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Internship proposal

Blind inverse problems in microscopy.

Topic

Most of the significant advances in imaging in recent years rely on the use of numerical computation. Examples include super-resolution in microscopy, compressed sensing in magnetic resonance imaging or radar interferometry for earth observation. These techniques share the prior need to design a mathematical model of the observation system. The acquired data is often described by an equation of the form $y = Ax$ where A is a linear operator that describes the physics of the acquisition. Recovering x then requires inverting the operator. Although these imaging modalities have allowed solving problems beyond reach in the past, their expansion is often severely limited by an important problem: it is impossible to accurately control imaging conditions, resulting in errors on the operator A that make the numerical methods unreliable.

The main objective of this project is to develop theories and algorithms to overcome these difficulties and apply them to microscopy and astronomical imaging.

This problem is one of the most important current challenges in the field of inverse problems. Depending on the candidate interest, it will be possible to concentrate the efforts on rather theoretical issues or practical issues.

Theory

The supervisors have begun investigating various open questions in this field. Two of them will be explored.

Recently [4], Kech and Krahmer provided necessary and sufficient conditions guaranteeing the ability to recover the operator A and the signal x from the measurements y. This implies studying the global injectivity of a bilinear mapping. The setting is however too restricted for practical applications and we plan to study the local injectivity of the mapping.

A second trail is about the design of optimization algorithms to recover A and x. A standard approach [1] consists in lifting the problem on the space of rank-1 matrices. A traditional approach to solve the resulting problem consists in relaxing the rank 1 constraint by penalizing the nuclear norm. We recently observed that simpler projected gradient descents [3] tend to produce better results. Following [5], we plan to explore the use of continuous relaxations, which discard spurious minimizer and simplify the minimization.

Applications

The supervisors of this project have begun a collaboration with the Centre de Biologie Intégrative in Toulouse (CBI), to improve the resolution of microscopes of the imaging facility. In particular, they are interested in the field of Single Molecule Localization Microscopy (SMLM) [2] and Structured Illumination microscopy [6].

The dominant approach to solve the associated inverse problems consists in assuming that the observation operator has been perfectly calibrated before the acquisition. This hypothesis is seldom satisfied and we expect to significantly improve the reconstructions by using a finer modelling of the system and an identification of the operator.

In addition, real improvements of the microscopes might result in new collaborations with the biologists to answer practical problems.

Practical aspects

We are looking for a highly motivated student, willing to continue with a PhD thesis, with a background in electrical engineering (signal/image processing, harmonic analysis) or mathematics (optimization, probability an statistics, geometry). Strong abilities in mathematics or computer sciences will be appreciated. A taste for optics or biological problems is a plus since applied projects might emerge.

If the candidate is successful, this internship will be pursued by a PhD. Various funding sources will be considered (ADI, FRM, Labex CIMI, doctoral school, ANR, ANITI).

This internship will take place either at IRIT or at INSA (in Toulouse, France), depending on the candidate's interest. It will be co-supervised by Emmanuel Soubies (CR Toulouse) and Pierre Weiss (CR Toulouse). In addition, the candidate will have access to the CBI (Centre de Biologie Intégrative) with Thomas Mangeat for an access to the microscopy resources and biological problems.

Do not hesitate to contact us for more information.

Bibliography

[1] Ali Ahmed, Benjamin Recht, and Justin Romberg. Blind deconvolution using convex programming. *IEEE Transactions on Information Theory*, 60(3):1711–1732, 2013.

[2] Valentin Debarnot, Paul Escande, Thomas Mangeat, and Pierre Weiss. Learning low-dimensional models of microscopes. *IEEE Transactions on Computational Imaging*, 2020.

[3] Valentin Debarnot and Weiss Pierre. Blind inverse problems with isolated spikes. *ArXiv*, 2020.

[4] Michael Kech and Felix Krahmer. Optimal injectivity conditions for bilinear inverse problems with applications to identifiability of deconvolution problems. *SIAM Journal on Applied Algebra and Geometry*, 1(1):20–37, 2017.

[5] Emmanuel Soubies, Laure Blanc-Féraud, and Gilles Aubert. A continuous exact \ell_0 penalty (cel0) for least squares regularized problem. *SIAM Journal on Imaging Sciences*, 8(3):1607–1639, 2015.

[6] Emmanuel Soubies and Michael Unser. Computational super-sectioning for single-slice structured-illumination microscopy. *IEEE Transactions on Computational Imaging*, 5(2):240–250, 2018.