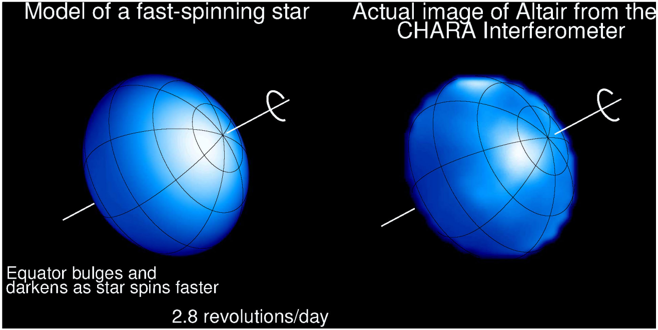

FIGURE – Altair seen by CHARA (Monnier et al. 2007).

The actual state of ESTER models

An idealization/simplification

- We consider a lonely rotating star
- We are interested in long time-scales
- We discard all magnetic fields.

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$$
\begin{cases}
\Delta\phi = 4\pi G\rho \\
\rho T\vec{v}\cdot\vec{\nabla}S = -\mathrm{Div}\vec{F} + \varepsilon_* \\
\rho(2\vec{\Omega}_* \wedge \vec{v} + \vec{v}\cdot\vec{\nabla}\vec{v}) = -\vec{\nabla}P - \rho\vec{\nabla}(\phi - \frac{1}{2}\Omega_*^2 s^2) + \vec{F}_v \\
\mathrm{Div}(\rho\vec{v}) = 0.
\end{cases}
\quad (1)
$$

$$\left\{\begin{array}{l} P \equiv P(\rho, T) \quad \mathrm{OPAL} \\ \kappa \equiv \kappa(\rho, T) \quad \mathrm{OPAL} \\ \varepsilon_* \equiv \varepsilon_*(\rho, T) \quad \mathrm{NACRE} \end{array}\right. \tag{2}$$

The energy flux

$$\vec{F} = -\chi_r \vec{\nabla} T - \frac{\chi_{\rm turb} T}{\mathcal{R}_M} \vec{\nabla} S$$

The transport of momentum

$$\begin{aligned}\vec{F}_v = \mu \vec{\mathcal{F}}_\mu(\vec{v}) \quad &= \quad \mu \left[\Delta\vec{v} + \frac{1}{3}\vec{\nabla}\left(\vec{\nabla}\cdot\vec{v}\right) + 2\left(\vec{\nabla}\ln\mu\cdot\vec{\nabla}\right)\vec{v} \right.\\ &\left. +\vec{\nabla}\ln\mu \times (\vec{\nabla}\times\vec{v}) - \frac{2}{3}\left(\vec{\nabla}\cdot\vec{v}\right)\vec{\nabla}\ln\mu \right] .\end{aligned}$$

or any mean-field expression of the Reynolds stress.

Boundary conditions

- On pressure

$$P_s = \frac{2}{3}\frac{\overline{g}}{\overline{\kappa}}$$

- On the velocity field

$$\vec{v} \cdot \vec{n} = 0 \quad \text{and} \quad ([\sigma]\vec{n}) \wedge \vec{n} = \vec{0}$$

- On temperature (black body radiation)

$$\vec{n} \cdot \vec{\nabla} T + T/L_T = 0$$

The last touch

$$\int_{(V)} r \sin\theta \rho u_\varphi \, dV = L$$

or

$$v_\varphi(r = R, \theta = \pi/2) = V_{\rm Eq}$$

The ESTER code

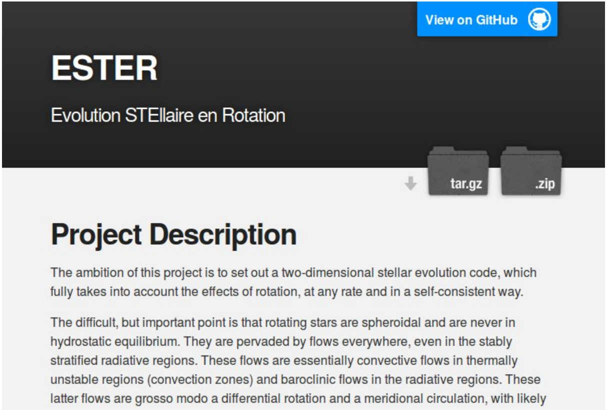
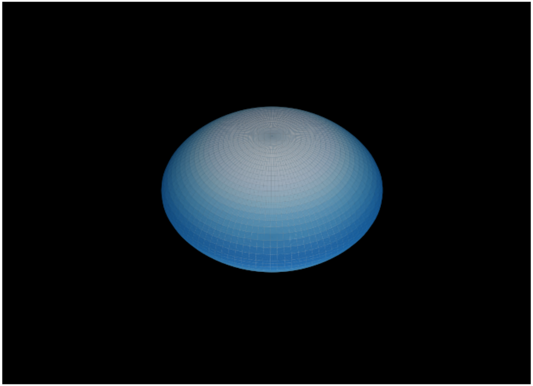


FIGURE – Freely available on the www



Gravity darkening exponent : $T_{\rm eff} \propto g_{\rm eff}^{\beta}$

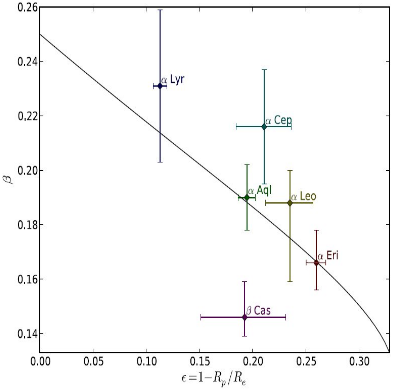


FIGURE – Observed values of β and a simple model of Espinosa Lara & Rieutord (2011).

Models of nearby stars

We have modeled 8 stars of intermediate mass :

Star		M ($M_\odot$)	V_{eq}(km/s)
Altair	α Aql	1.9	286
Alderamin	α Cep	1.9	265
Ras Alhague	α Oph	2.2	242
	δ_A Vel	2.27 & 2.43	150 & 143
Vega	α Lyr	2.4	205
Regulus	α Leo	4.1	335
Achernar	α Eri	6.5	339

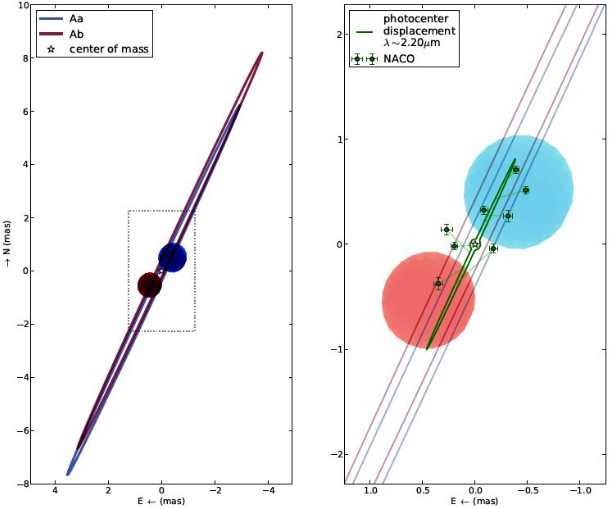
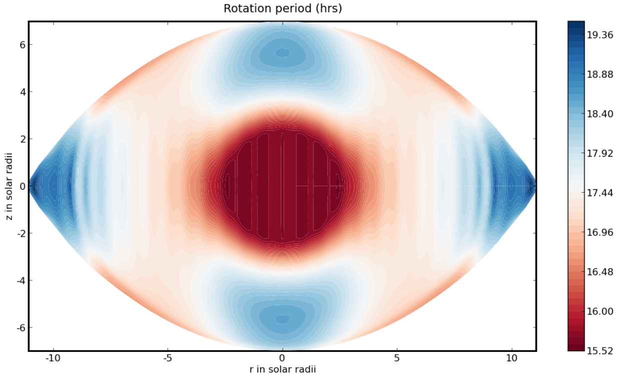


FIGURE – The orbit of delta vel (Kervella et al. 2013).

Star	Delta Velorum Aa		Delta Velorum Ab	
	Obs.	Model	Obs.	Model
Mass ($M_\odot$)	2.43 ± 0.02	**2.43**	2.27±0.02	**2.27**
R_{eq} ($R_\odot$)	2.97±0.02	**2.95**	2.52±0.03	**2.52**
R_{pol} ($R_\odot$)	2.79±0.04	**2.77**	2.37±0.02	**2.36**
T_{eq} (K)	9450	**9440**	9560	**9477**
T_{pol}(K)	10100	**10044**	10120	**10115**
L ($L_\odot$)	67±3	**65.2**	51±2	**48.5**
V_{eq} (km/s)	143	**143**	150	**153**
P_{eq} (days)		**1.045**		**0.832**
P_{pol} (days)		**1.084**		**0.924**
$X_{env.}$		**0.70**		**0.70**
$X_{core}/X_{env.}$		**0.10**		**0.30**
Z		**0.011**		**0.011**

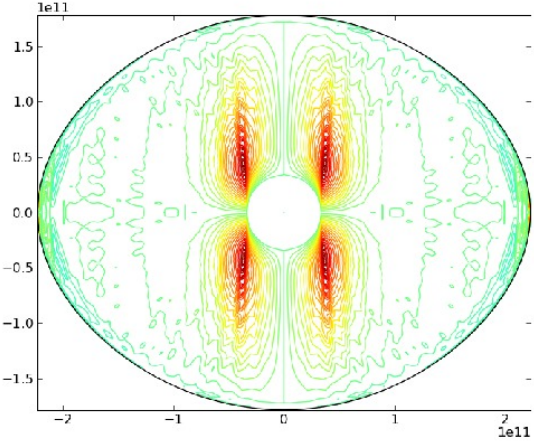
M=30$M_\odot$ at 98% of critical angular velocity



Espinosa Lara & Rieutord (2013) A&A,**552**,A35

Inside the stars : meridional circulation

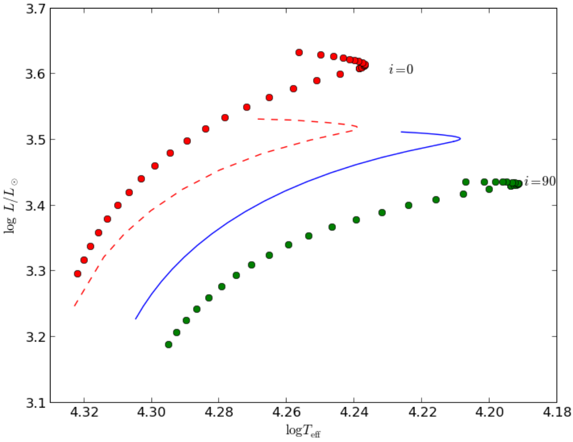
$M=5M_\odot$ at 70% of critical angular velocity



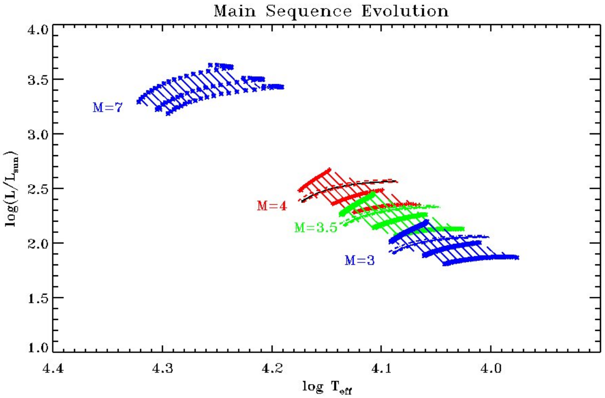
Espinosa Lara & Rieutord (2013)

Towards evolution

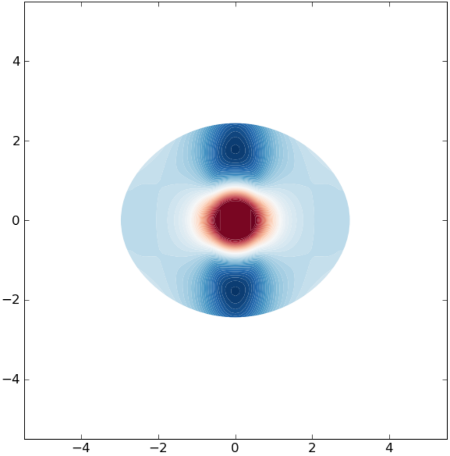
HR diagram track of a $7M_\odot$ star of constant angular momentum, starting at $\Omega/\Omega_k = 0.5$.



Towards evolution
HR diagram tracks at constant angular momentum



Evolution of a $5M_\odot$ star at constant angular momentum : heading to the Be state



Last developments and road map

- Portability improved, github management
- Documentation strongly improved (93 pages)
- Low mass stellar models under construction

Next :

1. Implement nuclear evolution on MS
2. Implement thermal evolution (PMS and post-MS)

We consider an isothermal atmosphere, which is cooling in some prescribed way. **The problem is 1D**

$$\rho\frac{\partial v_z}{\partial t} = -\frac{\partial P}{\partial z} - \rho g$$

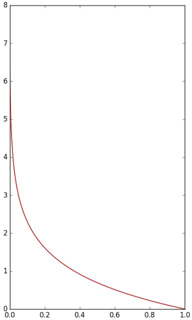
Assumptions : $\vec{v}$ small and viscosity negligible.

$$P = R_* T\rho \qquad \text{with} \qquad T \equiv T(t) \quad \text{prescribed}$$

Hydrostatic solution is

$$\rho = \rho_0 e^{-z/d}$$

where $d = R_* T/g$ is the scale height.



Thermal Evolution (2)

Atmosphere evolution is through ρ_0. How do we get it ?

The mass of the atmosphere is constant and so is its surface density Σ. This determine the evolution of ρ_0, namely

$$\Sigma = \int_0^\infty \rho(z)dz = \rho_0 d \tag{3}$$

since $d(t) = R_* T(t)/g$ is known.

What about the velocity ?
We start with masse conservation

$$\frac{\partial \rho v_z}{\partial z} = -\frac{\partial \rho}{\partial t}, \qquad \text{with} \qquad \rho = \rho_0 e^{-z/d}$$

or

$$\frac{\partial \rho v_z}{\partial z} = -\rho_0 e^{-z/d} \left(\frac{\dot{\rho}_0}{\rho_0} + \frac{\dot{d}z}{d^2} \right)$$

Thermal evolution (3)

However

$$\frac{\dot{\rho}_0}{\rho_0} + \frac{\dot{d}}{d} = 0$$

After some calculation we find :

$$v_z = \frac{\dot{d}}{d} z$$

where we used the boundary condition $v_z(0) = 0$.

The spherical case

We now consider a self-gravitating sphere of gas of uniform temperature. This is a polytrope $n = \infty$.

The evolution of temperature is supposed to be known.

We need to solve :

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) = 4\pi G\rho$$

$$-\frac{dP}{dr} - \rho\frac{d\phi}{dr} = 0$$

$$P = c^2\rho$$

where $c^2 = \mathcal{R}_* T(t)$ is the isothermal sound speed.

The spherical case

Hydrostatic equilibrium gives :

$$\rho = \rho_c \exp\left(\frac{\phi_c - \phi}{c^2}\right) \tag{4}$$

and

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) = 4\pi G\rho_c e^{-(\phi-\phi_c)/c^2} \tag{5}$$

This equation gives the dependence $\phi \equiv \phi(\rho_c)$ and ρ_c comes from

$$M = 4\pi\rho_c \int_0^\infty e^{-(\phi-\phi_c)/c^2} r^2 dr$$

M is the total mass. Now ρ is known everywhere.

The spherical case : the velocity field

Since the evolution of density is known, we deduce the velocity field that ensures mass conservation

$$\mathrm{Div}(\rho\vec{v}) = -\frac{\partial\rho}{\partial t} \Longleftrightarrow \frac{1}{r^2}\frac{\partial r^2\rho v}{\partial r} = -\frac{\partial\rho}{\partial t}$$

$$v(r) = -\frac{1}{r^2\rho}\int_0^r \frac{\partial\rho}{\partial t}x^2dx$$

The algorithm with the cartesian case

We wish to solve

$$\begin{cases} \partial_t \rho + \partial_z(\rho v_z) = 0 \\ \partial_z P + \rho g = 0 \\ \partial_t T = \kappa \Delta T - (P/\rho c_v)\partial_z v_z \\ P = \mathcal{R}_* \rho T \end{cases} \tag{6}$$

$$
\begin{cases}
\partial_z(\rho_n v_n^z) = -(\rho_n - \rho_{n-1})/\delta t \\
\partial_z P_n + \rho_n g = 0 \\
\kappa \Delta T_n - (P_n/\rho_n c_v)\partial_z v_n^z = (T_n - T_{n-1})/\delta t \\
P_n = \mathcal{R}_* \rho_n T_n
\end{cases}
\tag{7}
$$

which can be formally written

$$
\vec{F}(\vec{X}_n) = \vec{b}_{n-1}
$$

which can solved by a Newton-Raphson solver as the steady equation that have to be slightly modified.

Nuclear evolution

Let us consider a simplified version first.

$$\varepsilon_{\rm CNO} = 8.67\,10^{27} g\rho X X_{\rm CNO} T_6^{-2/3} e^{-152.28/T_6^{1/3}} \;\; {\rm cgs}$$

During the pp or CNO cycles, protons are converted into He and this release

$$Q = (4m_{\rm H} - m_{\rm He})c^2 = 4.3 \times 10^{-12} \; J$$

so that

$$\frac{dX}{dt} = \frac{4m_{\rm H}\varepsilon}{Q} = -K\rho X X(^{14}N) T_6^{-2/3} e^{-152.28/T_6^{1/3}}$$

1D model : Nuclear evolution

$$
\begin{cases}
\partial_r P + \rho \partial_r \phi = 0 \\
\Delta \phi = 4\pi G \rho \\
\mathrm{Div} \chi \vec{\nabla} T + \varepsilon(\rho, T) = \rho T (\partial_t s + V \partial_r s) \\
\partial_t X + V \partial_r X = -X \rho X(^{14}N) T_6^{-2/3} e^{-152.28/T_6^{1/3}} \\
\partial_t \ln \rho + V \partial_r \ln \rho = -\partial_r (r^2 V)/r^2 \\
P \equiv P(\rho, T, X, Z), \qquad \chi \equiv \chi(\rho, T, X, Z)
\end{cases}
\tag{8}
$$

1D model : Nuclear evolution

$$
\begin{cases}
\partial_r P_n + \rho_n \partial_r \phi_n = 0 \\
\Delta \phi_n = 4\pi G \rho_n \\
\delta t(\mathrm{Div}\chi \vec{\nabla} T_n + \varepsilon(\rho_n, T_n) - \rho_n T_n V_n \partial_r s_n) = (\rho T)_n (s_n - s_{n-1}) \\
\delta t(V_n \partial_r X_n + X_n \rho X(^{14}N) T_6^{-2/3} e^{-152.28/T_6^{1/3}}) = X_n - X_{n-1} \\
\delta t(V_n \partial_r \ln \rho_n + \partial_r (r^2 V_n)/r^2) = \ln \rho_n - \ln \rho_{n-1} \\
P \equiv P(\rho, T, X, Z), \qquad \chi \equiv \chi(\rho, T, X, Z)
\end{cases}
\tag{9}
$$

We note that in the 1D-problem, $\delta t V = \delta \xi$, namely the displacement during δt solely comes into play.

2D model : Angular momentum evolution

The foregoing equation generalize immediately in 2D. But there is the question of the angular momentum evolution.

In steady case this is determined by

$$s\frac{\partial\Omega^2}{\partial z} = \frac{(\vec{\nabla}P \wedge \vec{\nabla}\rho)_\varphi}{\rho^2}$$

$$\mathrm{Div}(\rho s^2\Omega\vec{w}) = \mathrm{Div}(\mu s^2\vec{\nabla}\Omega) \qquad \text{and} \qquad \mathrm{Div}(\rho\vec{w}) = 0$$

Time-evolution of the local angular momentum needs to be taken into account.

$$\partial_t\vec{v} + \vec{\omega}\wedge\vec{v} = -(\vec{\nabla}P)/\rho - \vec{\nabla}\phi + \vec{F}_{\rm visc}$$

the φ-component gives :

$$\partial_t(\rho s^2\Omega) + {\rm Div}(\rho s^2\Omega\vec{w}) = {\rm Div}(\mu s^2\vec{\nabla}\Omega)$$

while

$$\frac{\partial\omega_\varphi}{\partial t} + \overrightarrow{\rm Rot}(\overrightarrow{\rm Rot}\vec{w}\wedge\vec{w})_\varphi - s\frac{\partial\Omega^2}{\partial z} = \frac{(\vec{\nabla}\rho\wedge\vec{\nabla}P)_\varphi}{\rho^2} + \overrightarrow{\rm Rot}\vec{F}_v|_\varphi$$

2D model : angular momentum evolution

At first order we may say that

$$-s\frac{\partial\Omega^2}{\partial z} = \frac{(\vec{\nabla}\rho \wedge \vec{\nabla}P)_\varphi}{\rho^2}$$

and

$$\partial_t(\rho s^2\Omega) + \mathrm{Div}(\rho s^2\Omega\vec{w}) = \mathrm{Div}(\mu s^2\vec{\nabla}\Omega)$$

2D angular momentum evolution : Conclusion

The local time-evolution of angular momentum changes the meridional circulation that must adapt to the change of density. Hence

$$\rho\vec{w}\cdot\vec{\nabla}(s^2\Omega) = F_v(\Omega) - \rho\partial_t(s^2\Omega)$$

and

$$\mathrm{Div}(\rho\vec{w}) = -\partial_t\rho$$

But there is a subtlety :
What about the grid velocity ? In 1D lagrangian models **the velocity of the fluid ≡ the velocity of the grid**.

ESTER is a bit lagrangian because the grid follow the surface and the interfaces...