Shocks

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in

Laboratory and Astrophysical Plasmas

Plan of This Lecture

- Motivation
  - What is a shock?
  - Examples
  - Effects of shocks
- Fluid models of shocks
- Collisionless models of shocks
- Applications

What is a shock?

A shock is a thin transition layer in which bulk flow is dissipated as heat, increasing the entropy of the system [a].

- If Coulomb/molecular dissipation $\rightarrow$ collisional
- If dissipation is by wave/particle scattering or interaction with coherent electromagnetic fields $\rightarrow$ collisionless
- Properties
  - Upstream flow is super(sonic, fast magnetosonic, Alfvenic, slow magnetosonic).
  - Downstream flow is sub(sonic, fast magnetosonic, Alfvenic, slow magnetosonic).
  - The downstream gas is compressed.

[a] Not all this holds for *intermediate* shocks

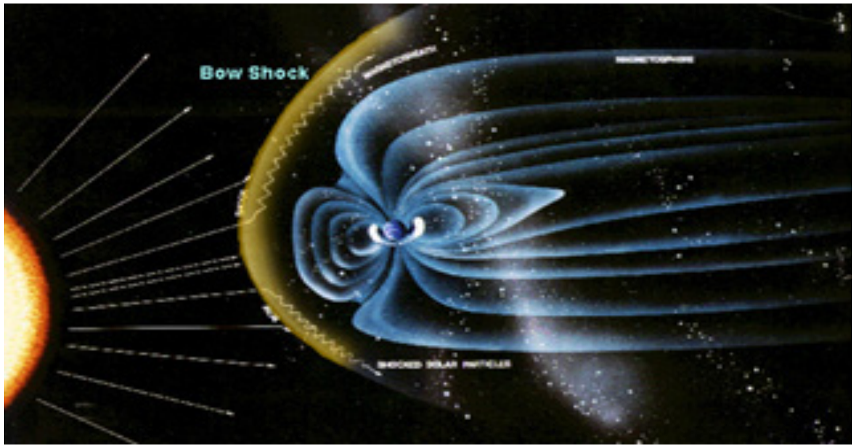
How do shocks form?

- Fast flow around an obstacle
- Nonlinear steepening of waves
- Accretion
- Explosions

Bowshocks

Image of LL Ori from the NASA/Hubble Heritage Team.

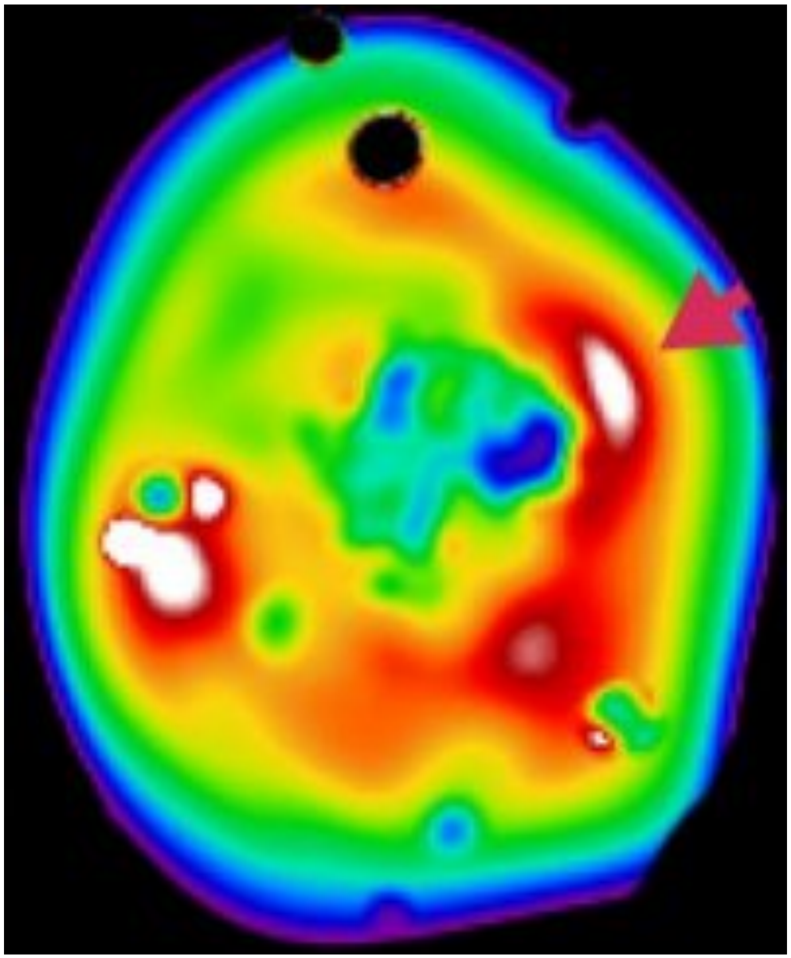
Schematic rendering of the terrestrial magnetosphere & bowshock.



Astronomical Image *vs* Artist's Conception

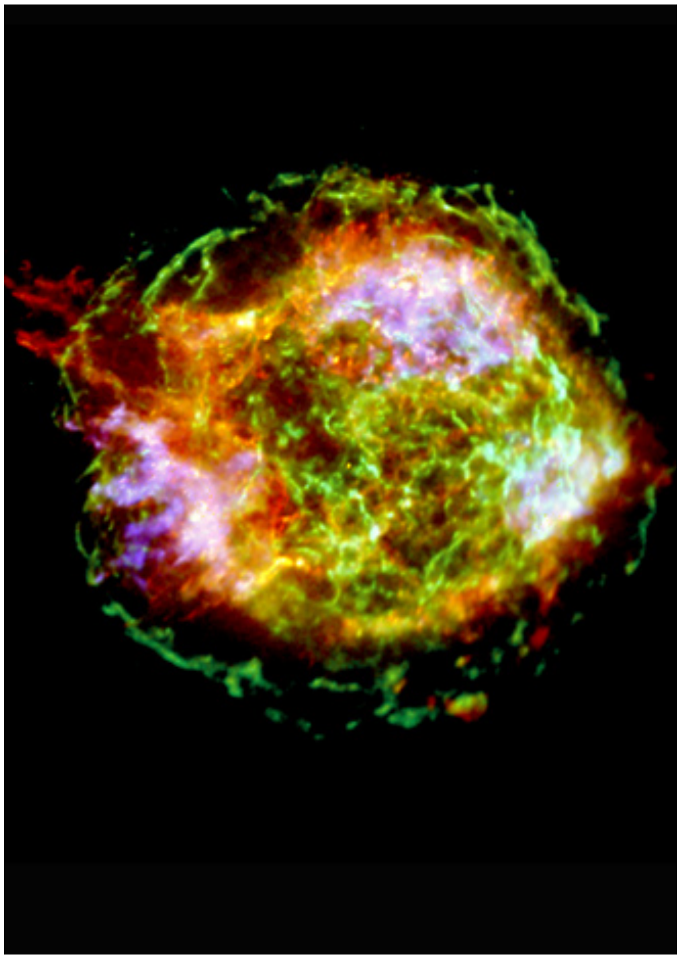
- Shock, except for a few in the lab & heliosphere, are diagnosed radiatively (measure n, T_e, B, thermal & nonthermal particles).
- Radiation can come from collisional or collisionless processes – important part of collisionless shock theory.
- Can affect shock structure through cooling & preheating.

Accretion Shocks

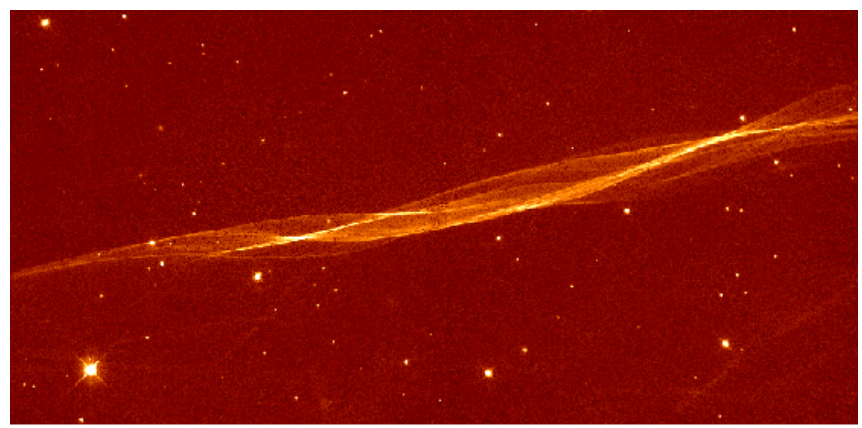


- X-ray image of shock heated gas in a galaxy cluster. The shock is driven by the merger of 2 clusters. The arrow shows a region of high entropy. Courtesy ESA.

Supernova Shocks



Left: X-ray image of the young supernova remnant Cas A (Chandra X-ray Observatory). Below: Small portion of the old supernova remnant Cygnus A (Hubble Space Telescope).



Steady, Planar Hydrodynamic Shocks

In the shock frame, the equations of continuity, momentum, and energy for a viscous fluid are

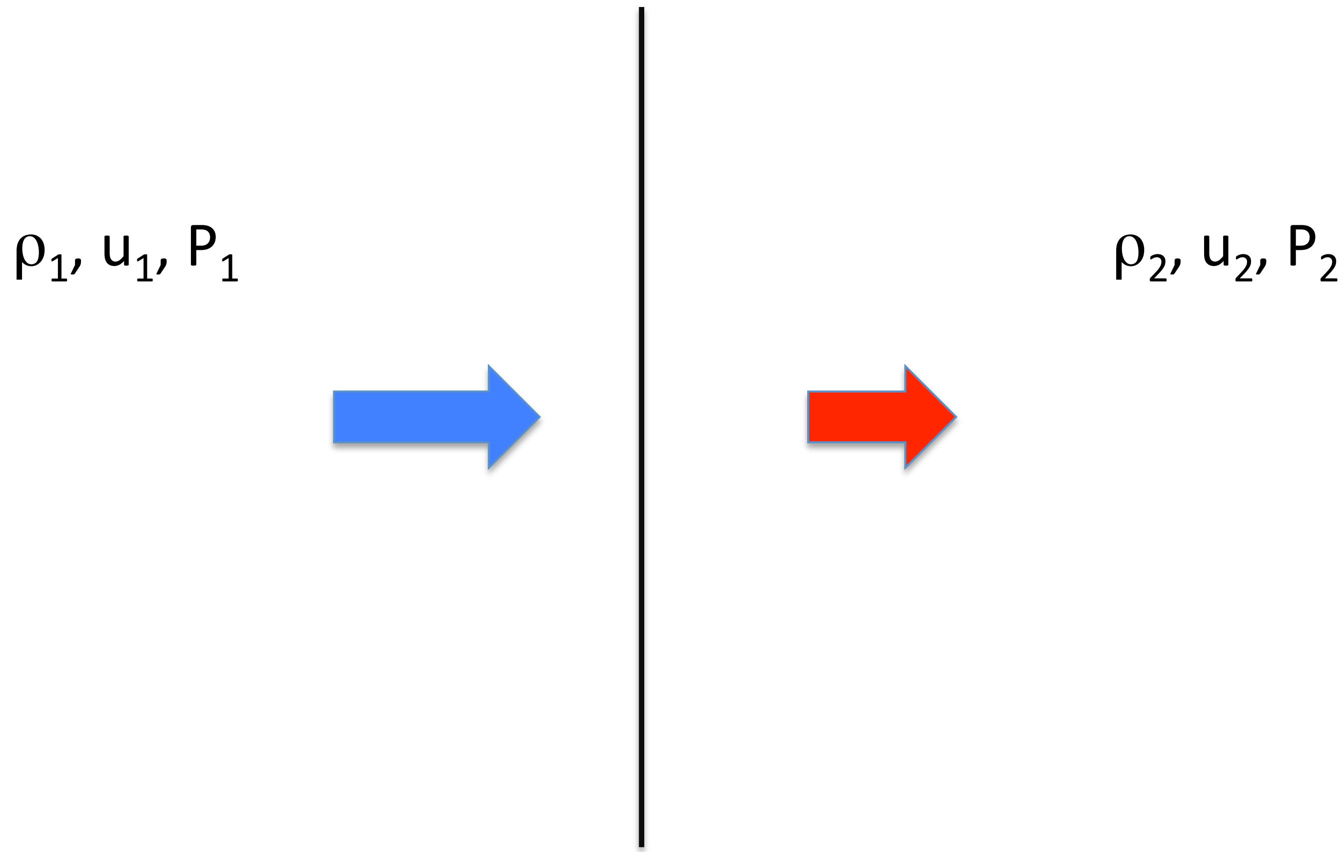
$$\frac{\partial}{\partial x}(\rho u) = 0,$$

$$\frac{\partial}{\partial x}\left(\rho u^2 + P + \pi\right) = 0,$$

$$\frac{\partial}{\partial x}\left[\left(\frac{1}{2}\rho u^2 + \rho U + P + \pi\right) u\right] = 0,$$

where $\pi \equiv -(4/3)\rho\nu\partial u/\partial x$. Integrate these equations over the shock layer & assume the upstream (1) & downstream (2) regions are uniform.

Basic Setup in the Shock Frame



Jump Conditions

Integration over the shock layer gives the Rankine-Hugoniot relations between upstream (unshocked) & downstream (shocked) quantities, based on conservation laws for mass, momentum, & energy:

$$\rho_1 u_1 = \rho_2 u_2,$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2,$$

$$\left(\frac{1}{2}\rho_1 u_1^2 + P_1 + \rho_1 U_1\right) u_1 = \left(\frac{1}{2}\rho_2 u_2^2 + P_2 + \rho_2 U_2\right) u_2.$$

Solutions of the Jump Conditions

Use conservation laws plus eqn. of state to solve for downstream (2) quantities in terms of upstream (1) quantities. This is just an algebra problem. When are the solutions physically realistic? If $\partial^2 V/\partial P^2 > 0$ then:

- Increase of entropy across the shock ($s_2 > s_1$) implies:
  - the upstream flow is supersonic ($M_1 \equiv u_1/c_{s1} > 1$)
  - the downstream flow is subsonic
  - $P_2 > P_1$

Limiting Forms in a Perfect Gas

Assume $P = (\gamma - 1)\rho U$. For $M_1 \gg 1$,

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} \rightarrow \frac{\gamma + 1}{\gamma - 1},$$

$$\frac{k_B T_2}{\mu} \rightarrow \frac{2(\gamma - 1)u_1^2}{(\gamma + 1)^2}.$$

For $M_1 - 1 \ll 1$,

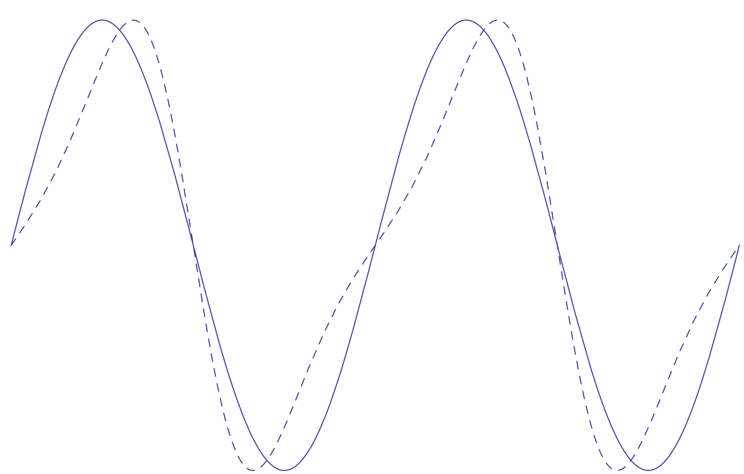
$$s_2 - s_1 = \frac{2}{3}\frac{k_B}{\mu}\frac{\gamma}{(\gamma + 1)^2}\left(M_1^2 - 1\right)^3.$$

Formation of a Weak Shock

An acoustic wave with $\hat{x}\delta v \sin k(x - c_s t)$ exemplifies a *simple wave* with v, ρ, etc. constant on $x(t)$ where

$$\frac{dx}{dt} = c_s + v = c_{s0} + \delta c_s + \delta v = c_{s0} + \frac{\gamma + 1}{2}\delta v.$$

- Heuristic argument: $\Delta(\delta v) \sim (\gamma + 1)\delta v \to$ different parts of the profile to converge at time $t_s \sim 2\pi/((\gamma + 1)k\delta v)$.
- Dissipation intervenes, forming a weak shock.

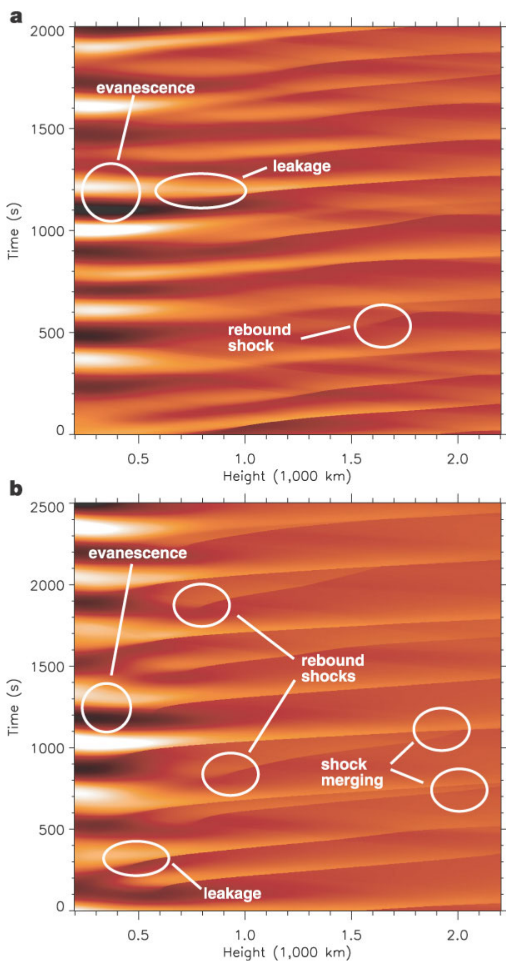


Consequences of Wave Steepening

- Very general feature of wave propagation.
- Waves propagating down a density gradient especially prone to breaking; conserved wave energy flux $\delta F_w \sim \rho \delta v^2 c_s \rightarrow \delta v \propto (\rho P)^{-1/4}$.
- Heat stellar chromospheres & coronae: recall

$$\Delta s \propto (M_1^2 - 1)^3 \propto \delta v^3.$$

Simulation of Chromospheric Shocks



- Upward propagation of oscillations in the solar photosphere drives weak shocks.
- Theory for chromospheric "spicules".

De Pontieu et al. 2004, *Nature* 430, 536

Evolving Shock: Sedov-Taylor Blast Wave

Impulsive energy injection E in a medium of density ρ drives a spherical shock with radius $R_s(t)$ & speed $V_s(t)$.

- Self similar solution once shock has swept up its own mass.
- Energy conservation $\rightarrow \rho V_s^2 R_s^3 \sim$ constant.

$$R_s = \left(\frac{2.02E}{\rho}\right)^{1/5} t^{2/5}$$

$$V_s = \frac{dR_s}{dt} = \frac{2}{5}\left(\frac{2.02E}{\rho}\right)^{1/5} t^{-3/5}.$$

- Model for interstellar shock driven by a supernova explosion.

Vorticity Production in Shocks

Evolution equation for vorticity $\omega \equiv \nabla \times \mathbf{v}$:

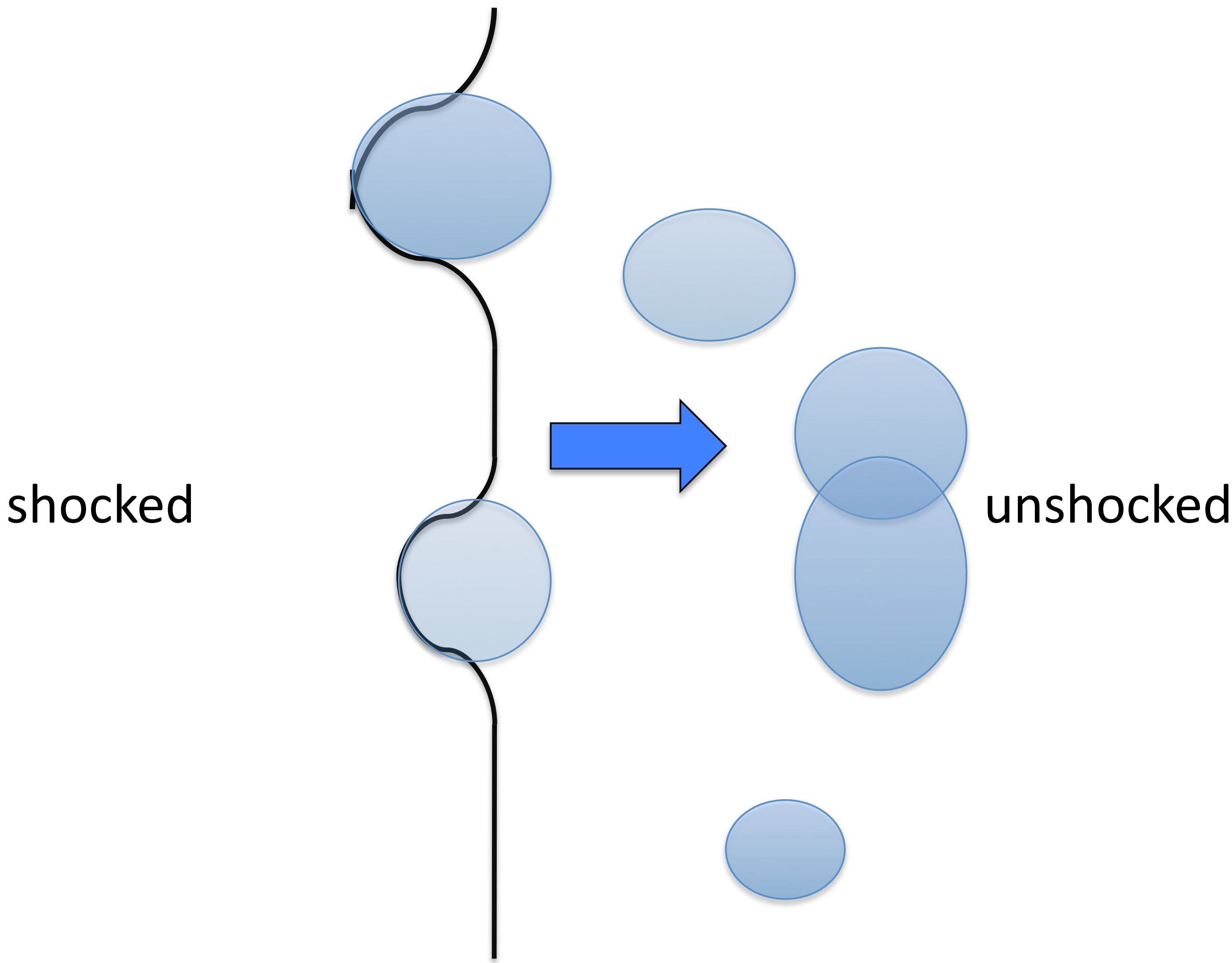
$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{\nabla P}{\rho}$$

Take curl:

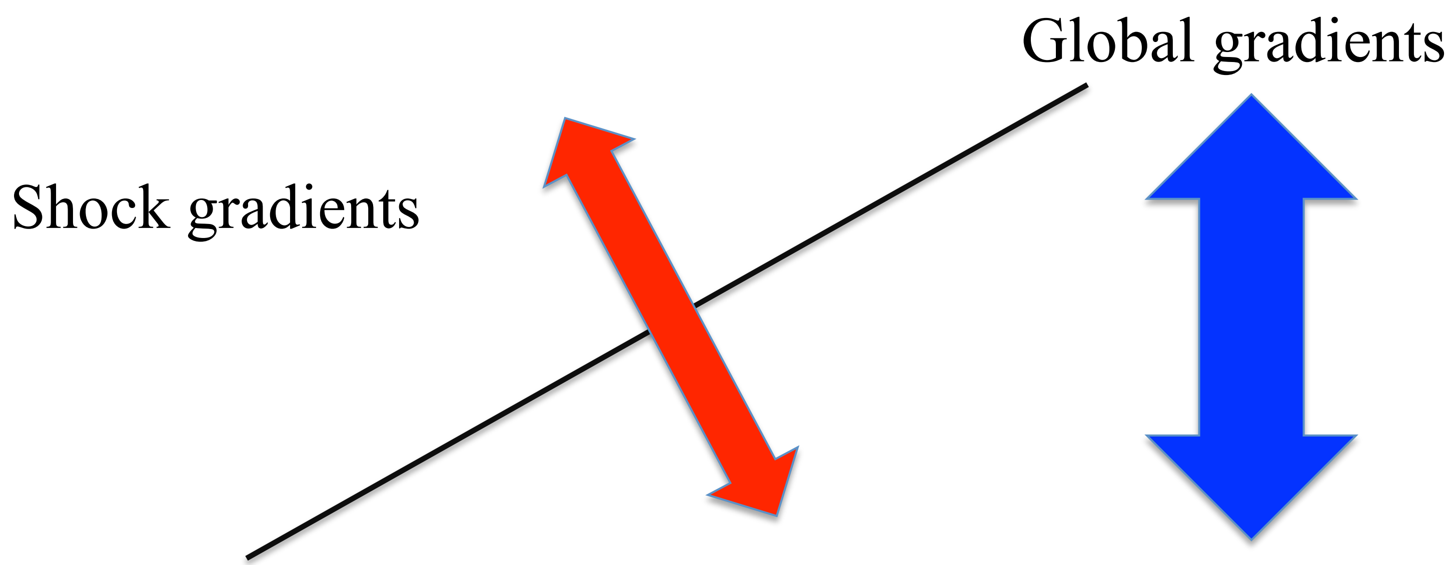
$$\frac{\partial \omega}{\partial t} + \nabla \times (\omega \times \mathbf{v}) = \frac{\nabla \rho \times \nabla P}{\rho^2}.$$

The LHS represents “freezing in" of vorticity; the RHS represents vorticity generation by baroclinicity.

Shock Entering a Clumpy Medium

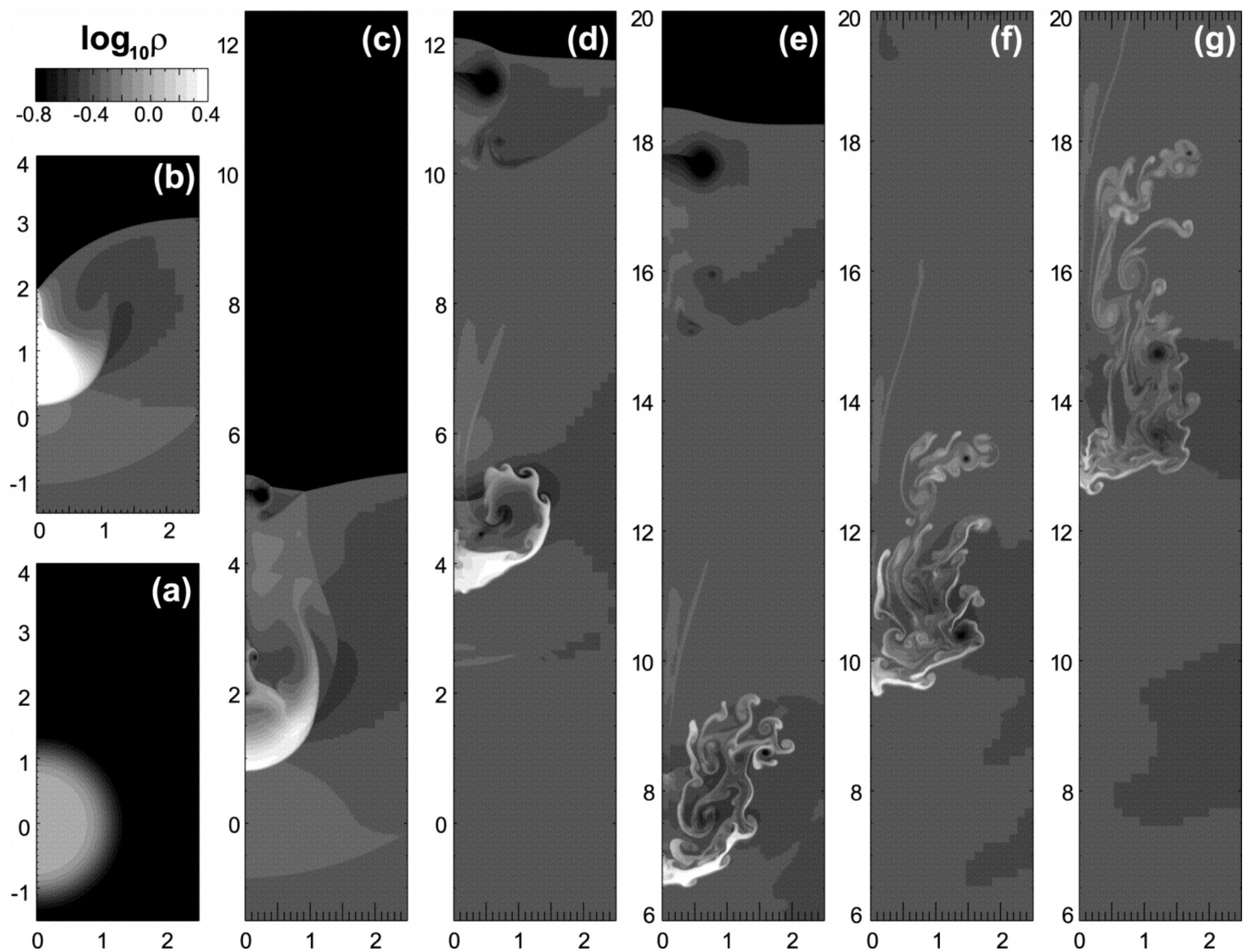


Close Up



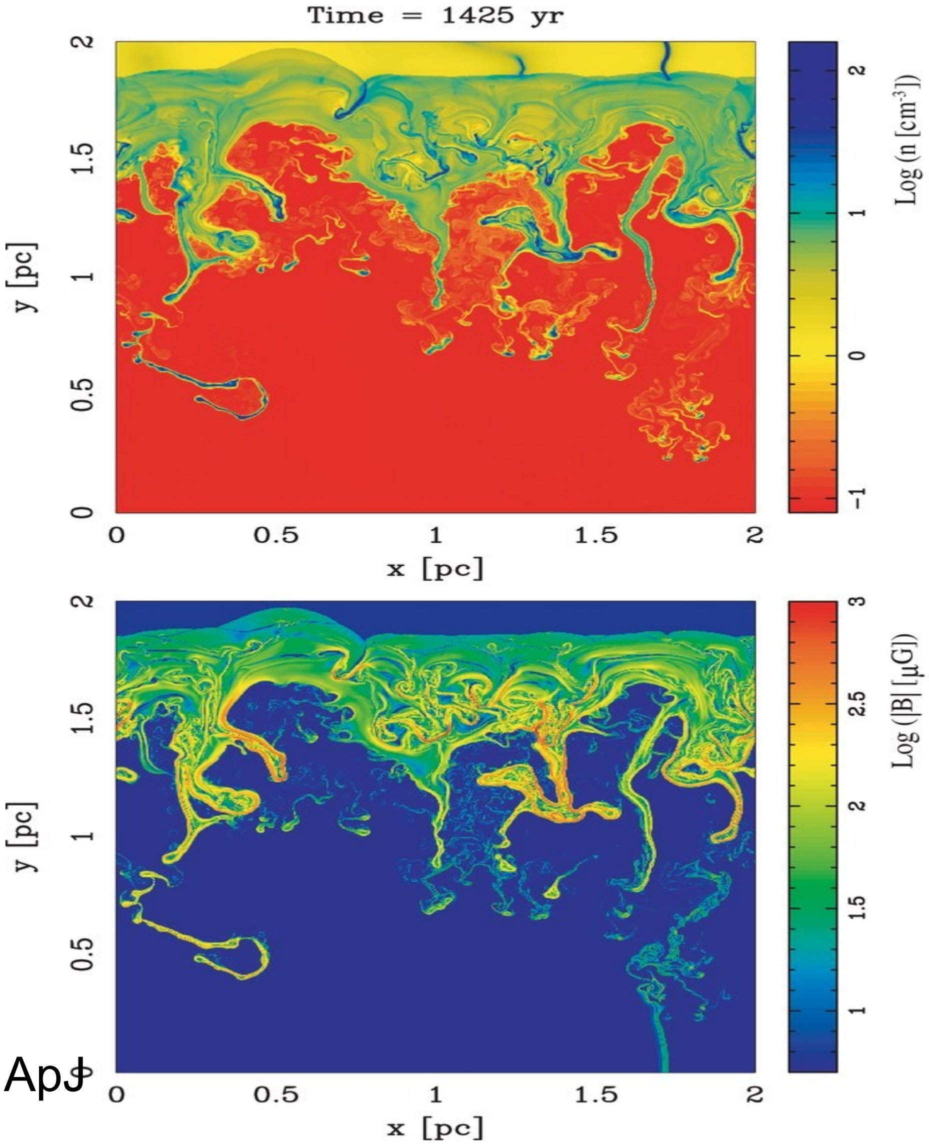
(shock gradients) X (global gradients) = baroclinicity

Interstellar Shock-Cloud Interaction



Nakamura et al. 2006 ApJS 164, 477

Magnetic Field Amplification by Clump-Induced Turbulence



Inoue et al (2009) ApJ 695, 825

MHD Shocks

- Expect identification with wave modes:
  - Fast magnetosonic
  - Slow magnetosonic
  - Intermediate/Alfvenic
- Expect density compression and entropy production.
- Expect the unexpected.

Electromagnetic Jump Conditions

From Maxwell's equations in the shock frame

$$\nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times (\mathbf{v} \times \mathbf{B}) = 0.$$

Integrate across the shock front:

$$(\mathbf{B}_2 - \mathbf{B}_1) \cdot \mathbf{n} = 0,$$

$$(\mathbf{u}_2 \times \mathbf{B}_2 - \mathbf{u}_1 \times \mathbf{B}_1) \times \mathbf{n} = 0,$$

where $\mathbf{n}$ is the shock normal.

Modified Fluid Equations

The continuity, momentum, and energy equations written in the frame of a steady shock are

$$\nabla \cdot (\rho \mathbf{u}) = 0,$$

$$\nabla \cdot \left(\rho \mathbf{u}\mathbf{u} + \mathbf{I} \left(P + \frac{B^2}{8\pi} \right) - \frac{\mathbf{B}\mathbf{B}}{4\pi} \right) = 0$$

$$\nabla \cdot \left[\left(\frac{1}{2} \rho u^2 + P + \rho U \right) \mathbf{u} + \frac{B^2 \mathbf{u} - \mathbf{B} \cdot \mathbf{u}\mathbf{B}}{4\pi} \right] = 0.$$

Modified Fluid Jump Conditions

Integrating across the shock, the following quantities are continuous:

$$\rho \mathbf{u} \cdot \mathbf{n},$$

$$\rho \mathbf{u}(\mathbf{u} \cdot \mathbf{n}) + \left(P + \frac{B^2}{8\pi}\right) \mathbf{I} \cdot \mathbf{n} - \frac{\mathbf{B}(\mathbf{B} \cdot \mathbf{n})}{4\pi},$$

$$\left(\frac{1}{2}\rho u^2 + P + \rho U\right) \mathbf{u} \cdot \mathbf{n} + \frac{\mathbf{B}^2(\mathbf{u} \cdot \mathbf{n}) - (\mathbf{B} \cdot \mathbf{u})(\mathbf{B} \cdot \mathbf{n})}{4\pi}.$$

Perpendicular Shock

When $\mathbf{B} \cdot \mathbf{n} \equiv 0$

$$B_2 u_2 = B_1 u_1,$$

$$\rho_2 u_2 = \rho_1 u_1,$$

$$\rho_2 u_2^2 + P_2 + \frac{B_2^2}{8\pi} = \rho_1 u_1^2 + P_1 + \frac{B_1^2}{8\pi},$$

$$\left(\frac{1}{2}\rho_2 u_2^2 + \frac{\gamma P_2}{\gamma - 1} + \frac{B_2^2}{4\pi}\right) u_2 = \left(\frac{1}{2}\rho_1 u_1^2 + \frac{\gamma P_1}{\gamma - 1} + \frac{B_1^2}{4\pi}\right) u_1.$$

Properties of Perpendicular Shocks

- Must be supermagnetosonic; $u_1 > c_{ms1} \equiv (c_{s1}^2 + v_{A1}^2)^{1/2}$.
- Magnetic field reduces compression, but in the high Mach number limit

$$\frac{\rho_2}{\rho_1} = \frac{B_2}{B_1} \to \frac{\gamma+1}{\gamma-1},$$

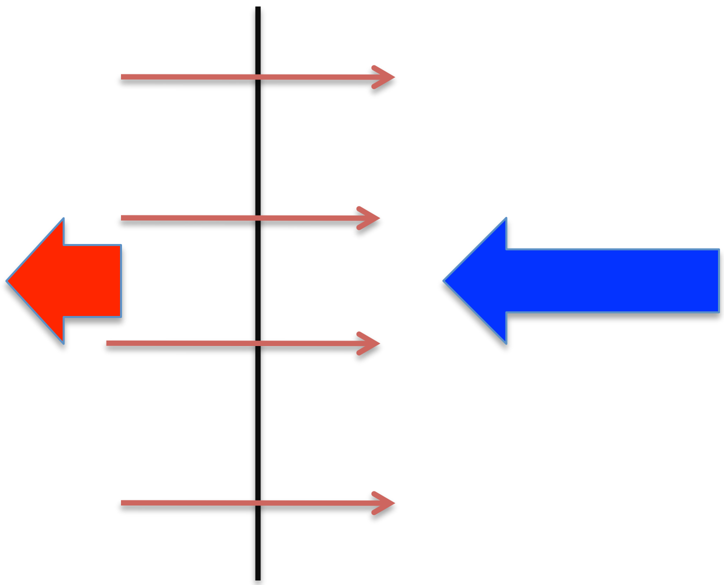
$$\frac{k_B T_2}{\mu} \to \frac{2(\gamma-1)u_1^2}{(\gamma+1)^2},$$

independent of B.

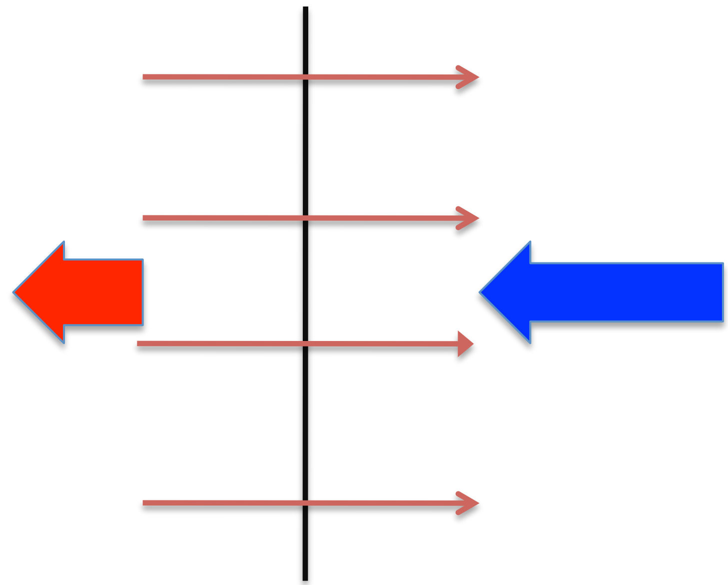
Fast, Slow, Intermediate Shocks

- Fast shocks
  - Are super-Alfvenic
  - Compress ρ and B
- Slow shocks
  - Are sub-Alfvenic
  - Compress ρ but weaken B
- Intermediate shocks transition from superAlfvenic to subAlfvenic flow.

Admissible Parallel Shocks



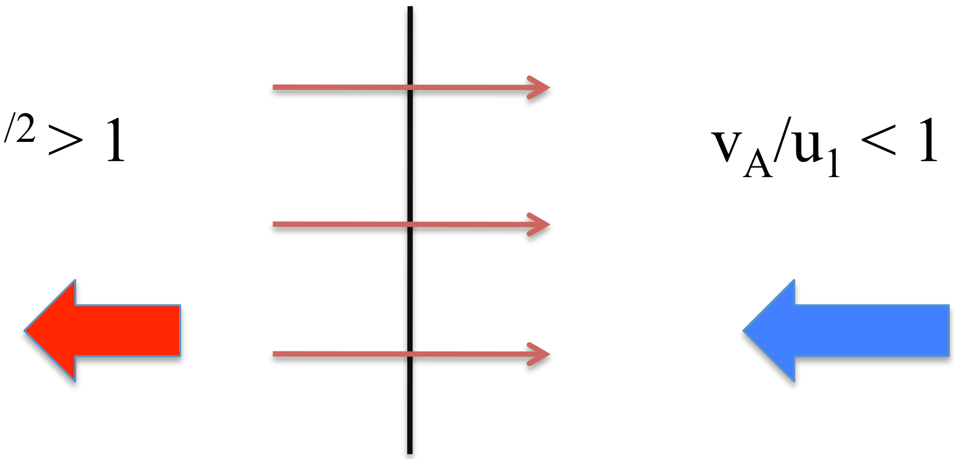
$v_A/u < 1$ on both sides. All Alfven waves emitted downstream are swept downstream.



$v_A/u > 1$ on both sides. Alfven waves emitted downstream can propagate upstream.

Inadmissible Parallel Shock

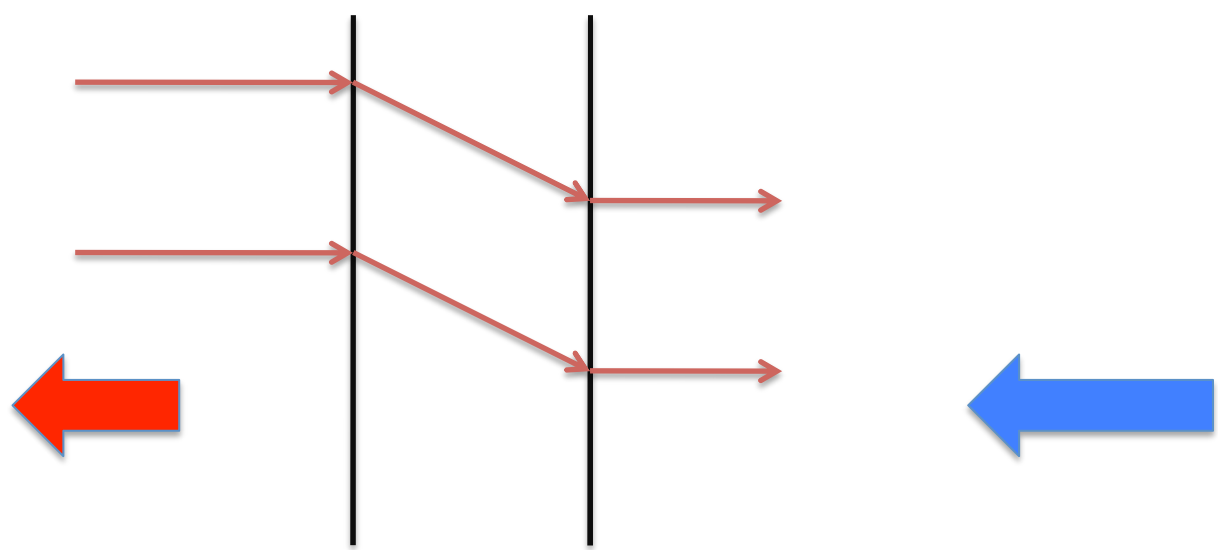
$v_A/u_2 = v_A/u_1(\rho_2/\rho_1)^{1/2} > 1$



$v_A/u_1 < 1$

Rightward propagating Alfven waves emitted downstream would accumulate at the shock front, enforcing a magnetic perturbation.

Evolved State of Inadmissible Shock



Transverse field in the intermediate region reduces the compression, enforcing vA/u < 1 in both the upstream & downstream regions.

The Structure of Shocks

In a neutral, ideal gas the natural lengthscale is the mean free path. Viscosity & heat conduction operate on this scale & set the shock thickness.

- Radiative shocks: rapid cooling enforces $T_2 \sim T_1$.
- Electrons *vs* ions: subshocks & precursors.
- Shocks in weakly ionized gas: magnetic/plasma precursors.

Radiative Shocks

Common in astrophysics.

- When the cooling length $u_2 \tau_{cool}$ is small, replaces the energy eqn. by $T_2 = T_1$.
- Quasiparallel fast shock (magnetic pressure unimportant): $\rho_2/\rho_1 \sim M_1^2$.
- Quasiperpendicular fast shock (magnetic pressure is important): $\rho_2/\rho_1 \sim \sqrt{2} M_{A1}$ for $\beta_1 \ll 1$.
- Unstable to Nonlinear Thin Shell Instability (Vishniac 1983).

Electrons vs. Ions: B=0

- Electric field keeps species together; $n_e \sim n_i$ on scales $> \lambda_D$.
- Ions carry the momentum; electrons are subsonic for $u_1 < v_e$.
- Electron & ion diffusivities D_e, D_i differ according to $D_e/D_i \sim (m_i/m_e)^{1/2}$

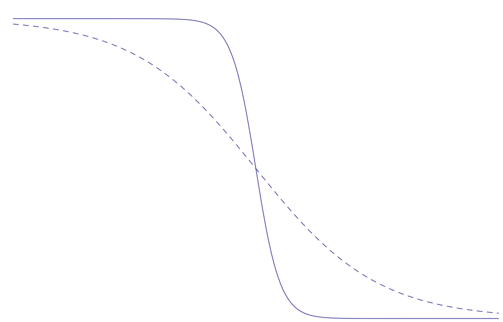


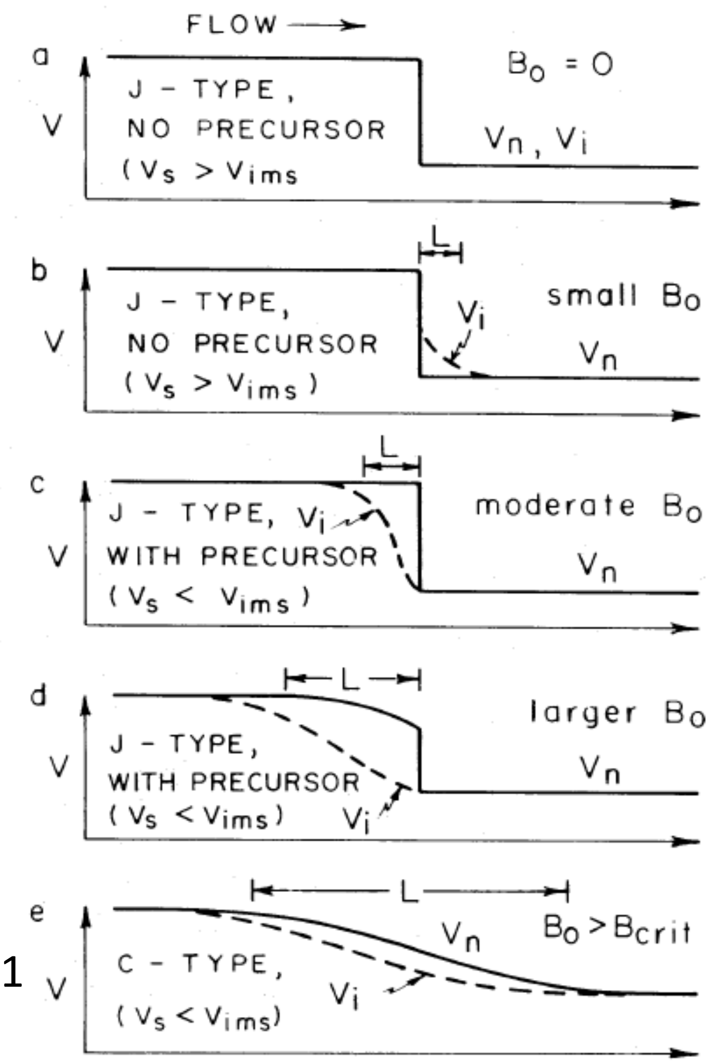
Figure 1: Sketch of T_i (solid) & T_e (dashed)

Shocks in Partially Ionized Gas

Consider perpendicular fast shocks in a weakly ionized gas e.g. $\rho_i/\rho_n \sim 10^{-6}$ in molecular clouds.

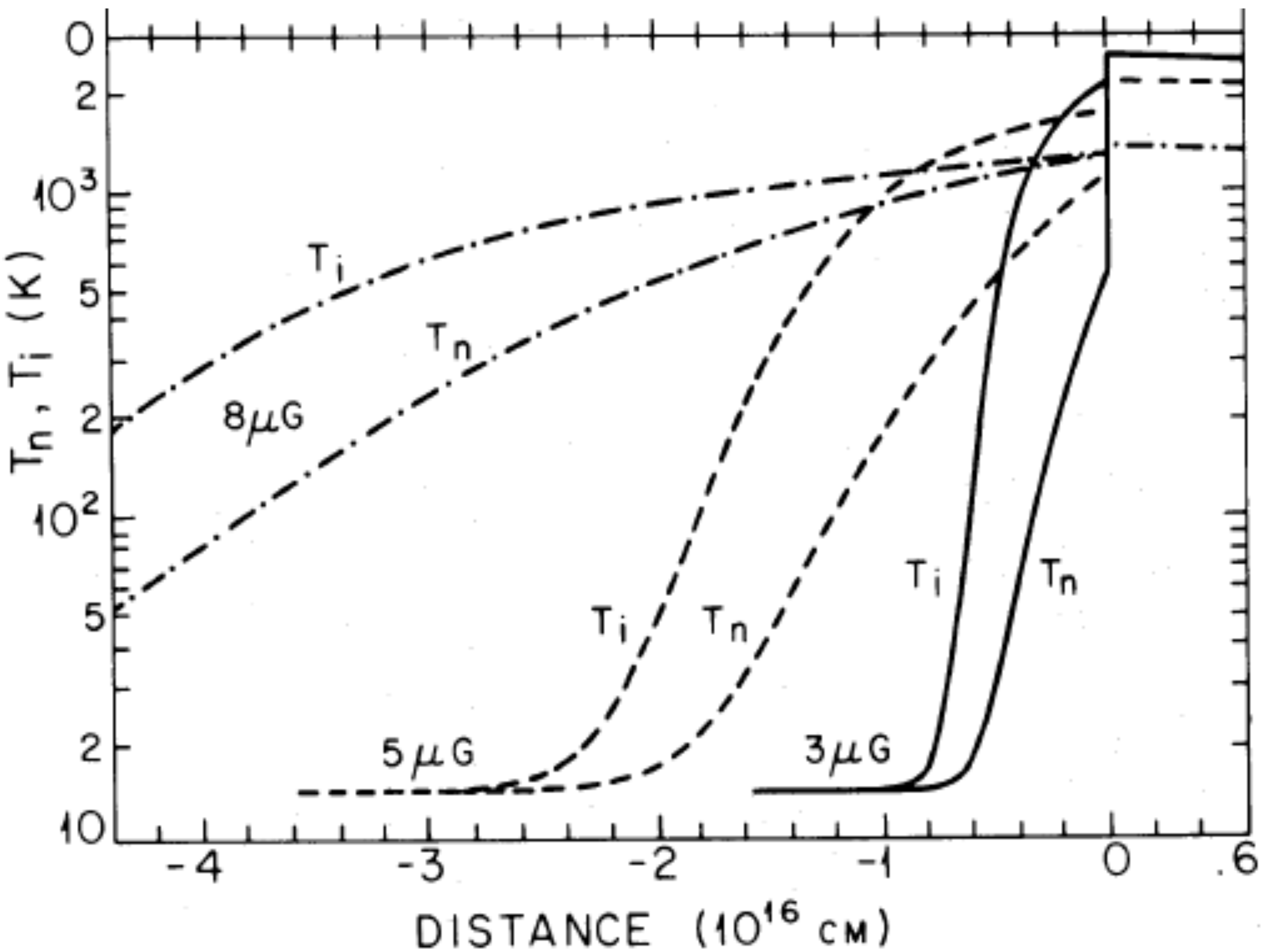
- If shock is supermagnetosonic in both plasma & neutrals, have jumps in both species.
- If shock is supermagnetosonic with respect to neutrals but submagnetosonic in the plasma, can have magnetic precursors
  - Plasma transition is smooth, neutrals shock.
  - Significant frictional heating in precursor
- If B is large enough, frictional heating and deceleration of the neutrals effects smooth transitions in both plasma and neutrals.

Fast Shocks in Weakly Ionized Gases



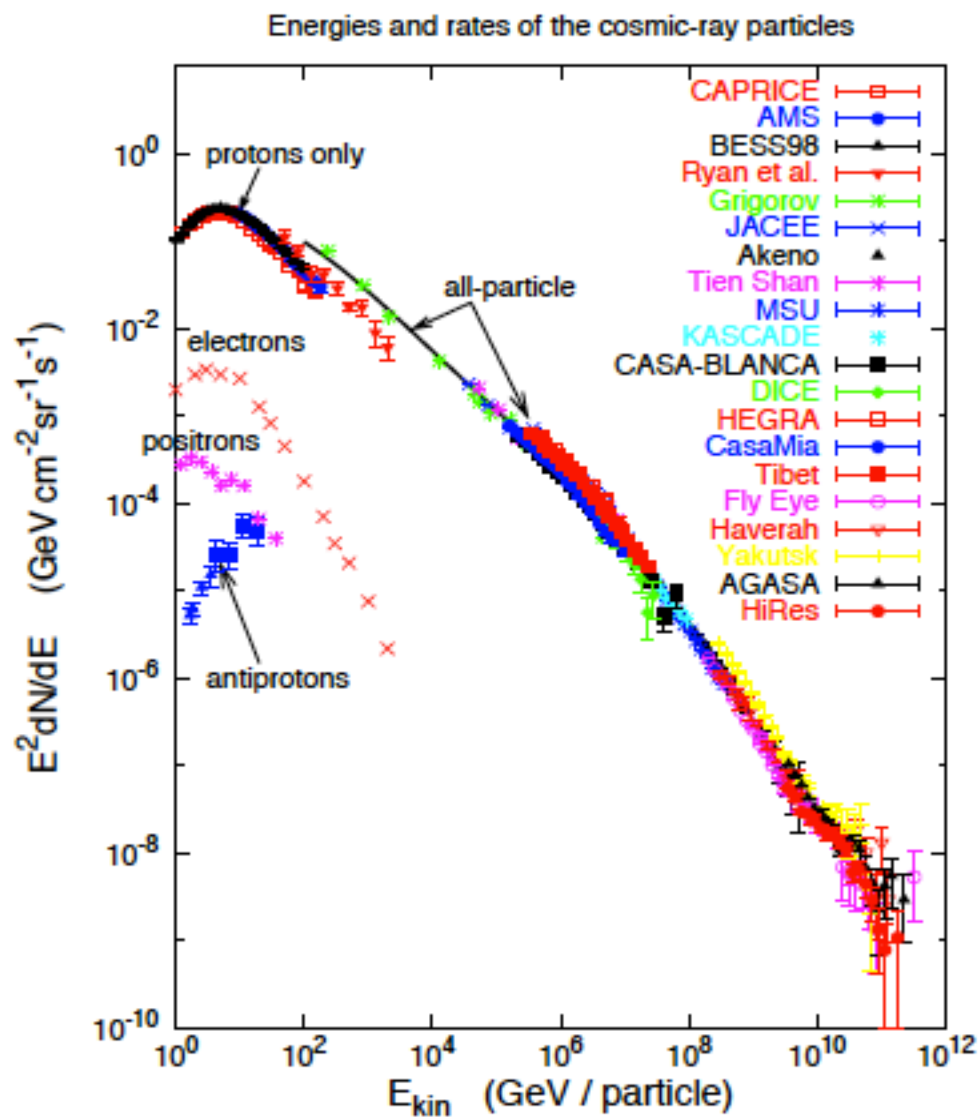
Draine 1980, ApJ 241, 1021

Temperatures in Shocks with Precursors



Draine 1980 ApJ 241, 1021

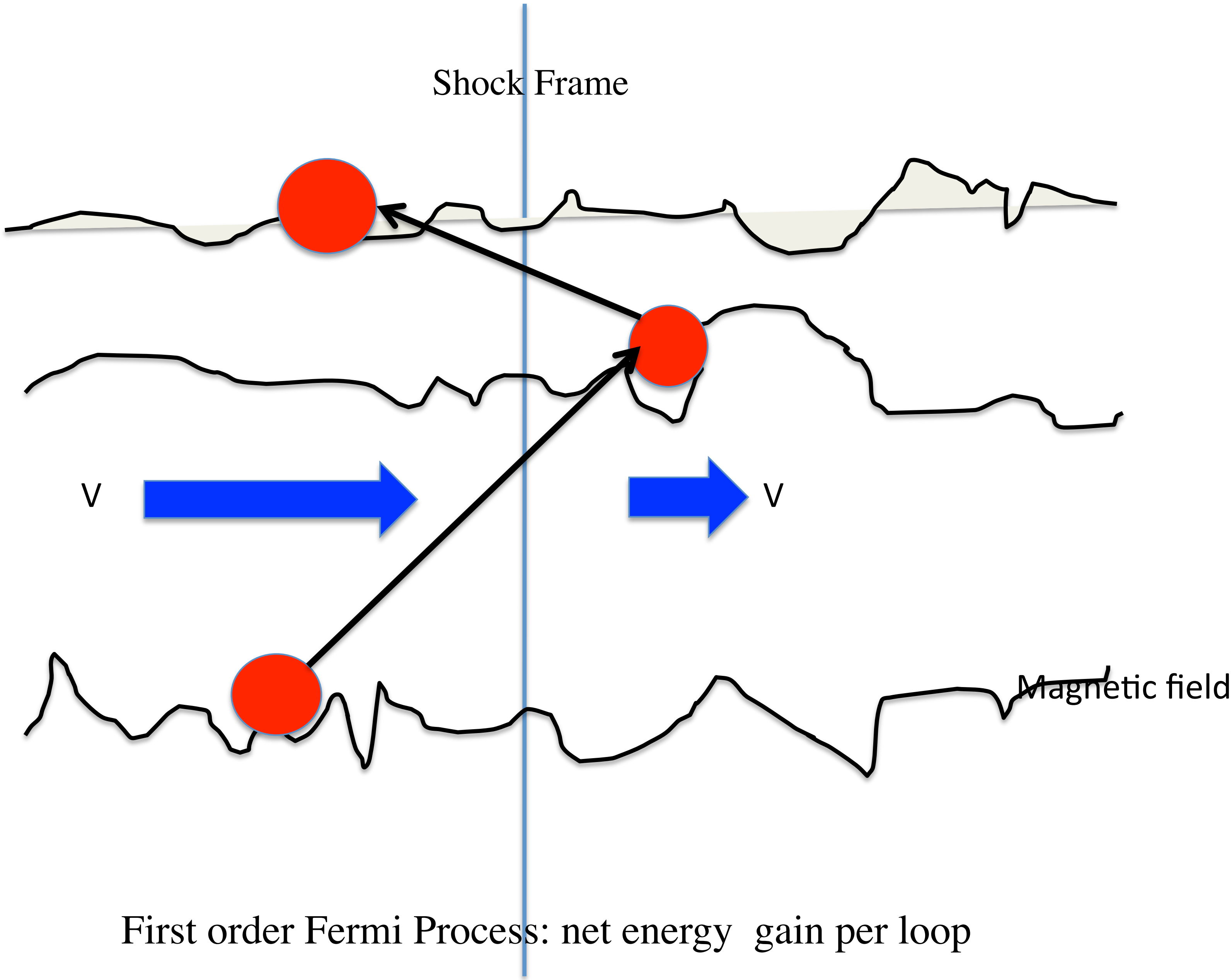
The Cosmic Ray Spectrum



Courtesy T. Gaisser

Cosmic Ray Acceleration at Shocks

- A supernova origin for cosmic rays first proposed by Baade & Zwicky in 1934.
- About 10% of supernova energy would maintain the Galactic cosmic ray pool.
- Mechanism should accelerate interstellar particles to reproduce observed composition.
- Mechanism should reproduce observed power law spectrum, at least up to the "knee" energy.



First order Fermi Process: net energy gain per loop

Basic Test Particle Theory

Proposed in 1977 - 1978 by Axford, Leer, Skadron, Krimsky, Bell, Blandford, Ostriker.

- Shock is a discontinuity with compression ratio $R \equiv \rho_2/\rho_1$, and scattering centers frozen in place.
- Cosmic rays scatter at rate ν with diffusion coefficient is $\kappa \sim c^2/\nu$ & remain isotropic.
- Steady state spectrum is power law

$$f(p) \propto p^{-3R/(R-1)}.$$

- For strong shocks with $R = 4$ this is close to observed value below knee.
- Upper limit imposed by shock size & lifetime.

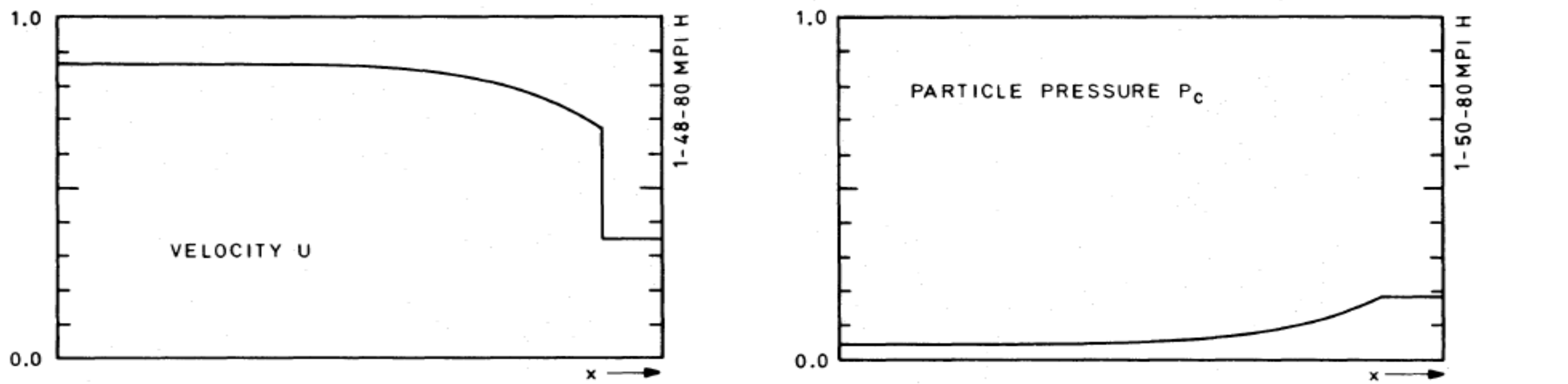
Cosmic Ray Hydrodynamics

- Cosmic rays destabilize gyroresonant Alfven waves with growth rate
$$\Gamma_{cr} \sim \omega_{cp} \frac{n_{cr}}{n} \left(\frac{v_D}{v_A} - 1 \right) .$$
- In a steady state, cosmic ray momentum & energy transferred to the waves is absorbed by the background.
- Cosmic rays exert a parallel force $-\nabla_{\parallel} P_{cr}$ and heat at rate $\mathbf{v}_A \cdot \nabla P_{cr}$.

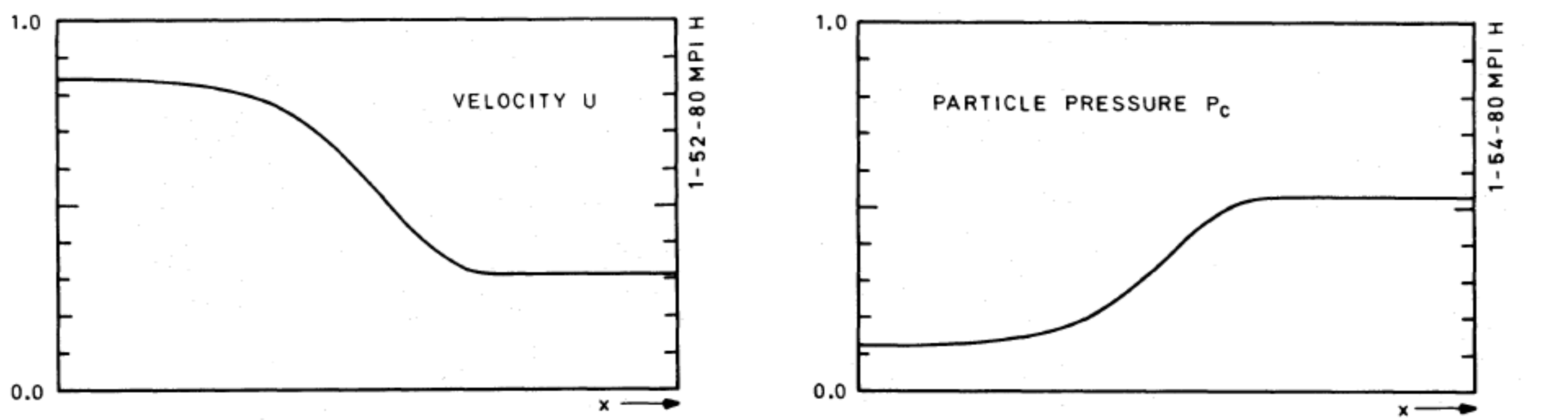
Cosmic ray Modified Shocks

- Upstream cosmic ray pressure gradient decelerates & heats the incoming gas, reducing M_1
- Downstream fluid energy is converted to cosmic ray energy.
- Shock is unstable to acoustic perturbations.

Shocks with Cosmic Ray Precursors



Two cosmic ray modified shocks parameterized by upstream cosmic ray pressure, from Drury & Voelk (1981) ApJ 248, 344.



Instability of CR-Modified Shocks

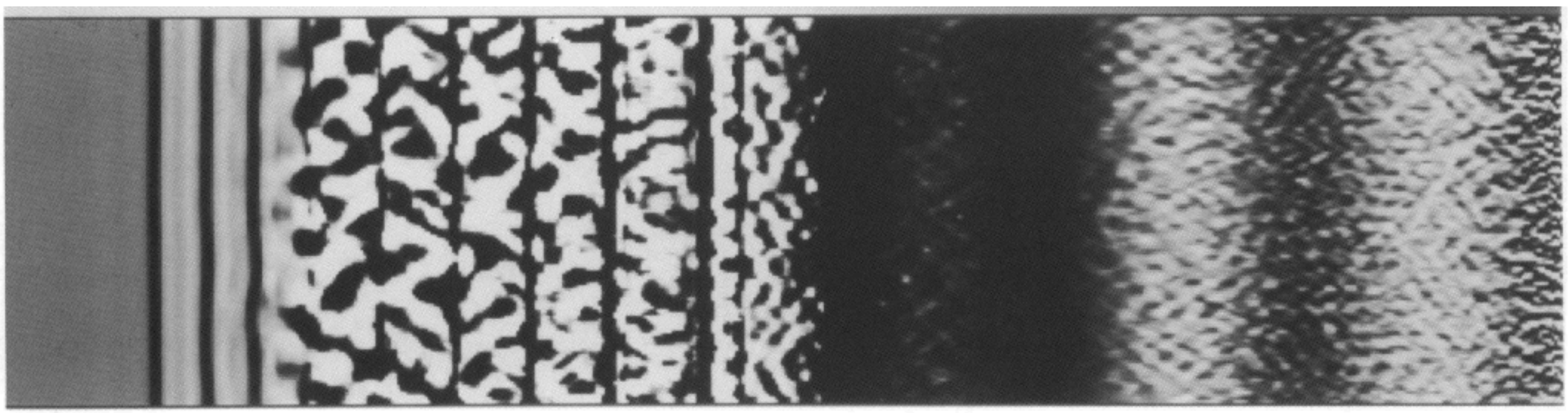
- Simplified treatment of the Drury instability (Drury & Falle 1986)

$$H = \mathbf{v}_A \cdot \nabla P_{cr} \sim \frac{v_A P_{cr}}{L}.$$

$$\frac{\delta H}{H} \sim \frac{\delta v_A}{v_A} - \frac{\delta L}{L} + \frac{\delta P_{cr}}{P_{cr}} = \frac{\delta \rho}{\rho}\left(\frac{1}{2} + 1 + \frac{4}{3}\right).$$

- The compressed phase of an acoustic wave receives extra heat.
- Diffusivity plays a role as well.

Cosmic Ray Driven Acoustic Instability



Compressive velocity field from unstable acoustic waves launched from the right.

Ryu et al. 1993, ApJ 405, 499

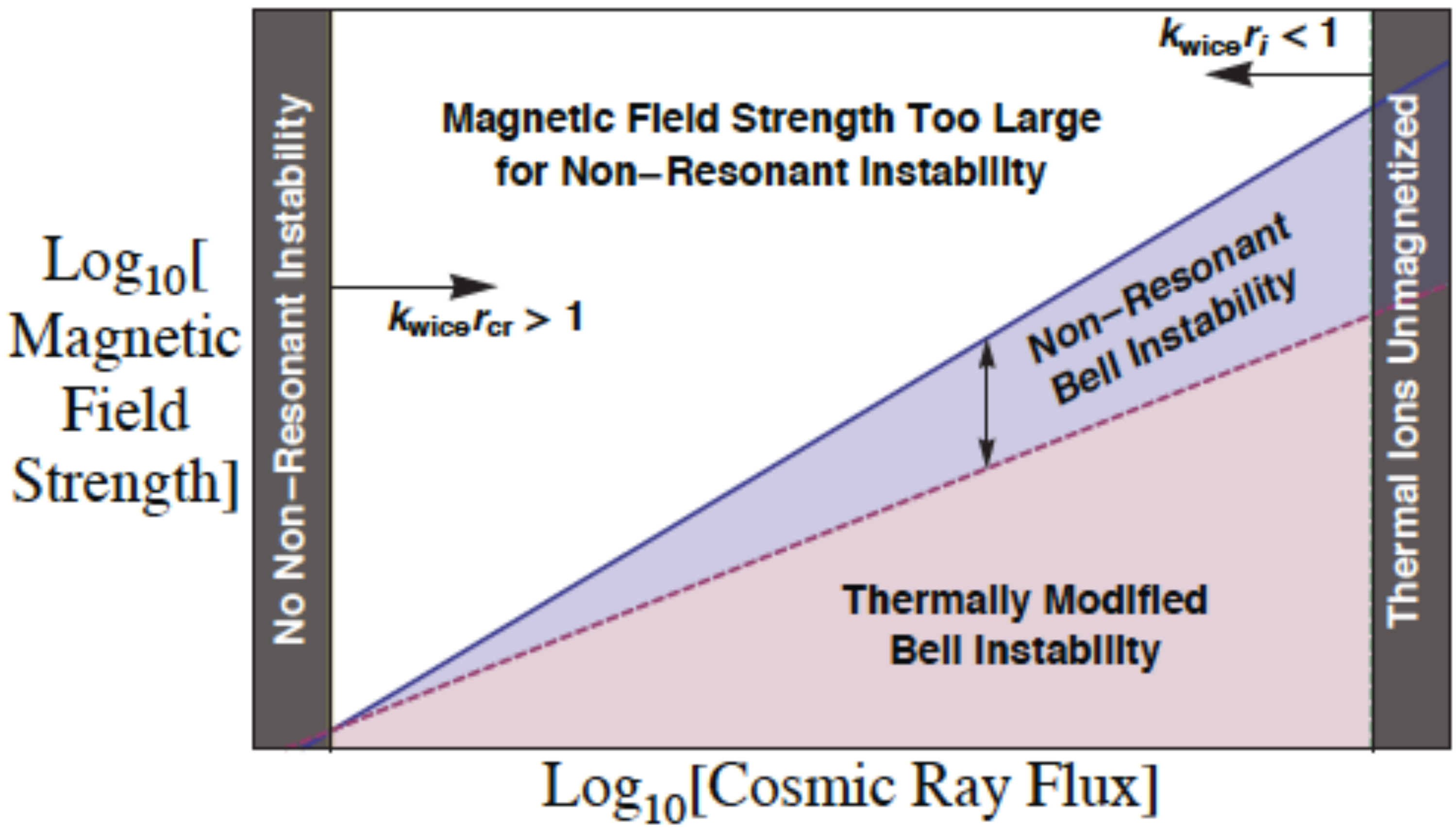
Small Scale Turbulence

- If

$$\frac{U_{cr}}{U_B}\frac{v_D}{c} > 1$$

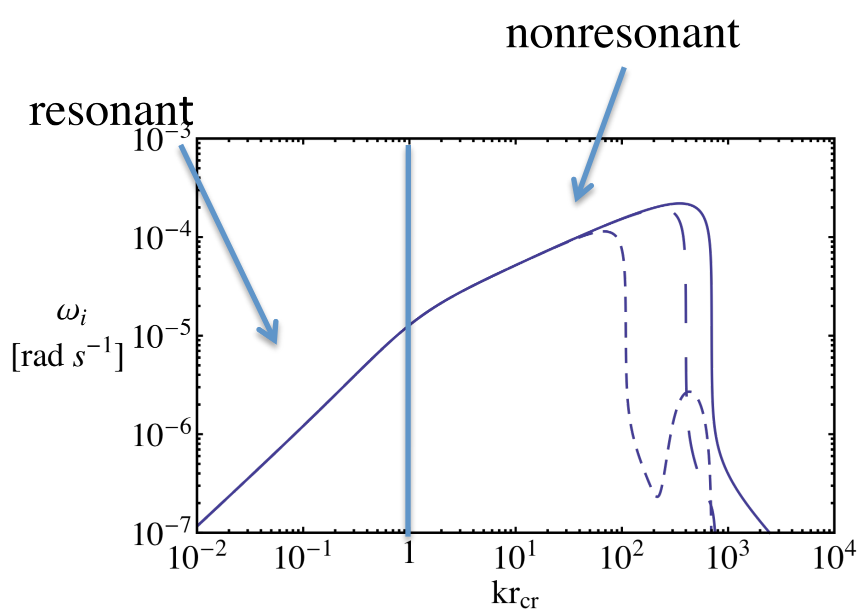
the gyroresonant streaming instability is replaced by a faster electromagnetic instability driven by the electron return current.

  - Filaments & amplifies the magnetic field
  - Observational evidence for strong B in young supernova remnants.



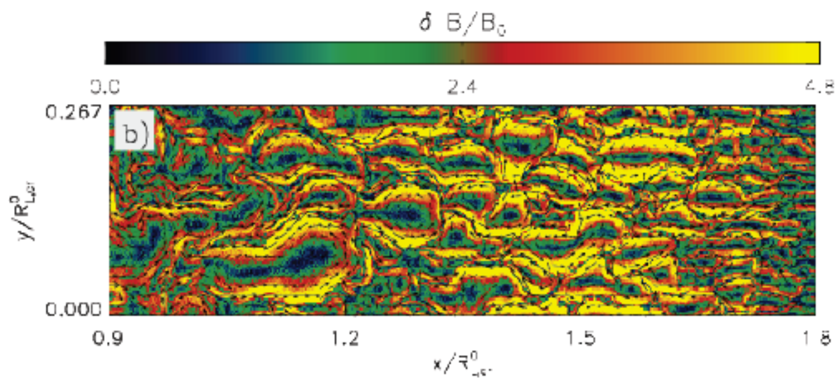
$k_{wice} r_i < 1$
Magnetic Field Strength Too Large
for Non–Resonant Instability
No Non–Resonant Instability
$k_{wice} r_{cr} > 1$
Non–Resonant
Bell Instability
Thermally Modified
Bell Instability
Thermal Ions Unmagnetized
Log$_{10}$[Magnetic Field Strength]
Log$_{10}$[Cosmic Ray Flux]

Rapid Growth to Nonlinear Amplitude



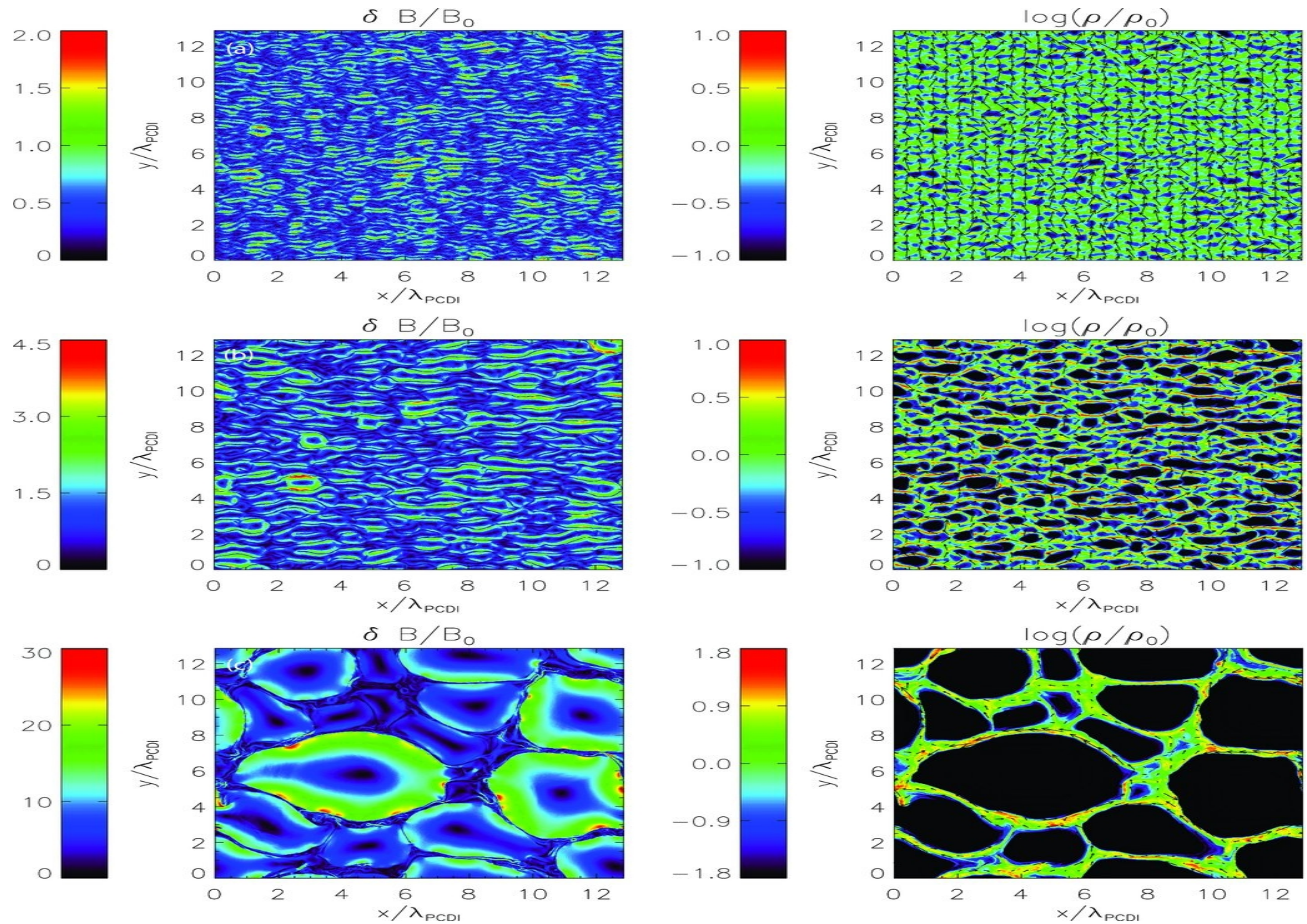
Linear growth rates for T = 0, (solid), T = 10^3 (long dashed), T = 10^6 (short dashed) (Zweibel & Everett 2010)

PIC simulation showing magnetic field growth in a shock layer.



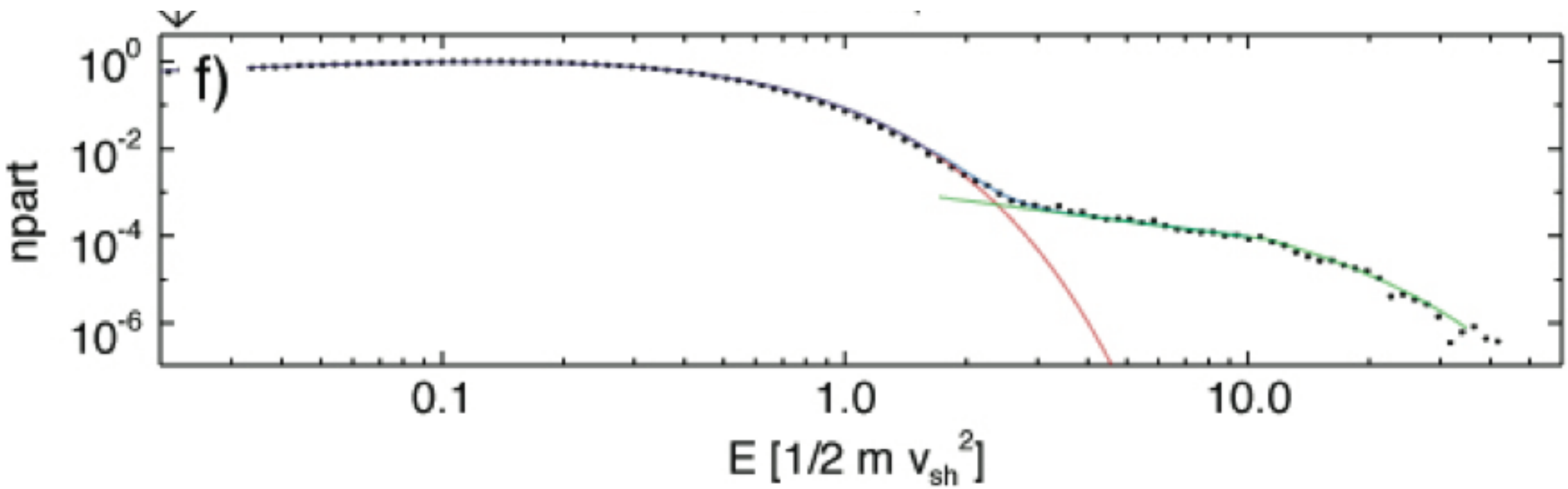
Riquelme & Spitkovsky 2010

Magnetic Field Amplification by Cosmic Ray Driven Instabilities in Shock Precursors



Riquelme & Spitkovsky (2010) ApJ 717, 1054

Hybrid Simulation of Shock Acceleration



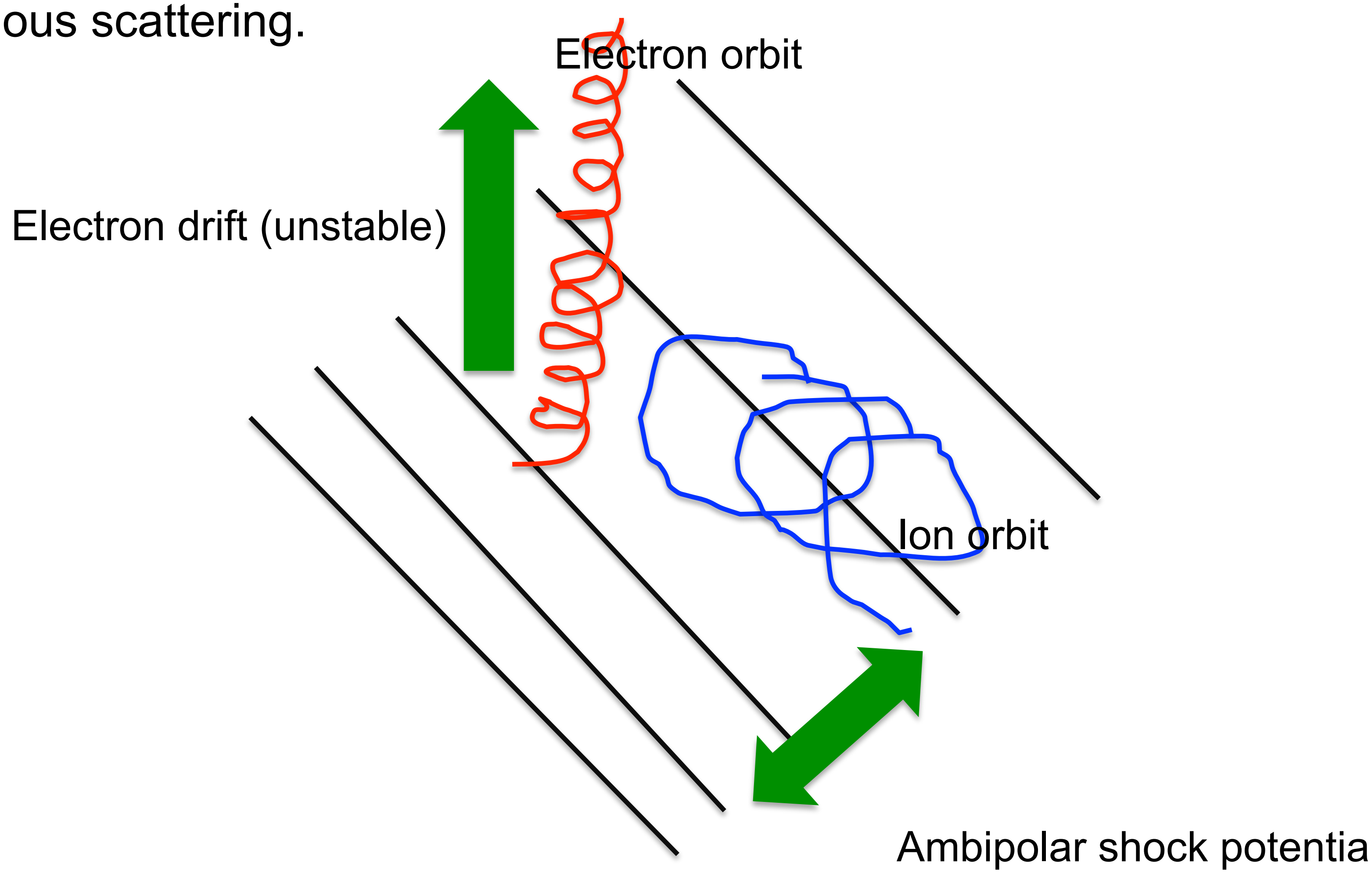
From Gargate & Spitkovsky (2012) ApJ

Collisionless Shocks

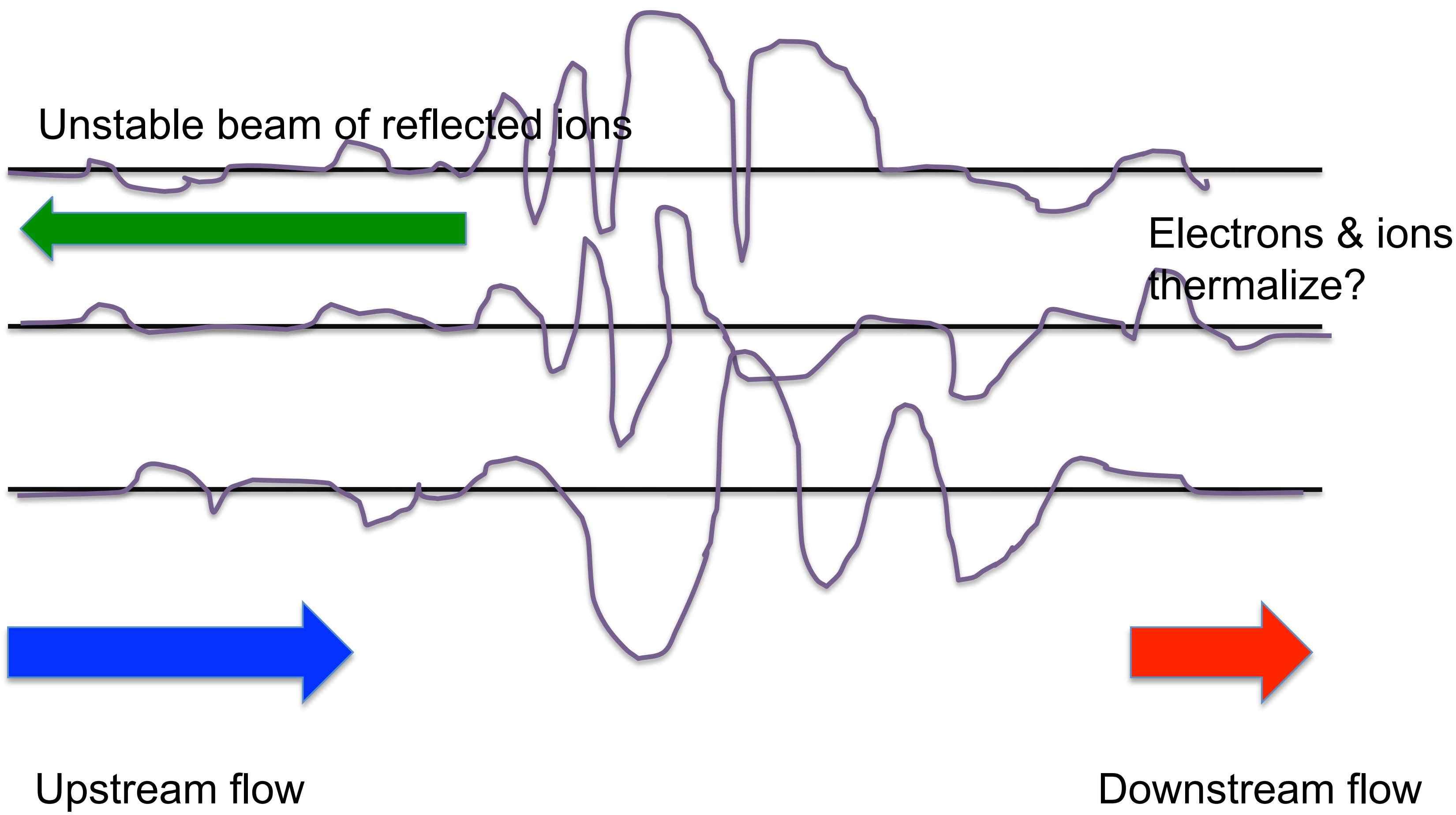
- Mean free path λ is too long to decelerate flow over distance required.
- For perpendicular shock, viscosity is suppressed by $(\omega_{ci}\tau_i)^2$.
- Collisionless shocks are subcritical or supercritical
  - In subcritical shocks, anomalous dissipation (wave-particle interactions) creates the shock transition
  - In supercritical shocks dissipation is inadequate & some of the incoming ions are reflected.
  - Critical M depends on β & θ but is ≤ 2.76

Schematic Subcritical Shock:

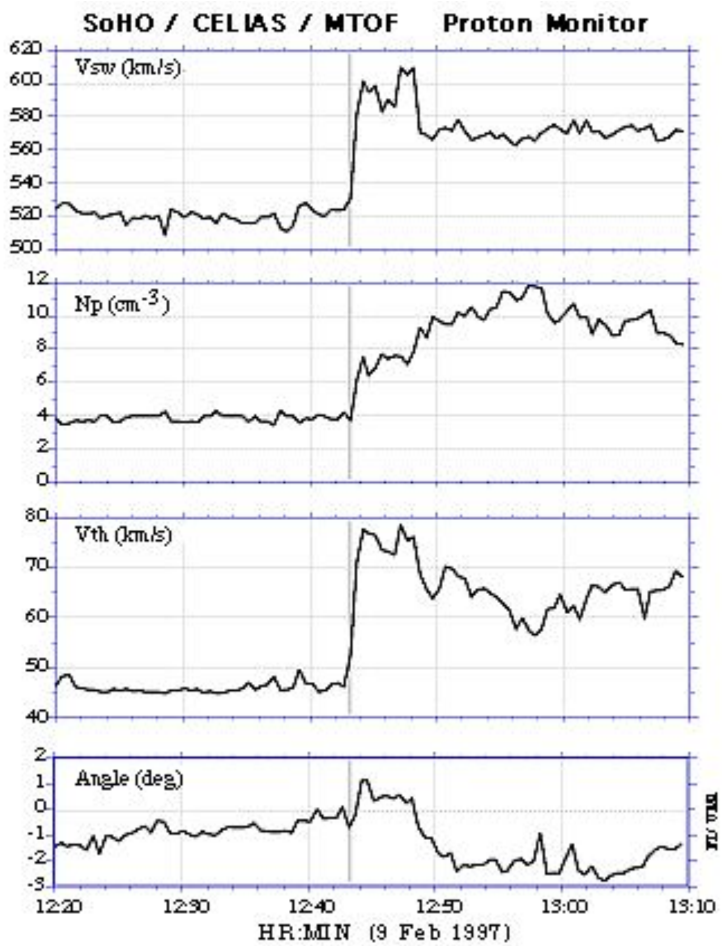
Ions & electrons are kept together by an electric potential, which causes an electron E x B drift that is unstable to microinstabilities, which provide anomalous scattering.



Supercritical Quasiparallel Shock

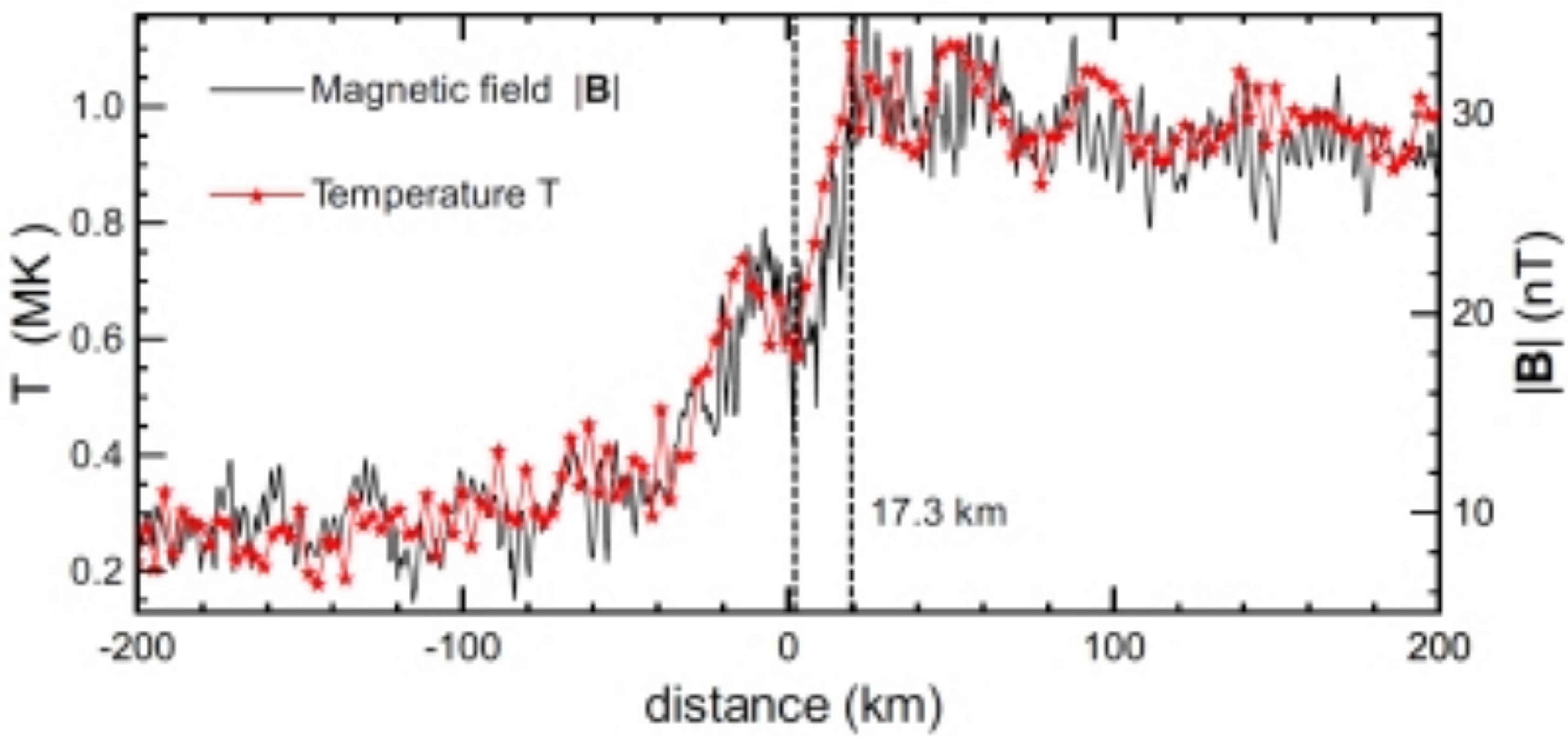


An Interplanetary Shock



Ipavich et al. 1998 JGR 103, 17205

Earth's Bowshock Measured by CLUSTER



Schwartz et al. 2011 PRL 107, 215002

Dreams of a Final Theory

- Geometry & boundary conditions imposed by global constraints.
- Resolve microscopic structure, understand dissipative processes.
- Derive electron and ion heating (thermal) and nonthermal (acceleration).
- Predict radiative signatures
- Understand time history including large scale instabilities.