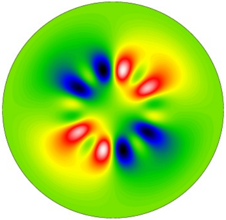
Non-adiabatic pulsations in ESTER models

D. R. Reese[1], M.-A. Dupret[2], and M. Rieutord[3]

[1]LESIA, [2]ULg, [3]IRAP

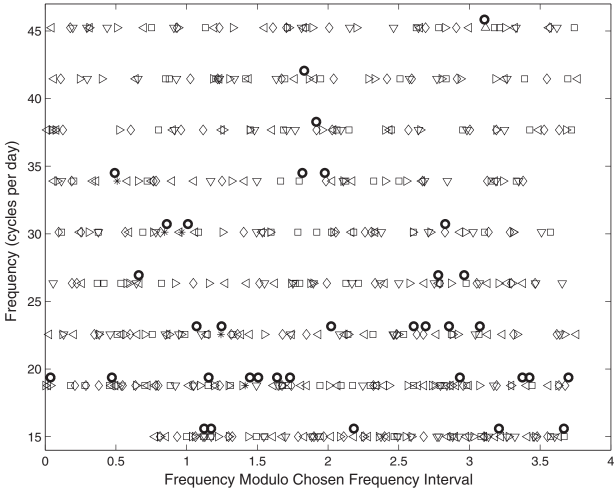
July 12, 2016



Introduction

The challenges in interpreting the pulsations of rapidly rotating stars

- theoretical challenges
  - 2D geometry – complicated formulas and numerically demanding
  - no automatic mode classification procedure
- lack of *simple* frequency patterns
  - p-modes: superposition of multiple independent patterns
  - g-modes: varying period separation + numerous inertial modes
- amplitudes are difficult to predict (classical pulsators)



(Deupree et al. 2012)

The benefits of non-adiabatic calculations

- find out which modes are excited
- consistent calculation of $\delta T_{\mathrm{eff}}/T_{\mathrm{eff}}$
  - amplitude ratios
  - phase shifts
  - line profile variations (LPVs)
  - mode identification

Previous 2D non-adiabatic pulsation calculations

Reference	Model	Pulsations
Lee & Baraffe (1995)	Chandrasekhar expansion	2 or 3 harmonics
Lee (2001)	Spherical	10 harmonics
Savonije (2005, 2007)	Spherical	2D calculations
Lee (2008)	Chandrasekhar expansion	4 harmonics

Comparisons with traditional approximation

- Savonije (2005, 2007): stabilising effect of Coriolis force
- Lee (2008): stabilising effect of centrifugal deformation

Previous 2D non-adiabatic pulsation calculations

Reference	Model	Pulsations
Lee & Baraffe (1995)	Chandrasekhar expansion	2 or 3 harmonics
Lee (2001)	Spherical	10 harmonics
Savonije (2005, 2007)	Spherical	2D calculations
Lee (2008)	Chandrasekhar expansion	4 harmonics

Comparisons with traditional approximation

- Savonije (2005, 2007): stabilising effect of Coriolis force
- Lee (2008): stabilising effect of centrifugal deformation

- in all cases, the effects of rotation are approximated
  $\Rightarrow$ there is a need for full 2D calculations, with 2D models

Necessary ingredients for 2D non-adiabatic pulsations

- 2D rotating models
  - hydrostatic equilibrium: adiabatic calculations
  - energy conservation equation: non-adiabatic calculations
- a 2D pulsation code which includes non-adiabatic effects

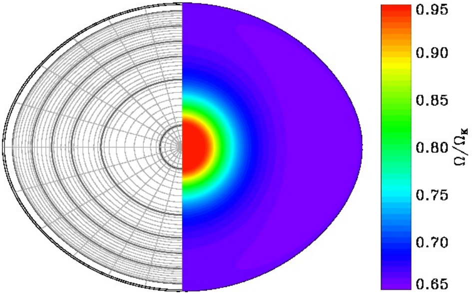
Necessary ingredients for 2D non-adiabatic pulsations

- 2D rotating models
  - hydrostatic equilibrium: adiabatic calculations
  - energy conservation equation: non-adiabatic calculations
- a 2D pulsation code which includes non-adiabatic effects

ESTER (models) + TOP (pulsations)

The ESTER code (a very brief overview)

- ESTER = Evolution STEllaire en Rotation
- fully includes centrifugal deformation
- satisfies energy conservation equation:
  - baroclinic (isobars ≠ isochores ≠ isotherms)
  - self-consistent 2D rotation profile



(Rieutord & Espinosa Lara, 2009)

The TOP pulsation code

- TOP = Two-dimensional Oscillation Program
- fully includes centrifugal deformation
- can handle baroclinic models
- includes non-adiabatic effects

http://johnmannophoto.com/blog/?p=103

Pulsation equations

Continuity equation (conservation of mass)

$$0 = \frac{\delta \rho}{\rho_o} + \vec{\nabla} \cdot \vec{\xi}$$

Poisson's equation

$$0 = \Delta \Psi - 4\pi G \left(\rho_o \frac{\delta \rho}{\rho_o} - \vec{\xi} \cdot \vec{\nabla} \rho_o \right)$$

$\delta\rho$ = Lagrangian density perturbation

ρ_o = equilibrium density profile

$\vec{\xi}$ = Lagrangian displacement

Ψ = Eulerian perturbation to the gravitational potential

Pulsation equations

Euler's equations (conservation of momentum)

$$
\begin{aligned}
0 \;=\;& [\omega + m\Omega]^2 \vec{\xi} - 2i\vec{\Omega} \times [\omega + m\Omega]\,\vec{\xi} - \vec{\Omega} \times \left(\vec{\Omega} \times \vec{\xi}\right) \\
-\;& \vec{\xi} \cdot \vec{\nabla}\left(\varpi \Omega^2 \vec{e}_\varpi\right) - \frac{P_o}{\rho_o}\vec{\nabla}\left(\frac{\delta P}{P_o}\right) + \frac{\vec{\nabla} P_o}{\rho_o}\left(\frac{\delta\rho}{\rho_o} - \frac{\delta P}{P_o}\right) - \vec{\nabla}\Psi \\
+\;& \vec{\nabla}\left(\frac{\vec{\xi} \cdot \vec{\nabla} P_o}{\rho_o}\right) + \frac{\left(\vec{\xi} \cdot \vec{\nabla} P_o\right)\vec{\nabla}\rho_o - \left(\vec{\xi} \cdot \vec{\nabla}\rho_o\right)\vec{\nabla} P_o}{\rho_o^2}
\end{aligned}
$$

ω = pulsation frequency

m = azimuthal order

Ω = rotation profile

ϖ = distance to the rotation axis

δP = Lagrangian pressure perturbation

Pulsation equations

Energy conservation equation

- unperturbed form:

$$\rho_o T_o \frac{dS_o}{dt} = \epsilon_o \rho_o - \vec{\nabla} \cdot \vec{F}_o$$

- perturbed form:

$$\begin{aligned} i\left[\omega + m\Omega\right] \rho_o T_o \delta S &= \epsilon_o \rho_o \left(\frac{\delta\epsilon}{\epsilon_o} + \frac{\delta\rho}{\rho_o} \right) - \vec{\nabla} \cdot \delta\vec{F} \\ &+ \vec{\xi} \cdot \vec{\nabla}\left(\vec{\nabla} \cdot \vec{F}_o\right) - \vec{\nabla} \cdot \left[\left(\vec{\xi} \cdot \vec{\nabla}\right) \vec{F}_o\right] \end{aligned}$$

$\delta\vec{F}$ = Lagrangian perturbation to the energy flux

δS = Lagrangian entropy perturbation

$\delta\epsilon$ = Lagrangian perturbation to the energy production

Pulsation equations

Energy flux

- total energy flux

$$\vec{F}_o = \vec{F}_o^{\mathrm{R}} + \vec{F}_o^{\mathrm{C}}$$

- unperturbed form of radiative energy flux:

$$\vec{F}_o^{\mathrm{R}} = -\frac{4acT_o^3}{3\kappa_o\rho_o}\vec{\nabla}T_o = -\chi_o\vec{\nabla}T_o$$

- perturbed form of radiative energy flux:

$$\begin{aligned}\delta\vec{F}^{\mathrm{R}} &= \left[(1+\chi_T)\frac{\delta T}{T_o} + \chi_\rho\frac{\delta\rho}{\rho_o}\right]\vec{F}_o^{\mathrm{R}} \\ &- \chi_o\left[T_o\vec{\nabla}\left(\frac{\delta T}{T_o}\right) + \vec{\xi}\cdot\vec{\nabla}\left(\vec{\nabla}T_o\right) - \vec{\nabla}\left(\vec{\xi}\cdot\vec{\nabla}T_o\right)\right]\end{aligned}$$

- frozen convection approximation:

$$\delta\vec{F}^{\mathrm{C}} \simeq \vec{0}$$

Pulsation equations

Equation of state, opacities, and nuclear reaction rates

$$
\begin{aligned}
\frac{\delta P}{P_o} &= \Gamma_1 \frac{\delta \rho}{\rho_o} + P_{\mathrm{T}} \frac{\delta S}{c_{\mathrm{v}}} = P_\rho \frac{\delta \rho}{\rho_o} + P_{\mathrm{T}} \frac{\delta T}{T_o} \\
\frac{\delta T}{T_o} &= \frac{\delta S}{c_{\mathrm{v}}} + (\Gamma_3 - 1) \frac{\delta \rho}{\rho_o} = \frac{\delta S}{c_{\mathrm{p}}} + \nabla_{\mathrm{ad}} \frac{\delta P}{P_o} \\
\frac{\delta \chi}{\chi_o} &= \chi_\rho \frac{\delta \rho}{\rho_o} + \chi_T \frac{\delta T}{T_o} \\
\frac{\delta \epsilon}{\epsilon_o} &= \epsilon_T(\omega) \frac{\delta T}{T_o} + \epsilon_\rho(\omega) \frac{\delta \rho}{\rho_o}
\end{aligned}
$$

- in what follows we will neglect $\delta\epsilon$

Pulsation equations

Boundary conditions

- in the centre: regularity conditions
- at infinity: gravitational potential perturbation goes to zero
- at the surface:

$$\nabla_{\mathrm{vert.}} \left(\frac{\delta P}{P_o} \right) = 0$$

$$4 \frac{\delta T}{T_o} = \frac{\delta F^{\mathrm{R}}}{F_o^{\mathrm{R}}}$$

Pulsation equations

Summary

- final result: a system of 10 equations with 10 unknowns:

$$\frac{\delta P}{P_o}, \quad \vec{\xi}, \quad \frac{\delta S}{c_\mathrm{p}}, \quad \delta \vec{F}^\mathrm{R}, \quad \frac{\delta T}{T_o}, \quad \Psi \tag{1}$$

- although some of these variables can be cancelled algebraically, they are needed to ensure good convergence

Work integral

- it is possible to derive an integral expression for the complex frequencies:

$$A\omega^2 + 2B\omega + C = 0$$

where

$$\begin{aligned} A &= \int_V \rho_0 \xi^2 \mathrm{dV}, \\ B &= \int_V \rho_0 \left[m\Omega\xi^2 - i\vec{\Omega}\cdot\left(\vec{\xi}\times\vec{\xi}^*\right)\right]\mathrm{dV}, \\ \Re(C) &= \text{a complicated expression} \\ \Im(C) &= -\int_V \Im\left\{\frac{\delta P\delta\rho^*}{\rho_0}\right\}\mathrm{dV} \end{aligned}$$

- From this we deduce the excitation rate:

$$\Im(\omega) = -\frac{\Im(C)}{2\left(A\Re(\omega)+B\right)}$$

Comparison with Lee & Baraffe (1995)

Lee & Baraffe (1995)	**Current work**
Model based on Chandrasekhar expansion	2D baroclinic model
Eulerian perturbations	Lagrangian perturbations
$\vec{F}^{\mathrm{C}} = 0$	$\delta\vec{F}^{\mathrm{C}} = 0$

Numerical implementation

- explicit expression in spheroidal coordinates
- projection onto spherical harmonics
- radial discretisation using Chebyshev polynomials

N_r	N_h	Memory (in Gb)	Time (in min)	Num. proc.
400	10	3.5		
400	15	7.9		
400	20	13.4	5	4
400	29	28.0	10	8
400	40	52.7	22	8
400	50	82.3	26	16

Estimated accuracy

- the problem is stiff: reduced numerical accuracy
- estimated accuracy based on variational expression:
  - frequencies: $\sim 10^{-4}$
  - excitation/damping rates: 10^{-2} to 10^{-1}

Description

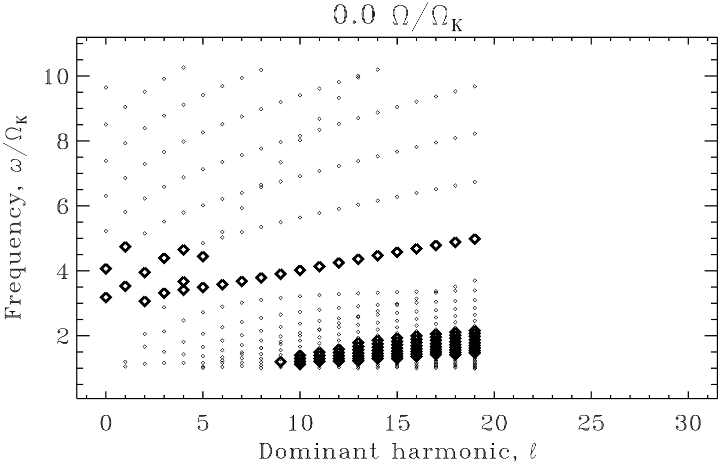
Model

- 9 $M_\odot$ models
- $\Omega = 0.0$ to $0.8\,\Omega_K$
- $z = 0.025$
- OPAL opacities

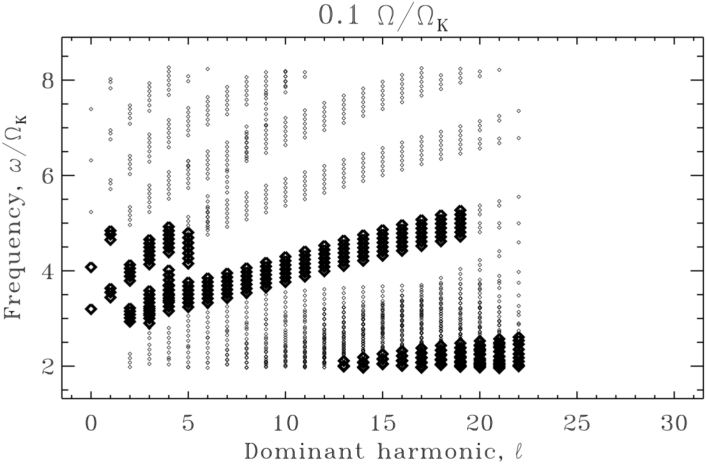
Modes

- β Cep type pulsations
- p and g modes
- excited by iron opacity bump at $\log(T) = 5.3$

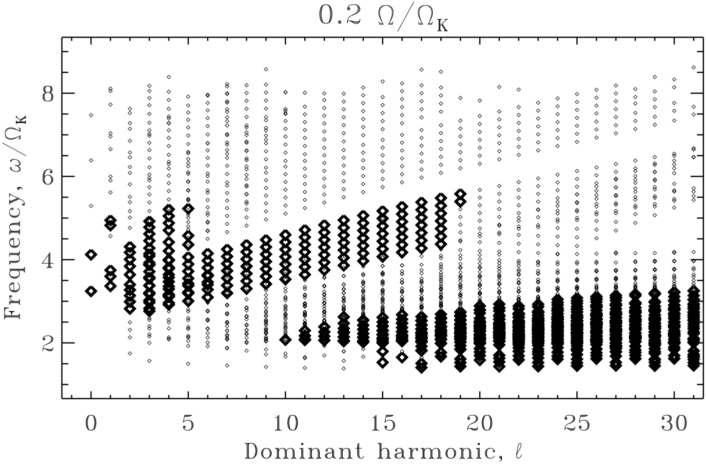
Frequencies



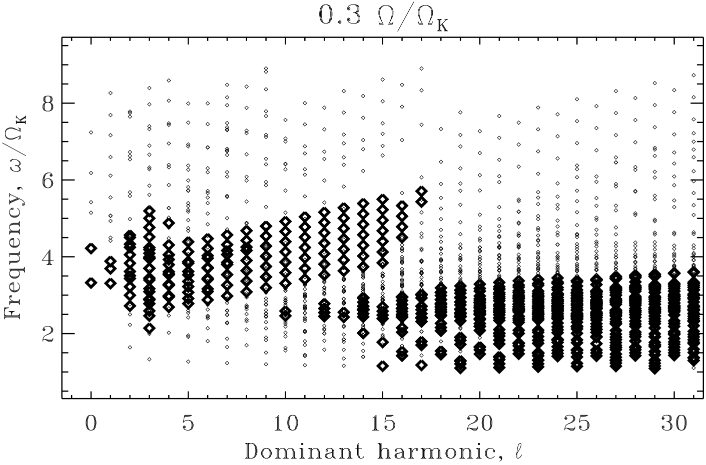
Frequencies



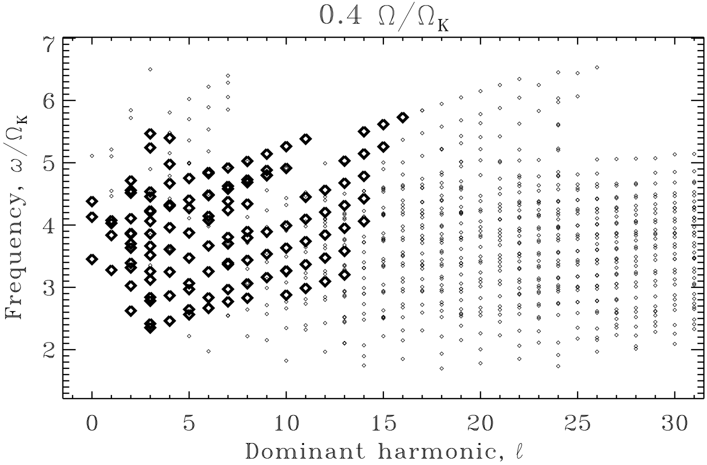
Frequencies



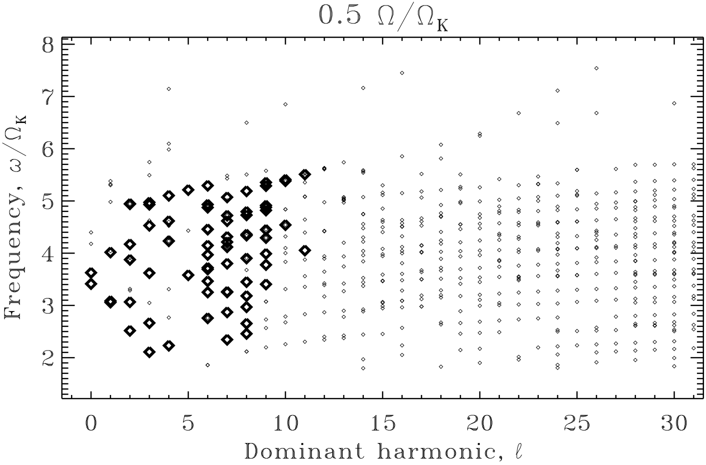
Frequencies



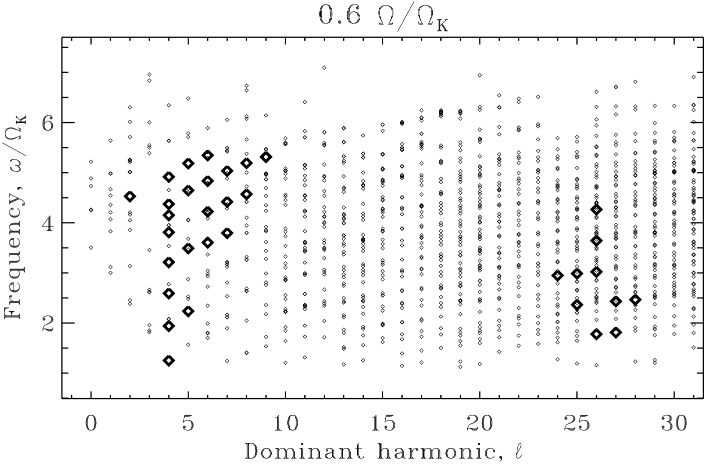
Frequencies



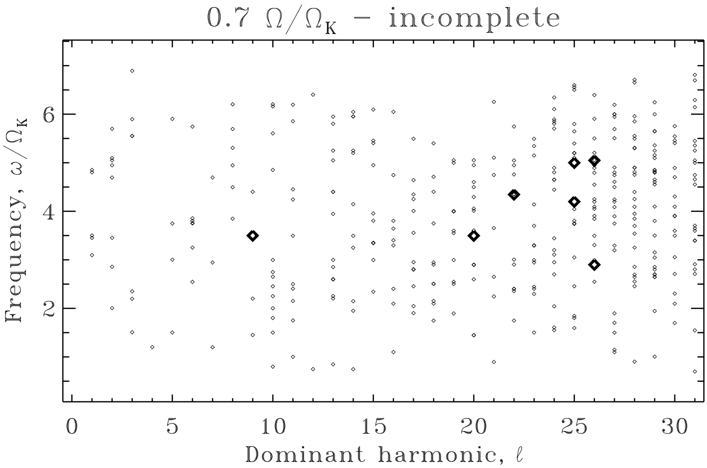
Frequencies



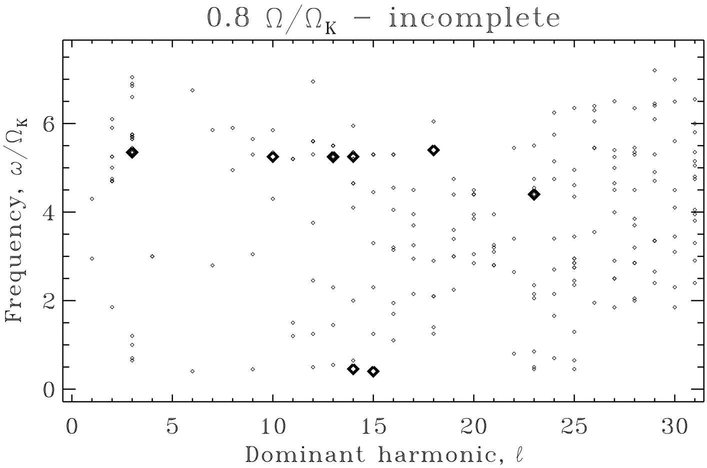
Frequencies



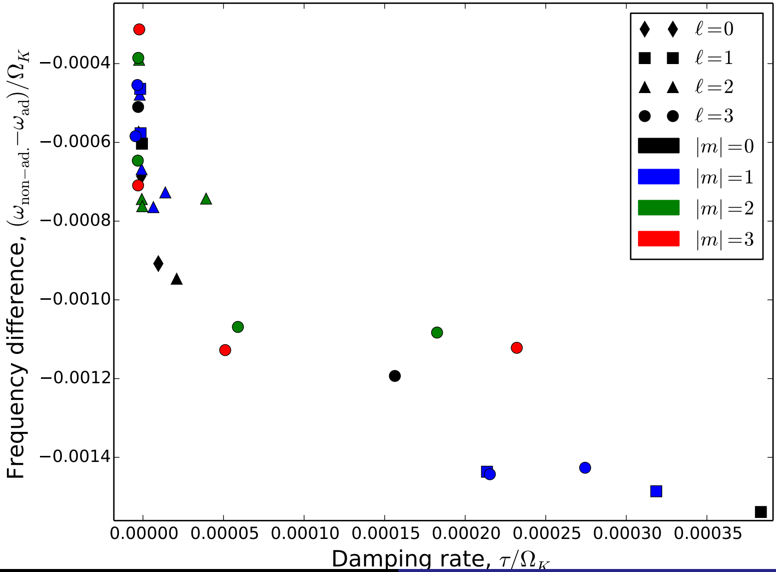
Frequencies

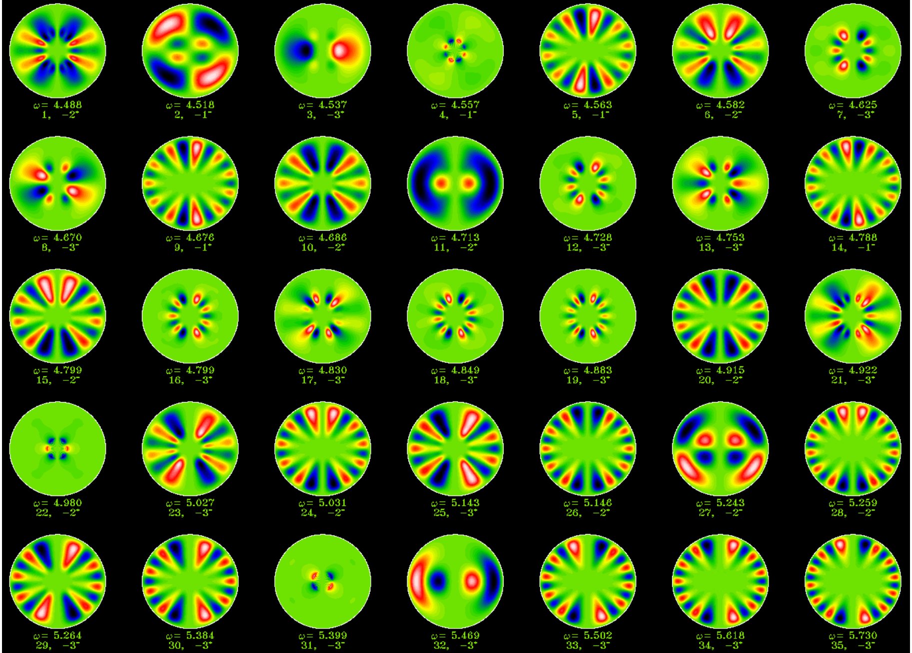


Frequencies

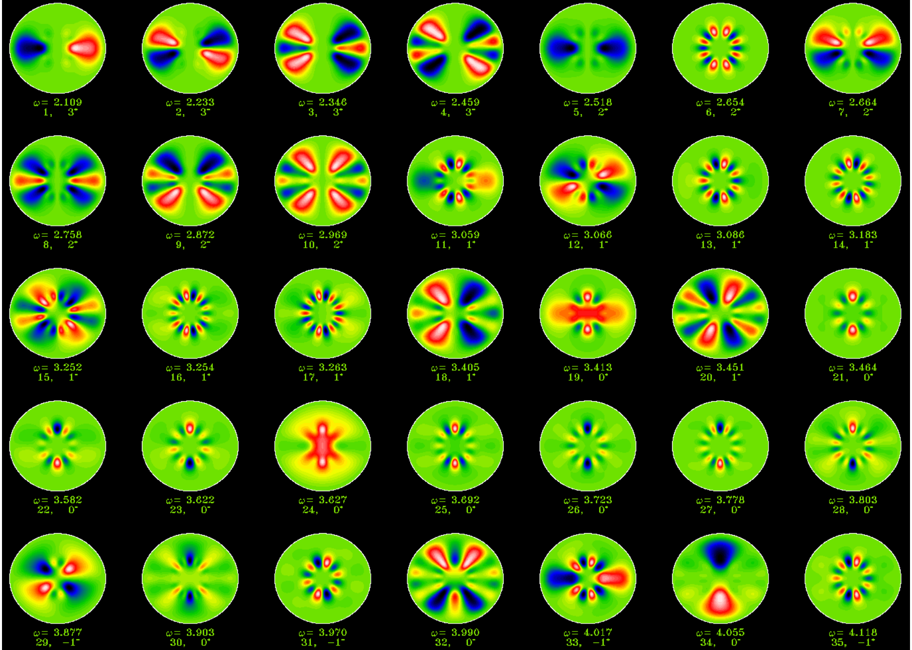


Damping rates

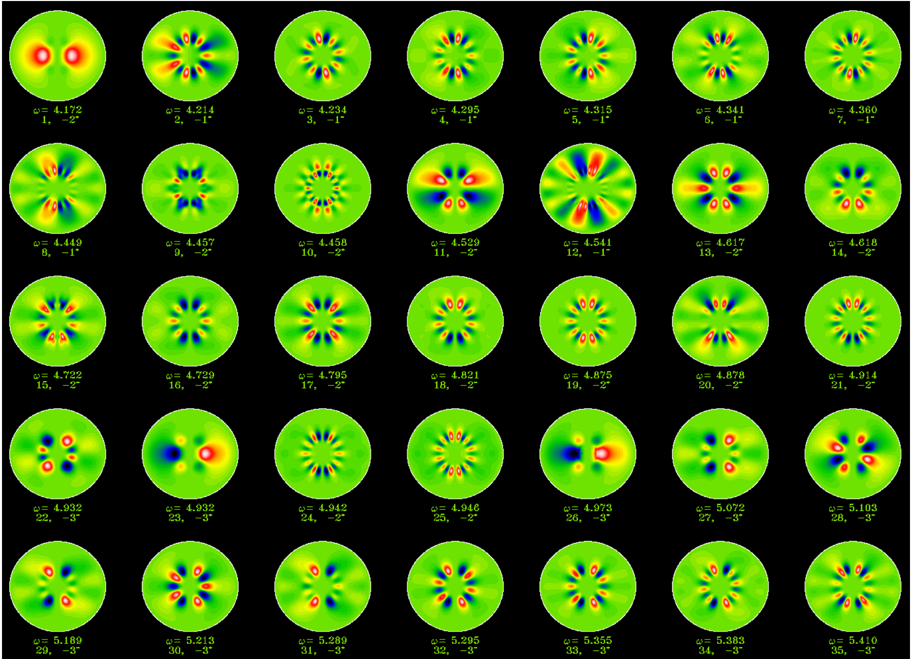




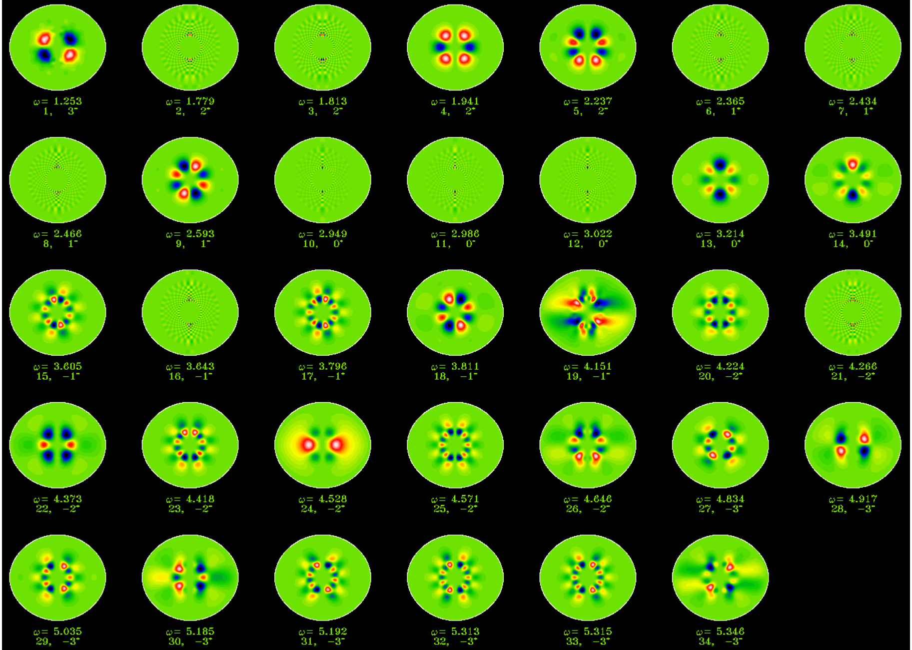
$\Omega = 0.4\ \Omega k$



$\Omega = 0.5\ \Omega k$

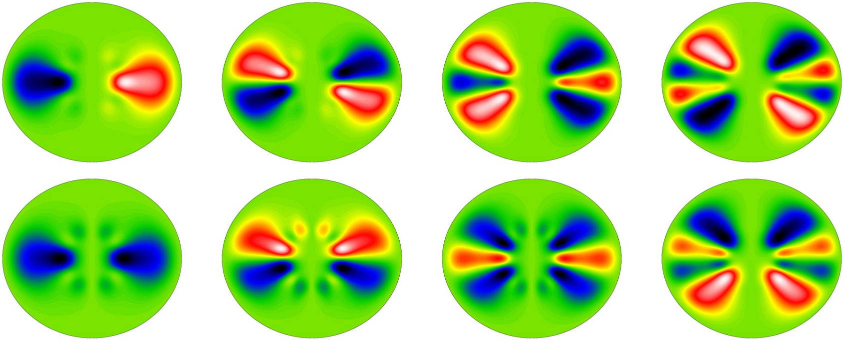


$\Omega = 0.5\ \Omega k$

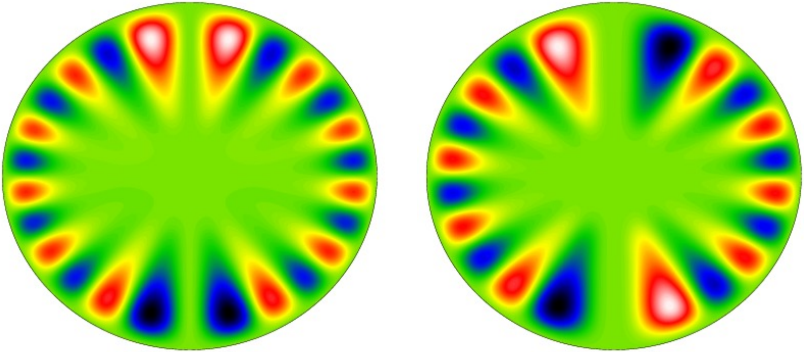


$\Omega = 0.6\ \Omega k$

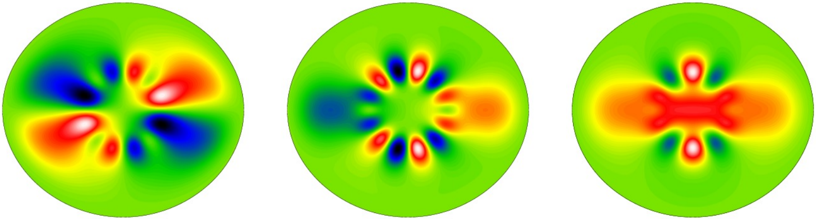
Island modes



Whispering gallery modes

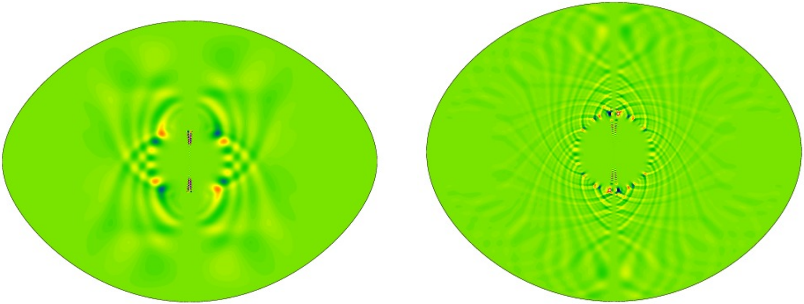


Mixed modes



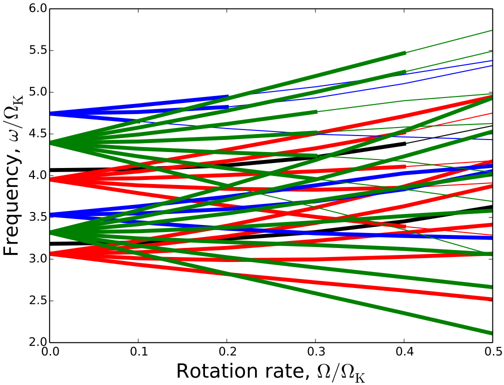
- see Ouazzani et al. (2015) for mixed modes in the adiabatic case

Rosette modes



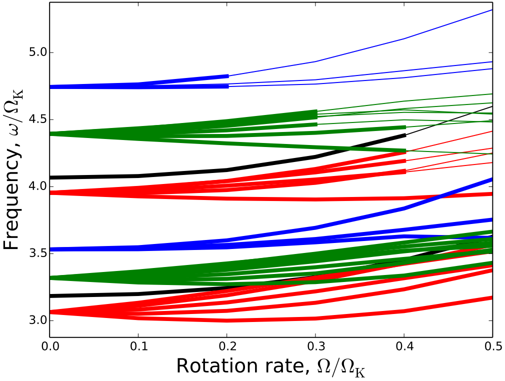
- also see Takata & Saio (2015) for non-adiabatic effects on Rosette modes and associated angular momentum transport

Multiplets – inertial frame



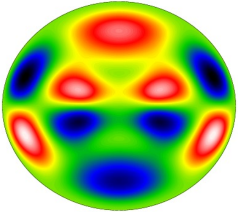
- prograde modes remain unstable longer
- Lee (2008) also found a preference for prograde modes

Multiplets – corotating frame

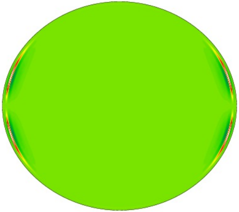


- prograde modes remain unstable longer
- Lee (2008) also found a preference for prograde modes

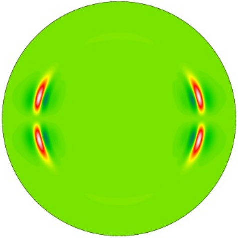
Work integral



Acoustic



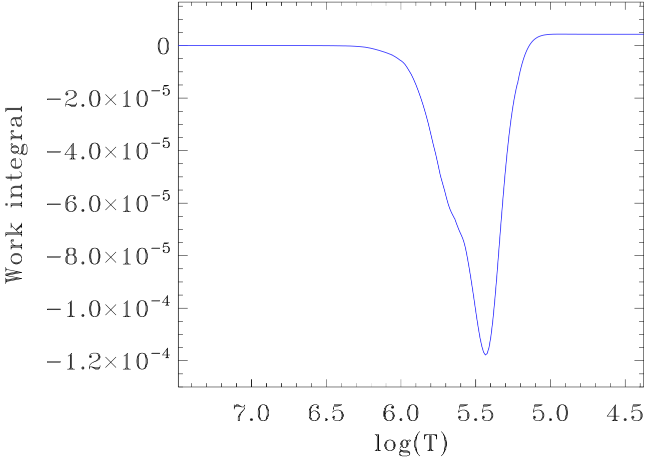
Work



Work (vs. log T)

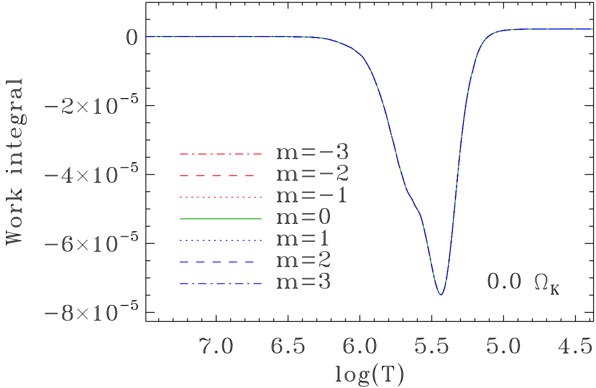
- red = driving regions
- blue = damping regions

Work integral



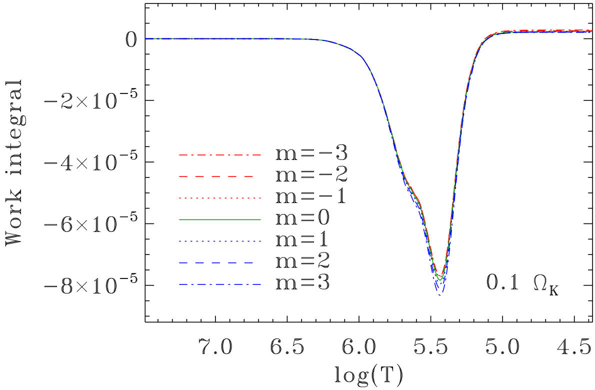
- obtained by integrating in horizontal direction + vertical anti-derivative

A multiplet



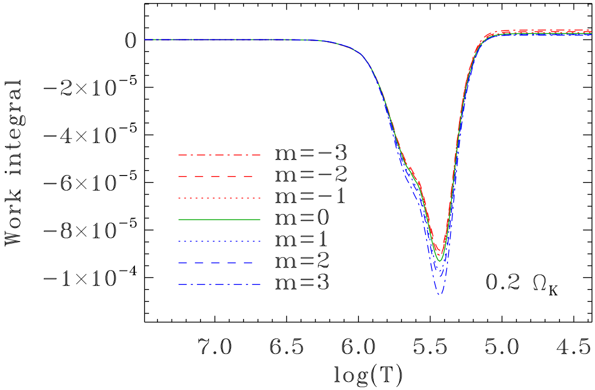
- rotation rate $= 0.0 \quad \Omega_K$, $\varepsilon = 0$

A multiplet



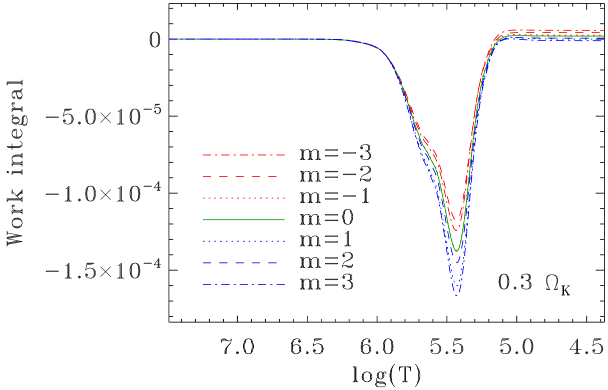
- rotation rate = 0.1 Ω_K, $\varepsilon = 4.9 \times 10^{-3}$

A multiplet



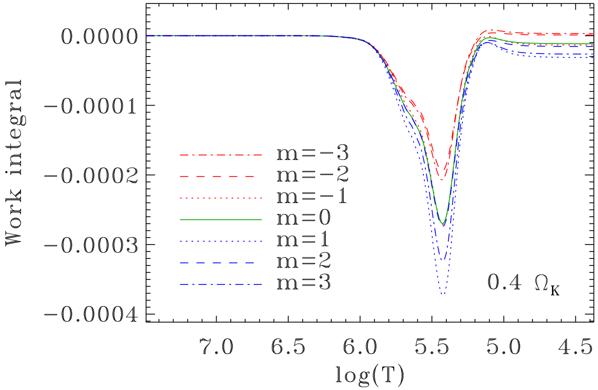
- rotation rate = 0.2 $\Omega_{\rm K}$, $\varepsilon = 1.9 \times 10^{-2}$

A multiplet



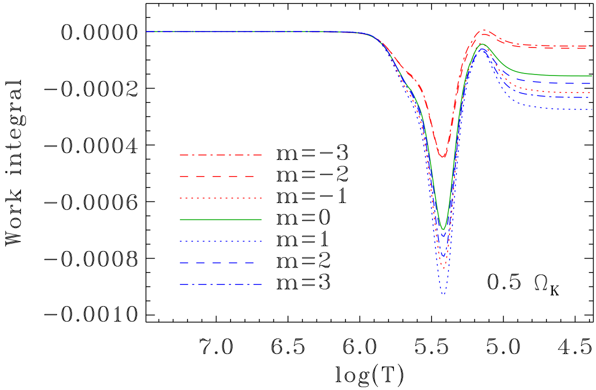
- rotation rate = 0.3 Ω_K, $\varepsilon = 4.3 \times 10^{-2}$

A multiplet



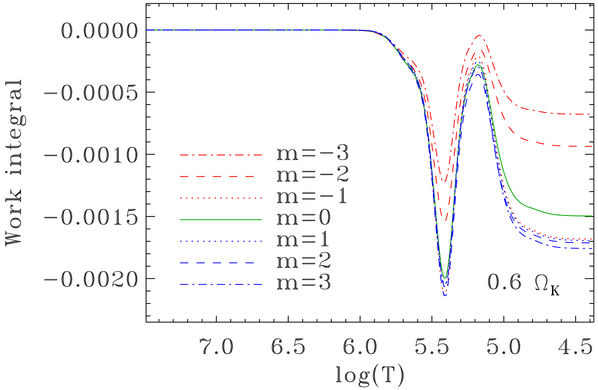
- rotation rate = 0.4 Ω_K, $\varepsilon = 7.4 \times 10^{-2}$

A multiplet



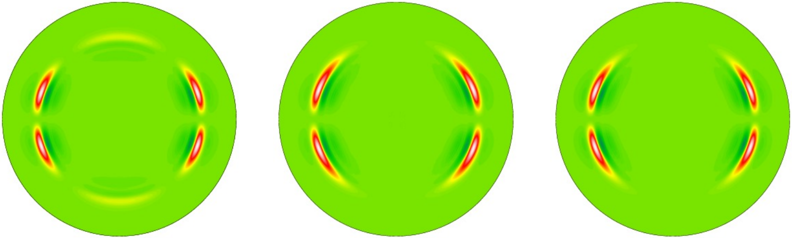
- rotation rate = 0.5 Ω_K, $\varepsilon = 11.2 \times 10^{-2}$

A multiplet



- rotation rate = $0.6\ \Omega_{\mathrm{K}}$, $\varepsilon = 15.5 \times 10^{-2}$

A multiplet



$\mathbf{m = 0}$
Stable

$\mathbf{m = -2}$
Excited

$\mathbf{m = 2}$
Stable

- rotation rate $= 0.4\ \Omega_{\mathrm{K}}$, $\varepsilon = 7.4 \times 10^{-2}$

Amplitude ratios

Previous works

- Daszyńska-Daszkiewicz et al. (2002, 2007), Townsend (2003)
  - non-adiabatic treatment
  - approximate treatment of rotation
- Reese et al. (2013) (see also Lignières et al. 2006, Lignières & Georgeot 2009)
  - full treatment of rotation
  - adiabatic calculations

Equations

- non-pulsating star:

$$\mathcal{I} = \iint_{\mathrm{Vis.Surf.}} I(g_{\mathrm{eff}}, T_{\mathrm{eff}}, \mu) \vec{e}_{\mathrm{obs.}} \cdot \vec{dS}$$

- pulsating star:

$$\begin{aligned}
\delta\mathcal{I} &= \iint_{\delta\mathrm{Vis.Surf.}} I(g_{\mathrm{eff}}, T_{\mathrm{eff}}, \mu) \vec{e}_{\mathrm{obs.}} \cdot \vec{dS} \\
&+ \iint_{\mathrm{Vis.Surf.}} \delta I(g_{\mathrm{eff}}, T_{\mathrm{eff}}, \mu) \vec{e}_{\mathrm{obs.}} \cdot \vec{dS} \\
&+ \iint_{\mathrm{Vis.Surf.}} I(g_{\mathrm{eff}}, T_{\mathrm{eff}}, \mu) \vec{e}_{\mathrm{obs.}} \cdot \delta(\vec{dS})
\end{aligned}$$

Equations

First term

$$\iint_{\delta S} \dots \vec{e}_{\mathrm{obs.}} \cdot d\vec{S} \propto \xi^2 \Rightarrow \text{ negligible}$$

Second term

$$\delta I = I \cdot \left(\frac{\partial \ln I}{\partial \ln T_{\mathrm{eff}}} \frac{\delta T_{\mathrm{eff}}}{T_{\mathrm{eff}}} + \frac{\partial \ln I}{\partial \ln g_{\mathrm{eff}}} \frac{\delta g_{\mathrm{eff}}}{g_{\mathrm{eff}}} \right) + \frac{\partial I}{\partial \mu} \delta\mu$$

- $\frac{\delta T_{\mathrm{eff}}}{T_{\mathrm{eff}}}$, $\frac{\delta g_{\mathrm{eff}}}{g_{\mathrm{eff}}}$, and $\delta\mu$ are deduced from the pulsation mode
- see next slide for I and its derivatives

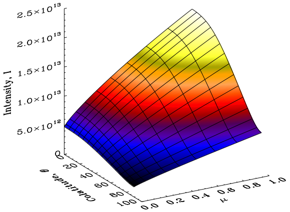
Third term

- $\delta(d\vec{S})$ is deduced from the Lagrangian displacement

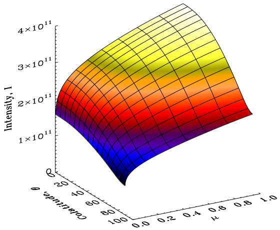
Intensities

$$I(T_{\rm eff}, g_{\rm eff}, \mu) = I_0(T_{\rm eff}, g_{\rm eff}) h(\mu, T_{\rm eff}, g_{\rm eff})$$

- $I_0(T_{\rm eff}, g_{\rm eff})$ from blackbody spectrum
- $h(\mu, T_{\rm eff}, g_{\rm eff})$ from Claret (2000)
- bolometric, Strömgren, and Johnson-Cousins photometric bands

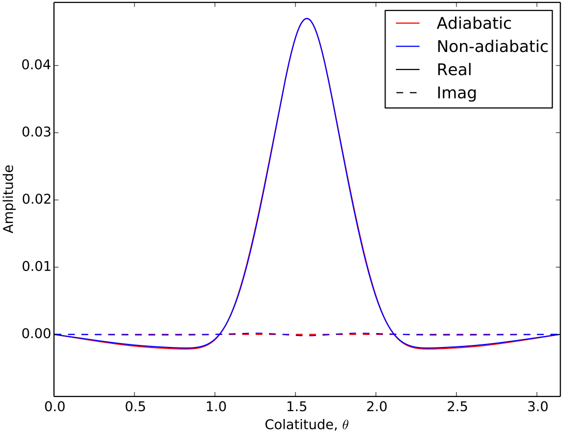


Bolometric



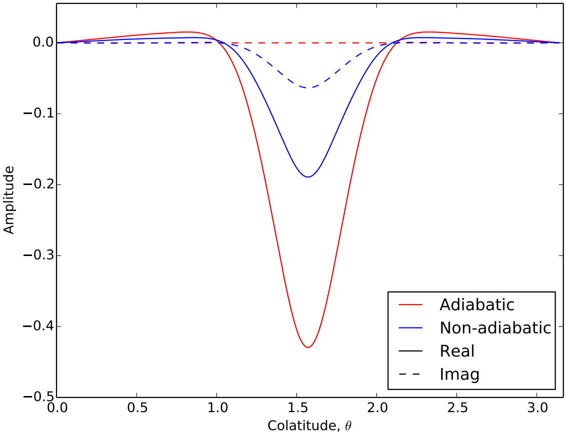
Strömgren, u

Various profiles



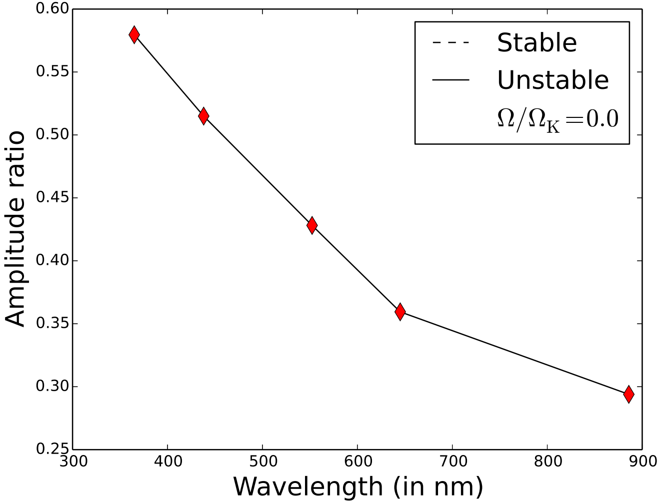
ξ_r

Various profiles

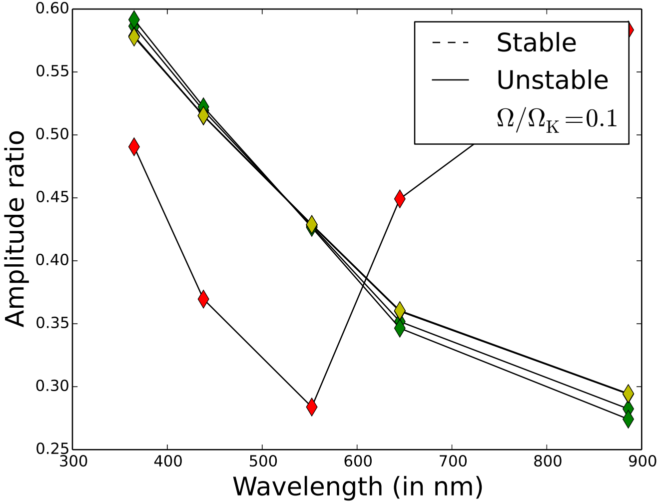


$$\delta T_{\mathrm{eff}} / T_{\mathrm{eff}}$$

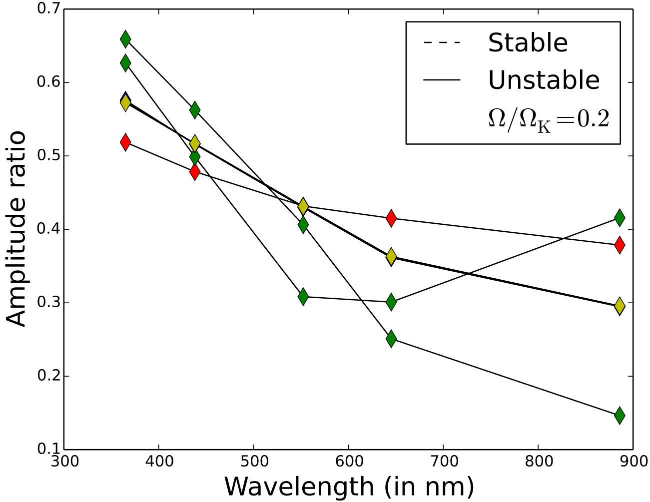
Amplitude ratios for an $\ell = 3$ multiplet ($i = 30^\circ$)



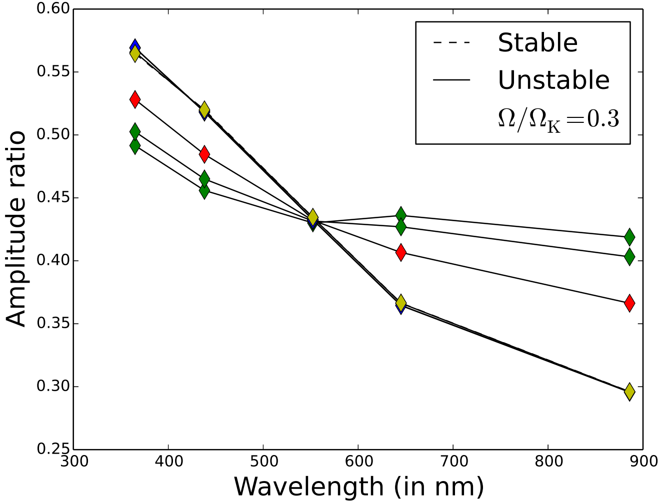
Amplitude ratios for an $\ell = 3$ multiplet ($i = 30^\circ$)



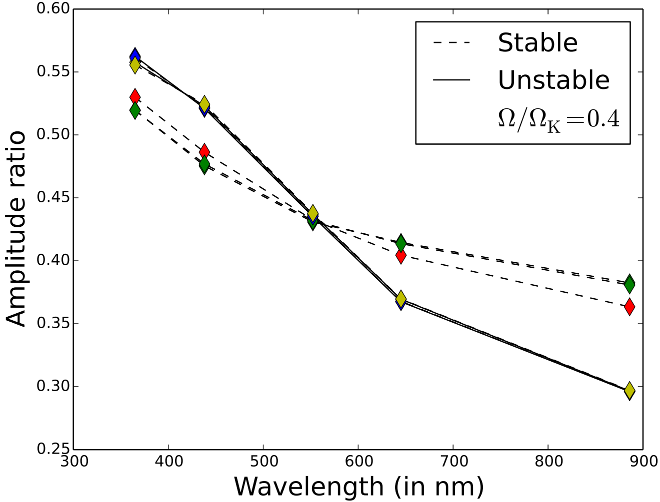
Amplitude ratios for an $\ell = 3$ multiplet ($i = 30°$)



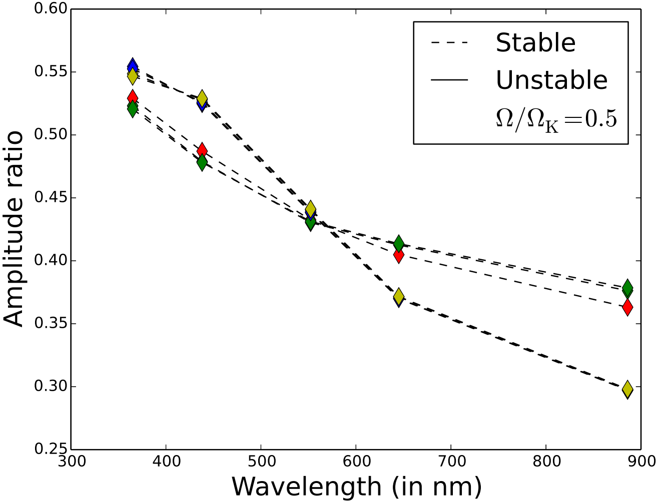
Amplitude ratios for an $\ell = 3$ multiplet ($i = 30^\circ$)



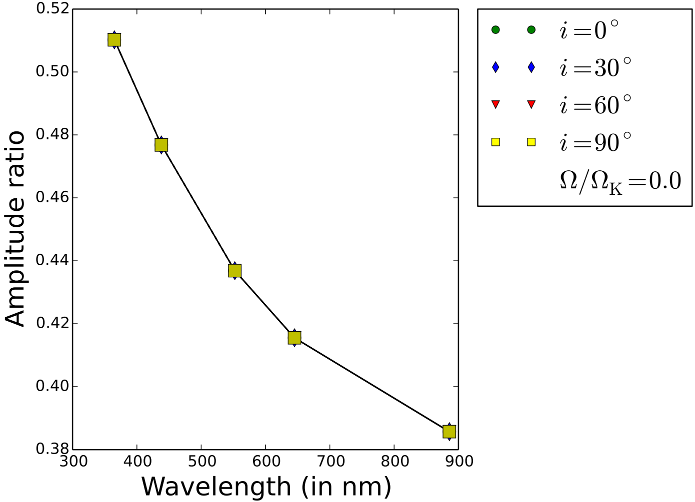
Amplitude ratios for an $\ell = 3$ multiplet ($i = 30°$)



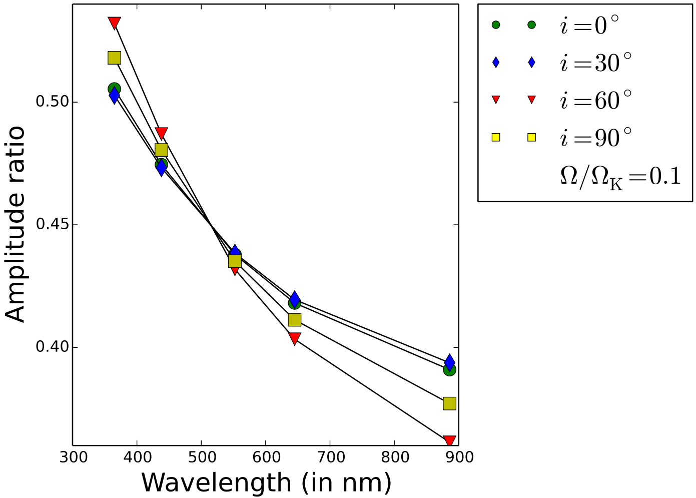
Amplitude ratios for an $\ell = 3$ multiplet ($i = 30^\circ$)



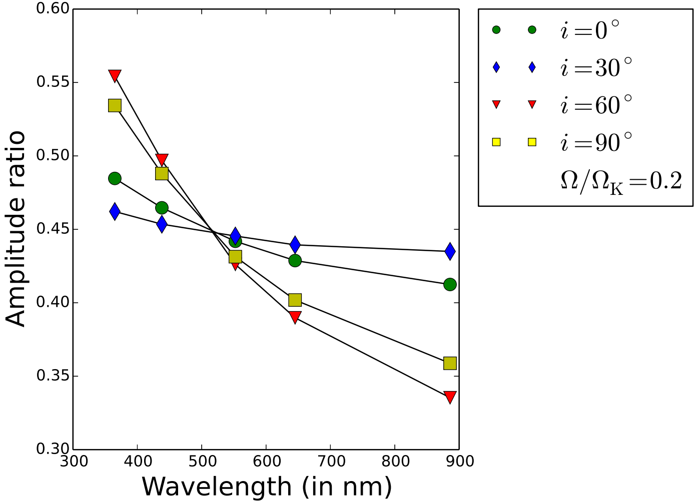
Amplitude ratios for an ($\ell = 2, m = 0$) mode



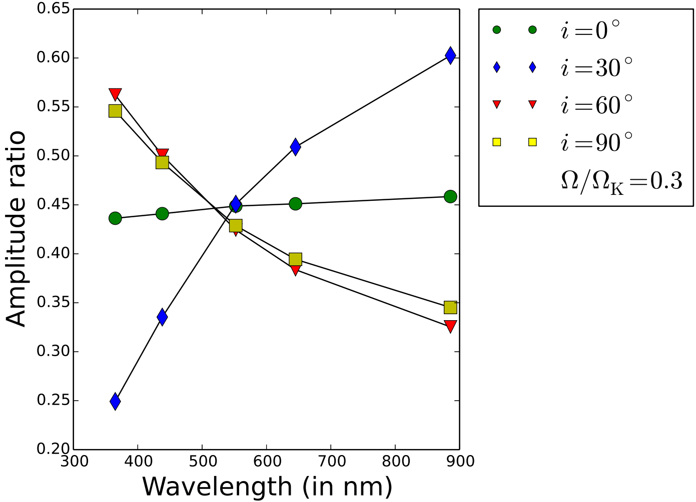
Amplitude ratios for an ($\ell = 2, m = 0$) mode



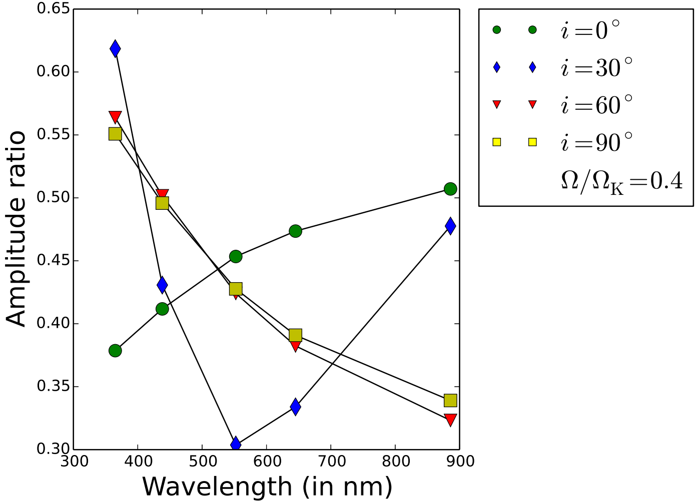
Amplitude ratios for an $(\ell = 2, m = 0)$ mode



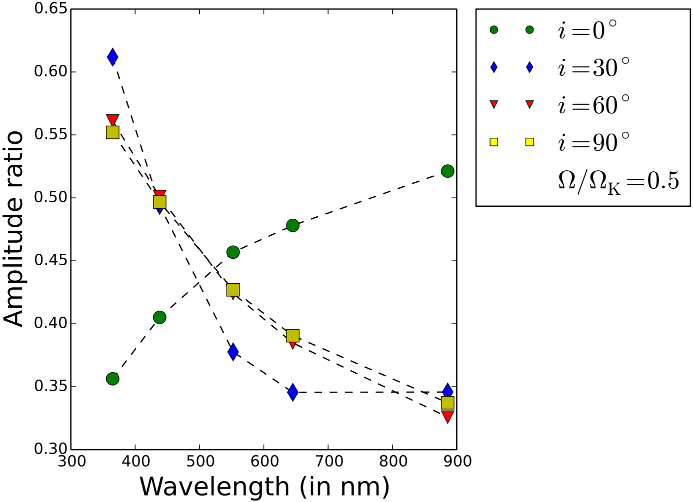
Amplitude ratios for an ($\ell = 2, m = 0$) mode



Amplitude ratios for an ($\ell = 2, m = 0$) mode



Amplitude ratios for an $(\ell = 2, m = 0)$ mode



Line Profile Variations (LPVs)

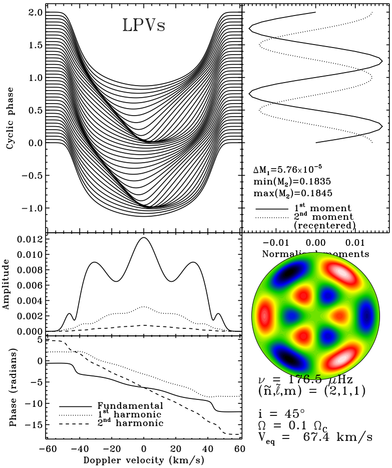
Previous works

- Clement (1994): 2D calculations
- Townsend (1997): the traditional approximation

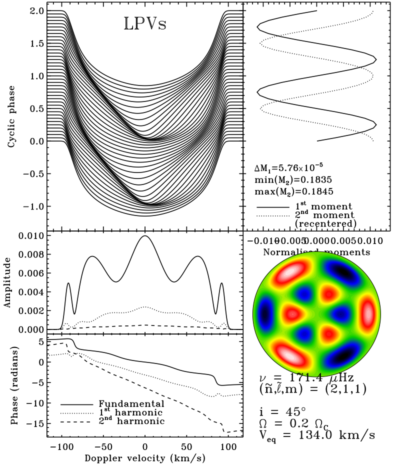
Description

- includes Doppler shifts and $\delta(d\vec{S})$
- δT_{eff} and δg_{eff} neglected
- use of blackbody spectrum (incl. gravity darkening)
- rudimentary description of limb darkening

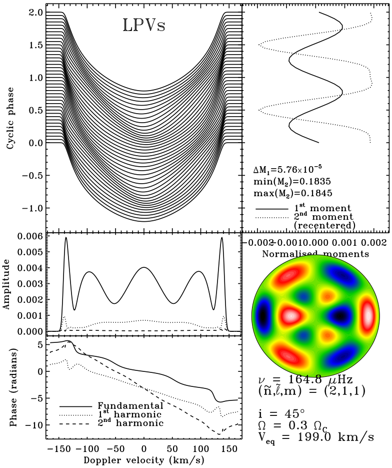
Increasing rotation rates



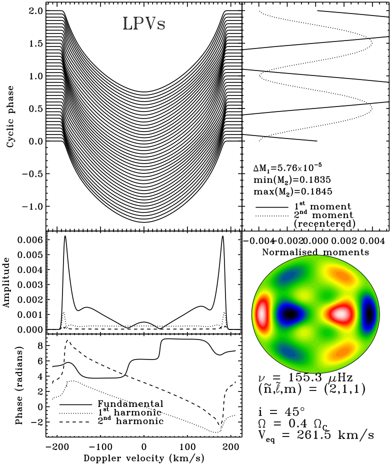
Increasing rotation rates



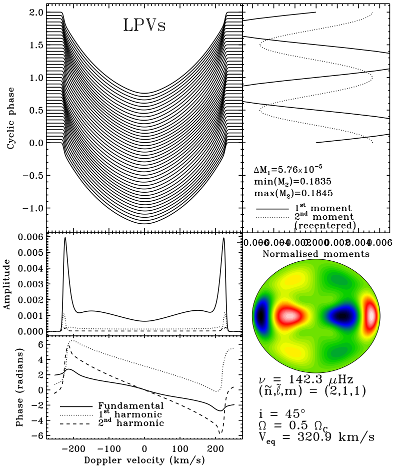
Increasing rotation rates



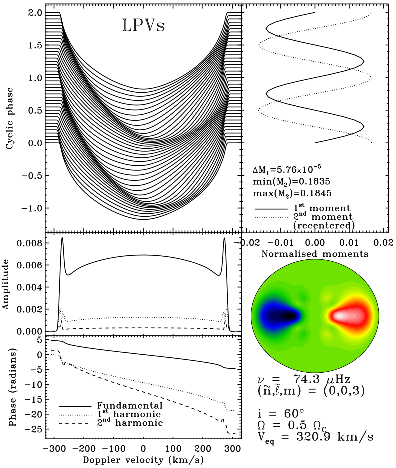
Increasing rotation rates



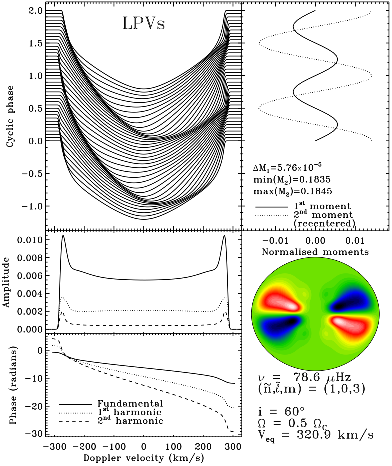
Increasing rotation rates



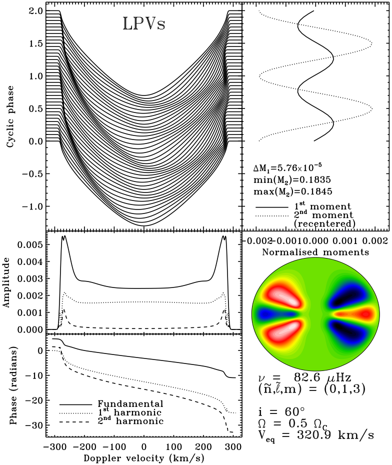
Increasing ℓ value



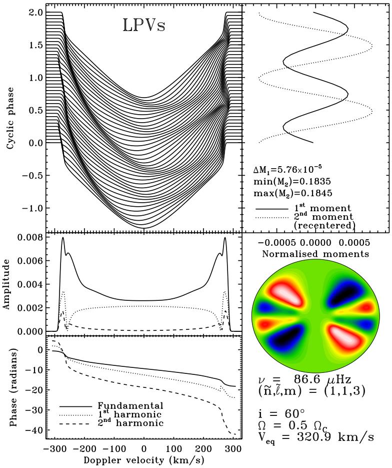
Increasing ℓ value



Increasing ℓ value



Increasing ℓ value



Conclusion

- important step forward:
  - can now predict which modes are unstable
  - can calculate amplitude ratios and LPVs

Prospects

- understand how rotation (de)stabilise modes
- what are the differences between prograde and retrograde modes
- include more realistic atmosphere
- identify modes