INSTABILITIES IN ASTROPHYSICS

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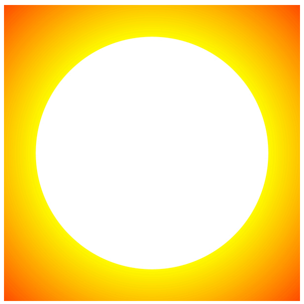
OUTLINE

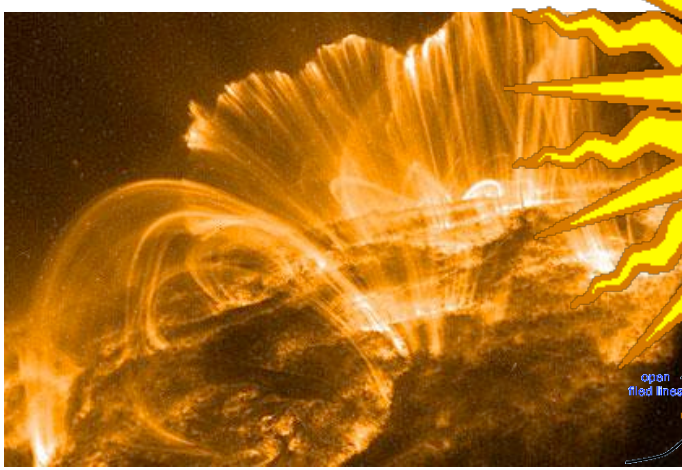
- Scope & structure of Plasma Astrophysics
- Role of Instabilities in Astrophysics
- Remarks on linear stage of instabilities
- Nonlinear development of instabilities
- Examples of important astrophysical instabilities.
- Summary

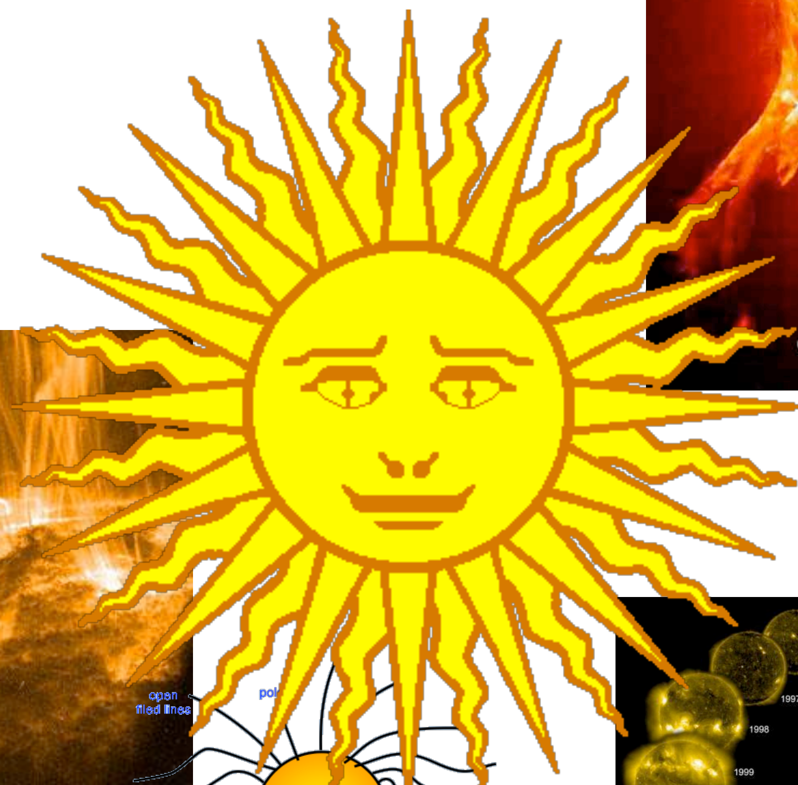
I. The Scope of Plasma Astrophysics:

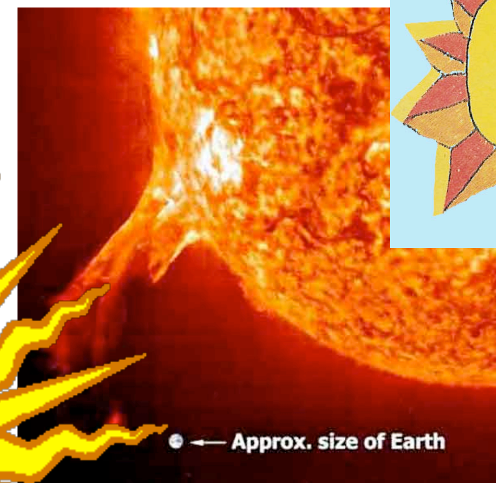
1.1. Plasma fills the Universe: **the Sun**

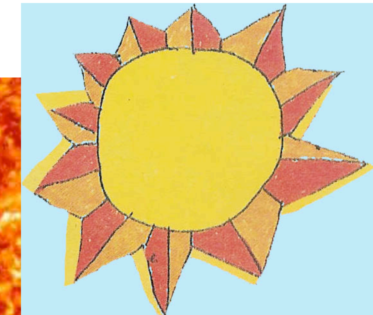
(image courtesy: Google Space Telescope)

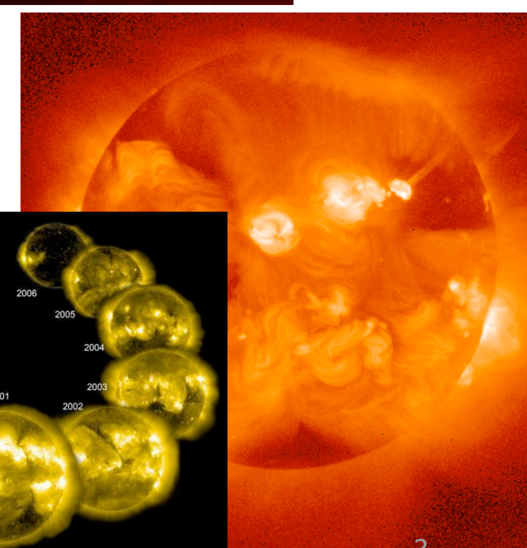


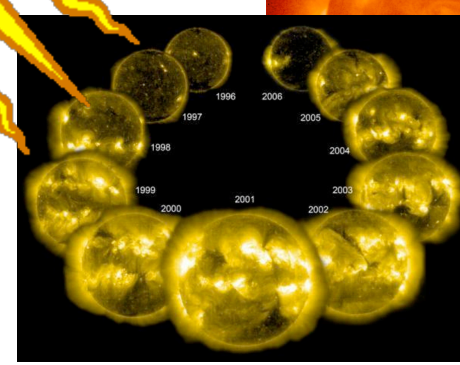


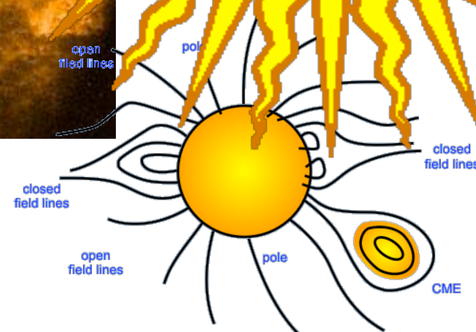








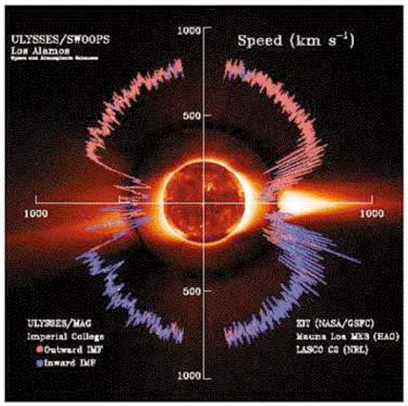




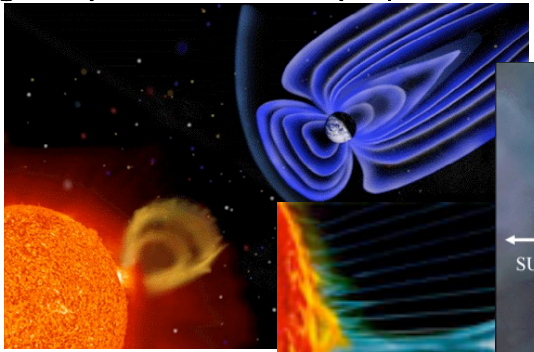
I. The Scope of Plasma Astrophysics:

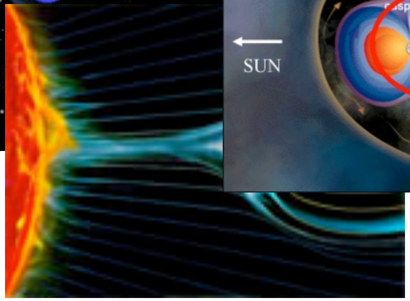
1.1. Plasma fills the Universe: Solar System

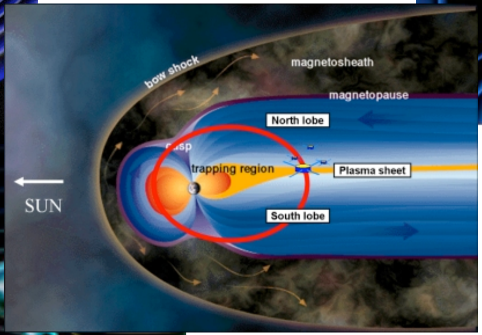
(image courtesy: Google Space Telescope)

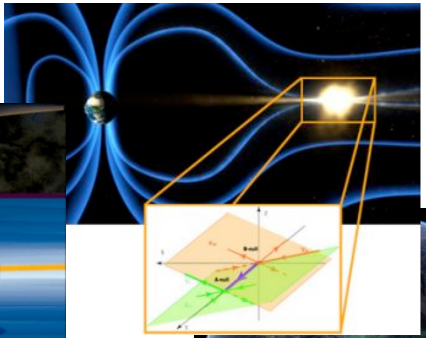


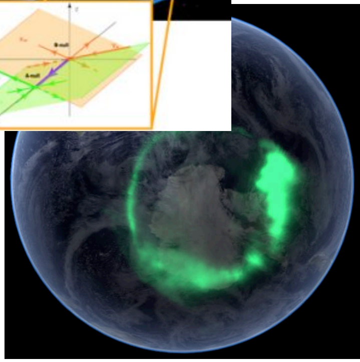
McComas, D.J., et al., Geophys. Res. Lett., 25, 1-4, 1998

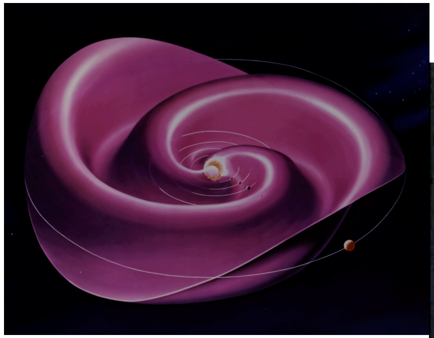


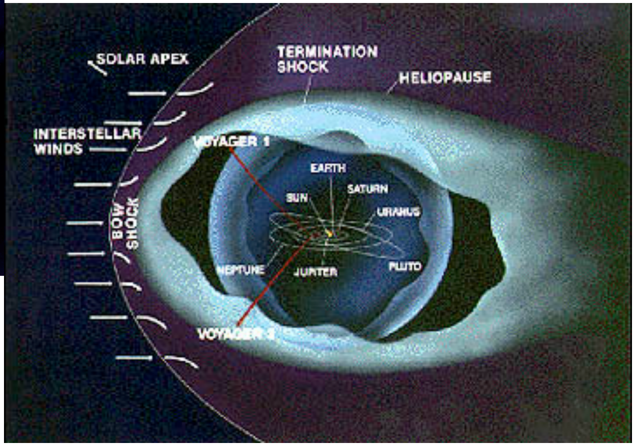


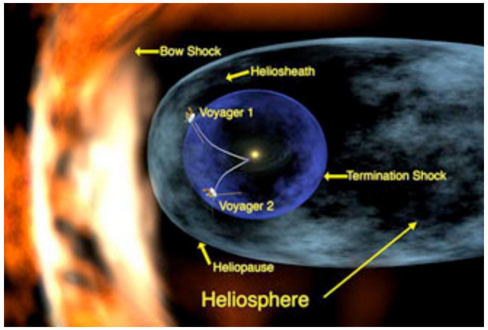






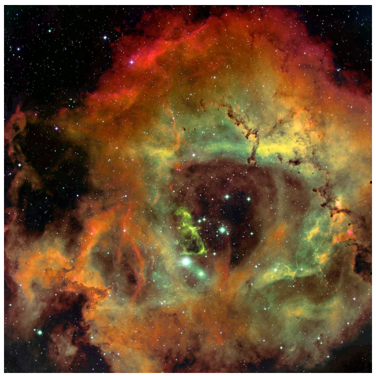


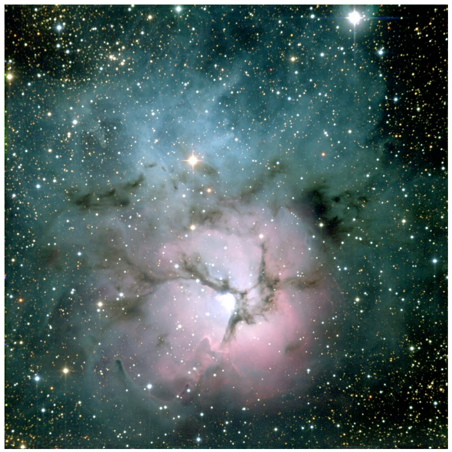


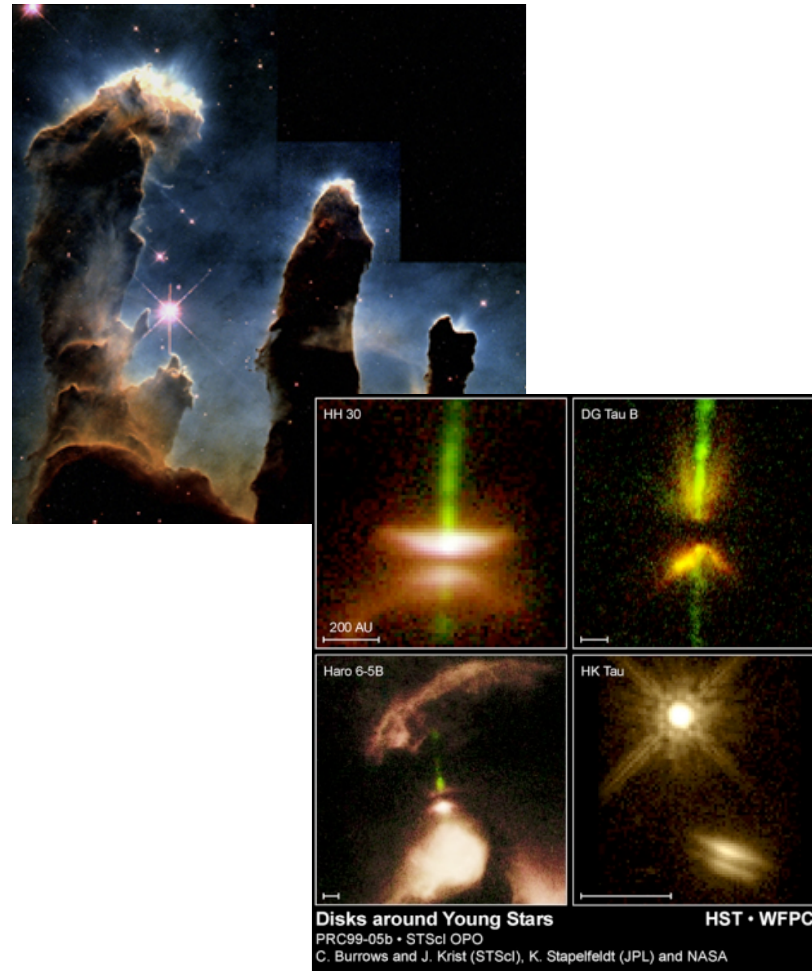


I. The Scope of Plasma Astrophysics:

1.1. Plasma fills the Universe: **Our Galaxy**







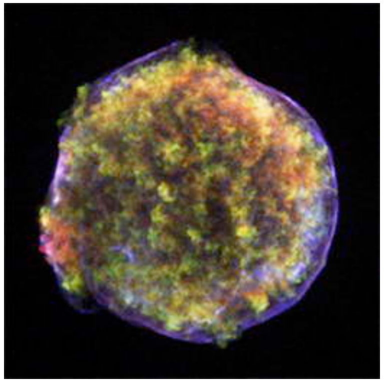
Young Stars

Interstellar Medium (ISM):
energy equipartition among:

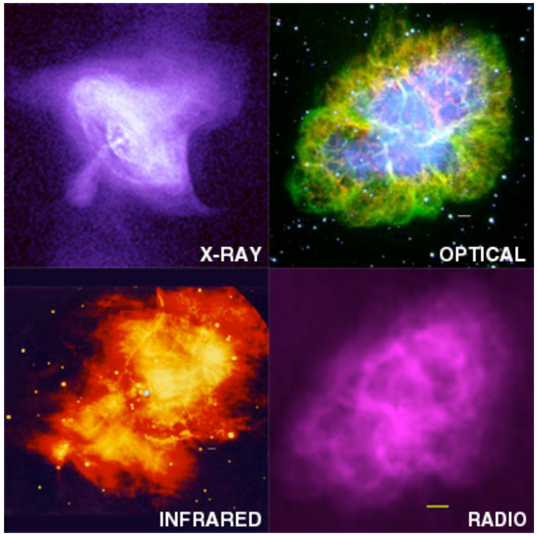
- thermal gas
- magnetic field
- turbulent kinetic energy
- cosmic rays
- stellar radiation

I. The Scope of Plasma Astrophysics:

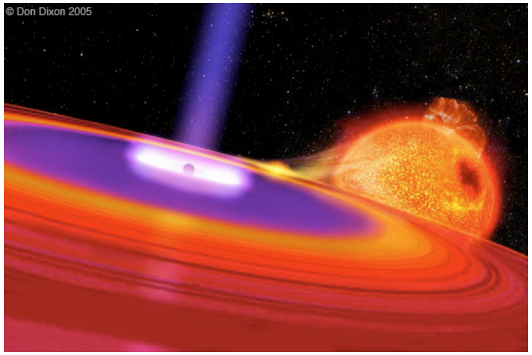
1.1. Plasma fills the Universe: Our Galaxy



Tycho SN remnant.



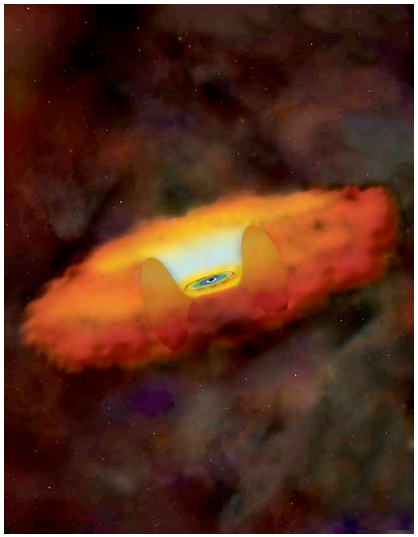
The Crab Nebula

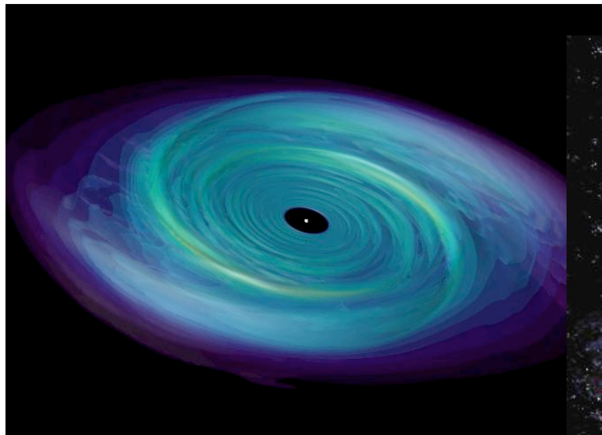


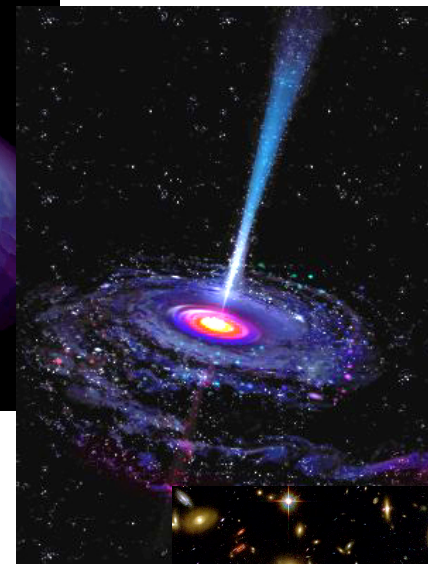
Accretion Disk in X-ray binary system

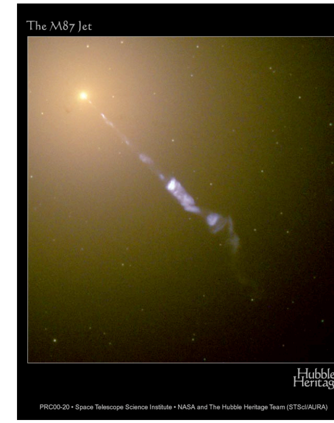
I. The Scope of Plasma Astrophysics:

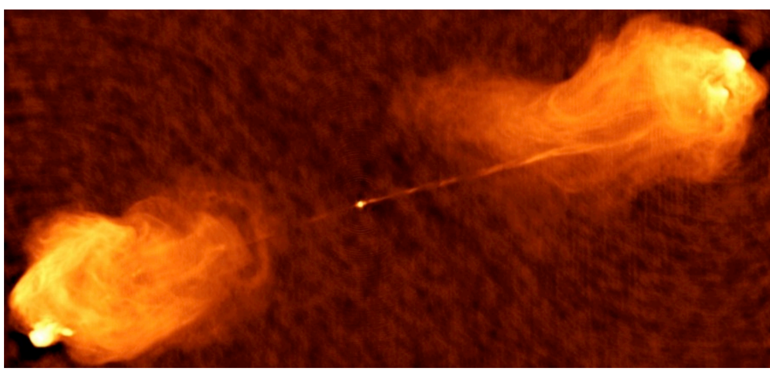
1.1. Plasma fills the Universe: **and beyond…**



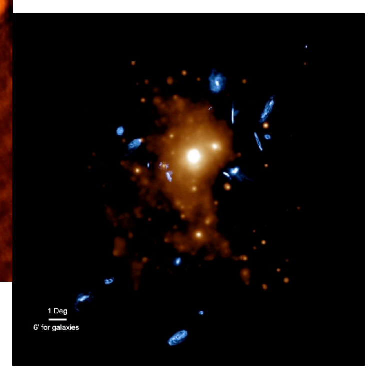








Cygnus A in radio



Virgo Cluster

Galaxy Cluster

I. Structure of Plasma Astrophysics:
I.2. Complexity of Cosmic Plasmas

("Cosmic" refers to space/solar and astrophysical plasmas)

Cosmic plasmas differ from laboratory and computational plasmas:

- Huge separation of scales, huge dynamic ranges.
- Lack of well defined initial conditions.
- One often does not have an isolated system (lack of well defined boundary conditions).
- Turbulent environments.

Cosmic plasmas are complex: many constituents/components!

Components …

Thermal gas/plasma

Nonthermal particles (CRs)

Magnetic field

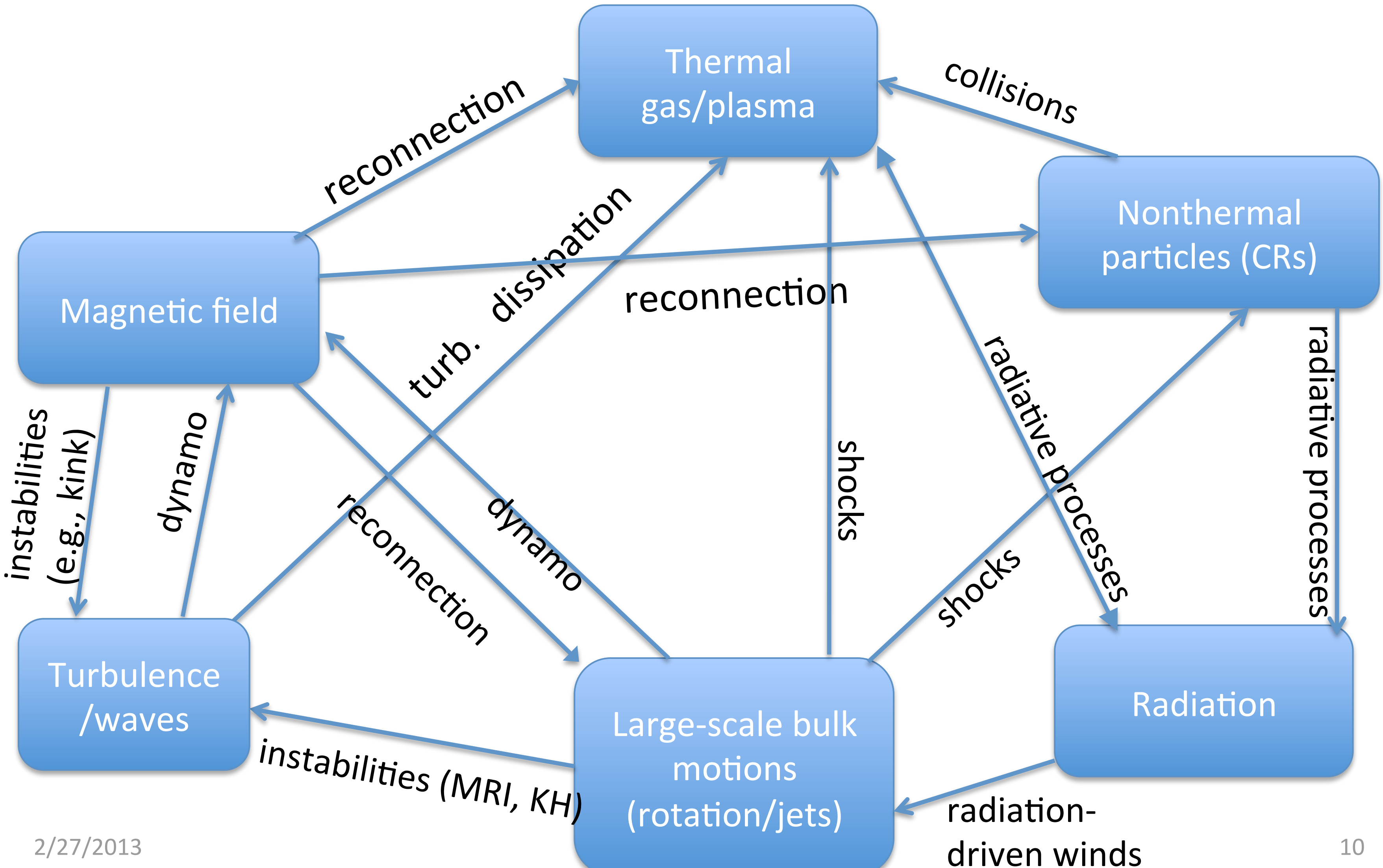
Turbulence /waves

Large-scale bulk motions (rotation/jets)

Radiation

... and Processes

Energy Exchange between plasma components: plasma **processes.**



Space and Astro plasmas are complex, multi-component systems

- In general, space/astro plasmas consist of several interacting ***constituents***:
  - thermal gas (neutral and ionized),
  - non-thermal particles/cosmic rays (CRs),
  - magnetic field,
  - small-scale turbulence/waves,
  - radiation (astro).

Often, some of these components are in equipartition (e.g., ISM).

- These ***constituents*** mutually interact, exchanging energy via ***physical processes***:
  - *Reconnection:* magnetic field → heat, nonthermal particles, bulk motions
  - *Turbulent dissipation*: turbulence → heat, nonthermal particles
  - *Shocks:* bulk motion → heat + nonthermal particles
  - *Dynamo:* rotation + turbulence → magnetic field
  - *Radiative processes:* heat, nonthermal particles → radiation

II. Role of Instabilities in Astrophysics

- Where do **instabilities** fit in this grand scheme of things?
- Instabilities necessarily involve energy exchange ("energy release"), they feed off of free energy in the system. Instabilities are "what makes great things happen", they are the mechanisms of *initiating* some of these energy exchange processes.

III. Remarks on linear stage of instabilities: 3.1. Role of equilibrium

- Stability is a mathematical concept that deals with behavior of dynamical systems when they are slightly perturbed from an **equilibrium**.
- Stability/instability is a property of an equilibrium!

One cannot talk about instability without specifying an equilibrium first!

- What is an equilibrium?

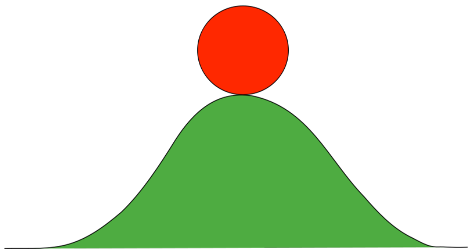
$$\frac{dx}{dt} = f(x)$$

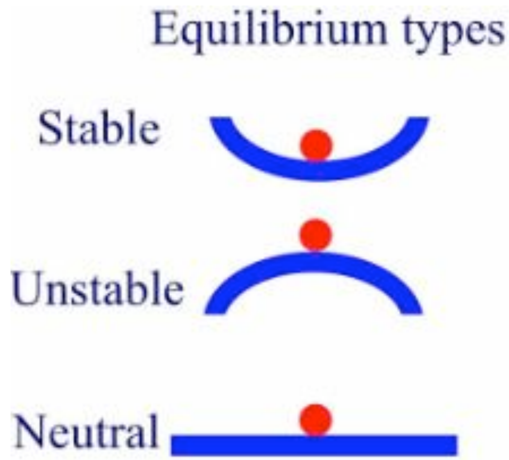
- Equilibrium state (equilibrium point): $x = x_0$ such that $f(x_0) = 0$.

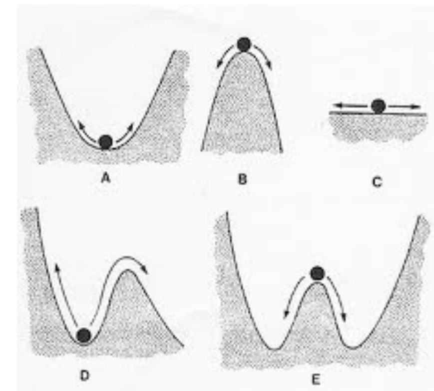
- <u>Stability:</u> if, for any sufficiently small initial deviation $|x-x_0| < \varepsilon$, there exist $\delta_\varepsilon > 0$ such that $|x-x_0|$ remains $< \delta_\varepsilon$, and $\delta_\varepsilon \rightarrow 0$ as $\varepsilon \rightarrow 0$, then the equilibrium point x_0 is ***stable***. Otherwise it is ***unstable***.

III. Remarks on linear stage of instabilities: 3.1. Role of equilibrium

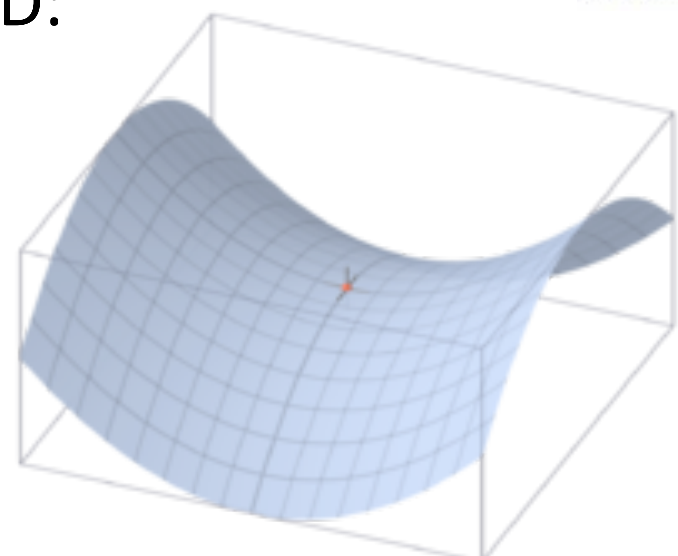
- 1D: ball on a hill:







- 2D:



Mechanical systems with several (*N*) degrees of freedom:

$$\frac{d\,\vec{x}}{d\,t} = \vec{f}(\vec{x})$$

- a vector equation, with $\vec{x}$ is a vector in the system's phase space:

$$\vec{x} = (q_1, q_2, \ldots, q_N, p_1, p_2, \ldots, p_N)$$

III. Remarks on linear stage of instabilities:
3.2. Stability in Fluid Mechanics

- In fluid mechanics (including plasma physics) what do we mean by equilibrium?

The state of the system is no longer described by just one or a few *numbers* (e.g., position of the ball), but rather by a several *functions* (in 1, 2, 3, …, 6, 7 D):

e.g., fluid variables in (3+1) space-time or (6+1)-D distribution function in kinetics.

- Thus, here the state of the system (e.g., equilibrium) is

a vector in the Hilbert space of functions, e.g., in MHD:

$$X(\vec{r},t) = \begin{pmatrix} \rho(\vec{r},t) \\ \vec{u}(\vec{r},t) \\ \vec{B}(\vec{r},t) \end{pmatrix}$$

- The small deviation from the equilibrium δ*X* is also a (vector) function.
- Formulation of the stability definition should involve |δ*X*|< δ.

But what is the proper choice of the norm of a vector in this space (L_1, L_2)?

III. Remarks on linear stage of instabilities:
3.3. Standard Linear Stability Analysis

- Consider a dynamical system: $$\frac{d\vec{x}}{dt} = \vec{f}(\vec{x})$$

with an equilibrium point x_0: $\vec{f}(\vec{x}_0) = 0$

- We are interested in the fate/evolution of infinitesimally small perturbations:

$$\vec{x} = \vec{x}_0 + \delta\vec{x}$$

- Linearization (Taylor-expand near x_0 to 1st order with respect to δx):

$$\frac{d\vec{x}}{dt} = \vec{f}(\vec{x}) = \vec{f}(\vec{x}_0) + \frac{\partial\vec{f}}{\partial\vec{x}}\Big|_{x_0} \cdot \delta\vec{x} = 0 + \frac{\partial\vec{f}}{\partial\vec{x}}\Big|_{x_0} \cdot \delta\vec{x} = \overleftrightarrow{A} \cdot \delta\vec{x}$$

where $\overleftrightarrow{A} = \frac{\partial\vec{f}}{\partial\vec{x}}\Big|_{x_0}$ is a Jacobian.

In Hilbert space, $\overleftrightarrow{A}$ is a (linear) operator (in general, differential).

Growth of unstable modes in linear regime

- Linearized equation of motion: $$\frac{d\,\delta\vec{x}}{dt} = A\cdot\delta\vec{x}$$
- Fourier representation: $\delta\vec{x}(t) \propto \vec{x}_1\, e^{-i\omega t} \Rightarrow \partial_t \to (-i\omega)$
- Then $$-i\omega\,\vec{x}_1 = \overleftrightarrow{A}\cdot\vec{x}_1$$ $$(i\omega\overleftrightarrow{I} + \overleftrightarrow{A})\cdot\vec{x}_1 = 0$$
- **Dispersion relation (DR):** $$\det\,(i\omega\overleftrightarrow{I} + \overleftrightarrow{A}) = 0$$
- Matrix ***A*** depends on parameters of equilibrium:

A = ***A***(n_0, B_0, …), but not on ω.

- Thus, DR is a polynomial algebraic equation for ω, of order n where n is the dimensionality of the vector space. E.g., for ball on 1D hill or valley --- two canonic coordinates: q and p (x and v), so we get quadratic equation for ω.

Linear stability

$$\frac{d\,\delta x}{d\,t} = f(x_0 + \delta x) = A\,\delta x \;\Rightarrow \delta x(t) = \delta x(0)\,e^{\gamma t}$$

Mathematical characteristics of the solution ω of the DR tell us whether that equilibrium is stable or not.

- $\omega = \omega_r + i\,\gamma$
- $\exp(-i\omega t) = \exp(-i\omega_r\, t) \times \exp(\gamma\, t)$.
- $\gamma>0$ – instability, $\gamma<0$ – stability.
- if $\omega_r=0$, then we have purely growing or decaying mode.
- Does it mean that instabilities in Nature grow as $e^{\gamma\, t}$?
- **NO**!

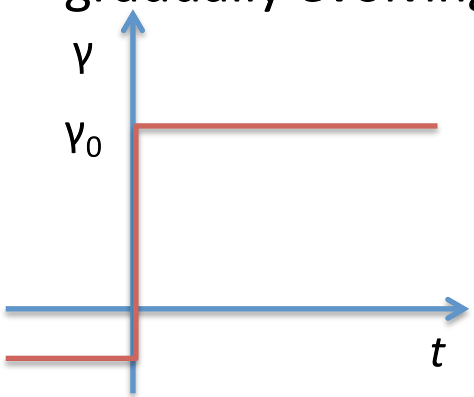
Linear growth of instabilities

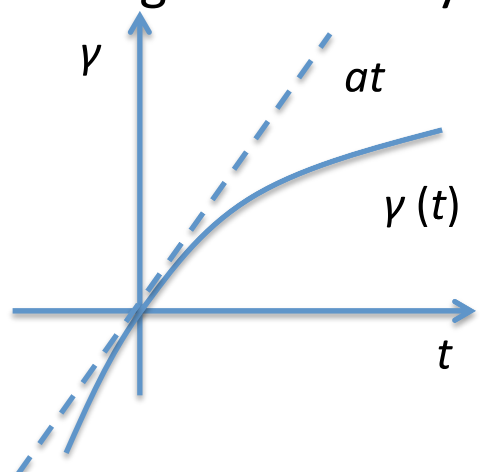
(thanks to Steve Cowley)

The above analysis assumes that we start with a highly unstable equilibrium. But how does Nature set it up in the first place?

Usually, it does not!

$\gamma = \gamma(t)$ --- gradually evolving equilibrium crossing the stability boundary.





No simple initial conditions in real life!

- $d\,\delta x/dt = \gamma(t)\,\delta x$
- gradually changing -- Taylor-expand near marginal stability $\boldsymbol{\gamma(t) = a\,t}$
- $d\,\delta x/dt = a\,t\,\delta x \rightarrow$ $\boldsymbol{\delta x \sim \exp(a\,t^2)}$.

IV. Nonlinear development of instabilities

- Linear theory is not the whole story… Linear stage lasts only for so long, but is then followed by nonlinear physics.
- What is the long-term, nonlinear fate of an instability?
- It depends….

Relaxation to a new, equilibrium/saturation

- no new free energy is supplied;
- new equilibrium has lower energy and lower symmetry
- examples: kink, tearing.
- benign

Development of turbulence

- free energy is supplied continuously (driven);
- affects global equilibrium profiles to lead to marginal stability
- examples: convection, MRI, ITG, firehose and mirror, Weibel
- modestly benign

System disruption

- No suitable lower-energy equilibrium available.
- The system runs away until the entire configuration is disrupted
- catastrophic, leads to big spectacular events.
- examples: kink –sawtooth, RT,

IV. Nonlinear development of instability:

4.1. Saturation: relaxation to nearby equilibrium.

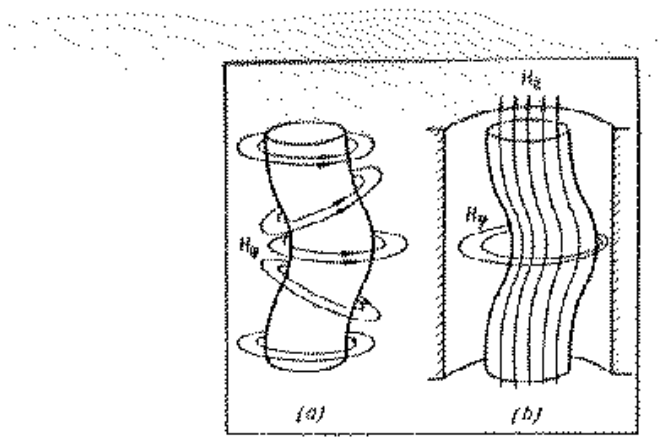
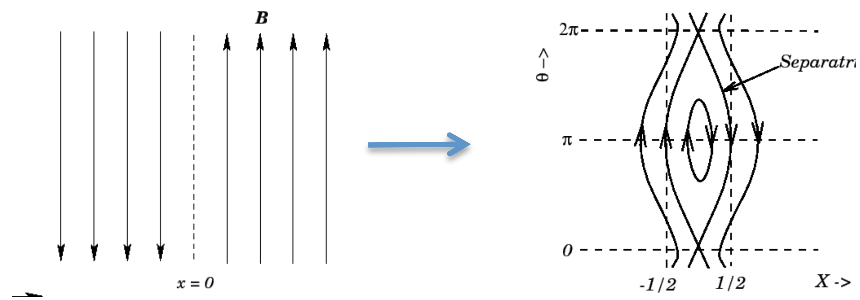
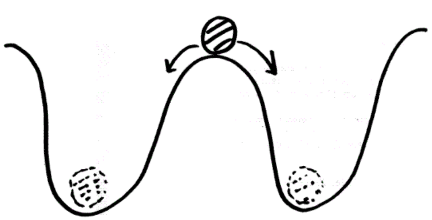


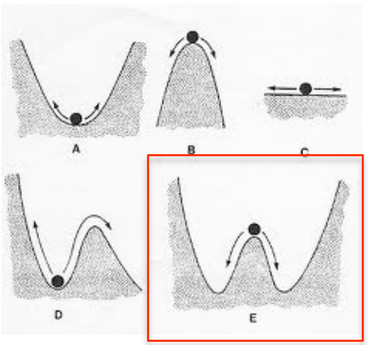
Figure 9: Mode=1 kink instability — From [6] Golant

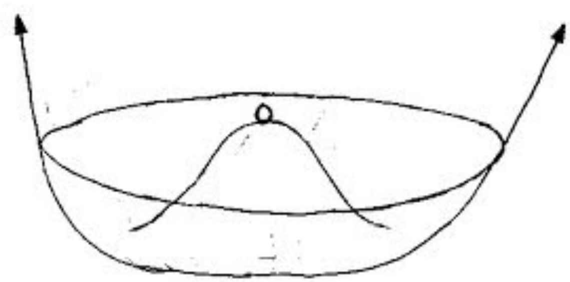


tearing instability

- Connection to symmetry breaking.





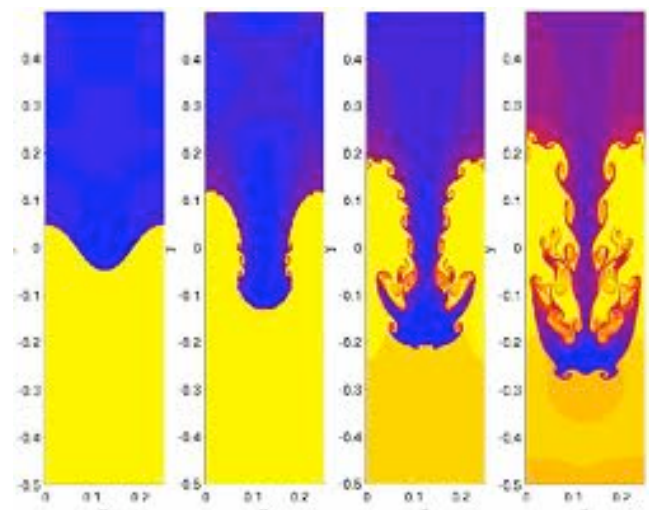


IV. Nonlinear development of instabilities:

4.2. Transition to Turbulence and Parasitic Instabilities:

- One person's trash is another's treasure:

a growing instability can sometimes itself be regarded as a (time-evolving) equilibrium, which can in turn become unstable.



- Examples:
  - secondary KH on RT fingers;
  - secondary parasitic instabilities

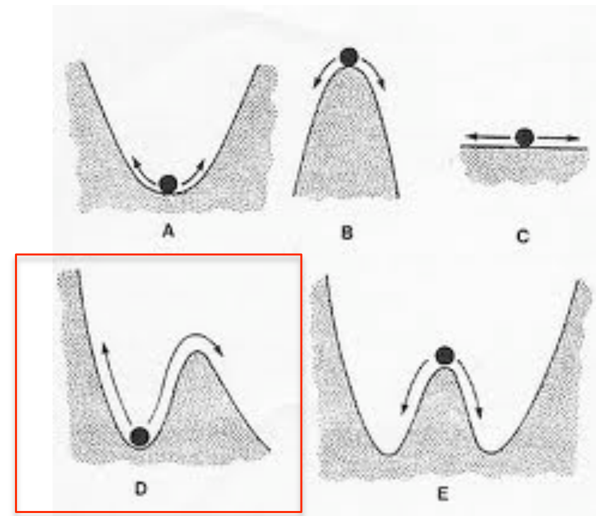
on MRI channel modes (later).

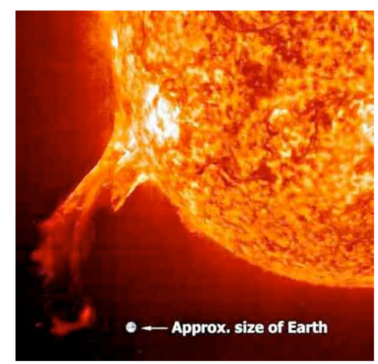
- This is how instabilities lead to turbulence…

IV. Nonlinear development of instabilities:

4.3. Catastrophic System Disruption

- System is trying to relax, to reach a new equilibrium lower energy state, but there is no such state nearby.
- accessible lower-energy is so far away that the system effectively disrupts before it reaches it.
- solar eruptions?
- tokamak disruptions (giant ELMs?)





V. Examples of Instabilities in Astrophysics: Macro- and Micro-instabilities

- Astronomical systems are astronomically large:

huge separation of scales:

system size $L >> d_{i,} \rho_i, \lambda_D$

[fundamental plasma length-scales that depend only on local physical parameters (*n,T, B*) but not on *L*].

- This discussion is relevant not only for instabilities but for all plasma processes in general.

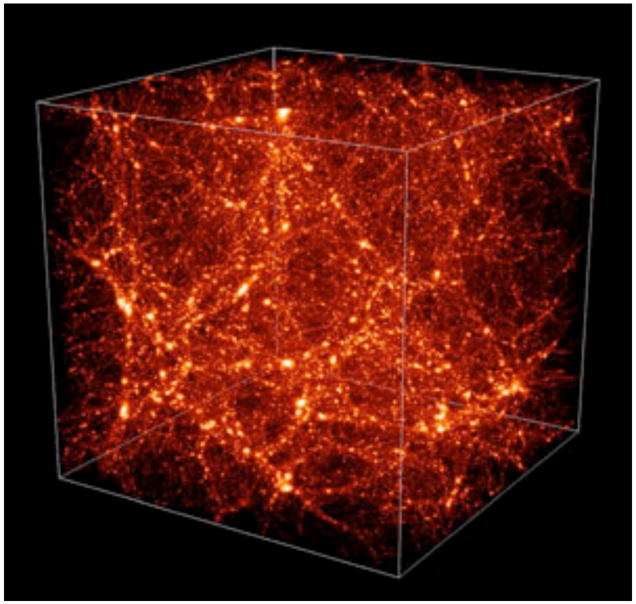
- Two classes of instability:
  - hydrodynamic/MHD --- macroscopic (up to system size L)
  - two-fluid/kinetic --- microscopic (on plasma basic scales ~ $d_{i,} \rho_i, \lambda_D$)

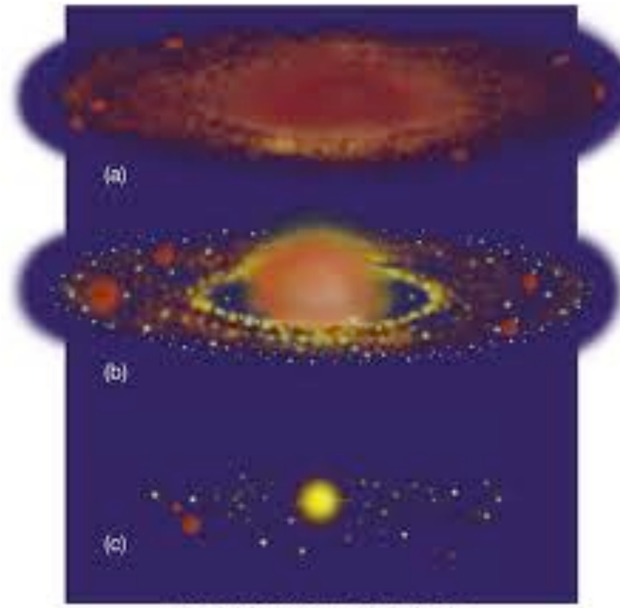
V. Examples of Instabilities in Astrophysics: V.1 Hydrodynamic instabilities

- Large-scale Fluid (macroscopic):
  - hydrodynamic (gravitational, Rayleigh-Taylor, Kelvin-Helmholtz, convective/buoyancy).
  - ideal-MHD (kink/sausage, MRI, Parker)
  - resistive-MHD (tearing)
- Small-scale (2-fluid and kinetic, microscopic):
  - electromagnetic kinetic Weibel
  - Pressure-anisotropy-driven (firehose, mirror)
  - electrostatic kinetic streaming: Buneman, ion-acoustic, bump-on-tail.

V.1 Hydrodynamic instabilities:

Gravitational (Jeans) instability





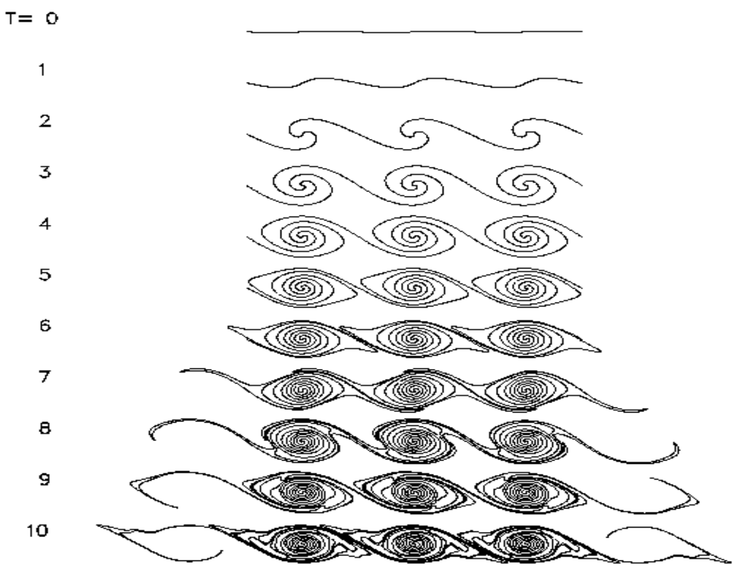
Jeans Mass:

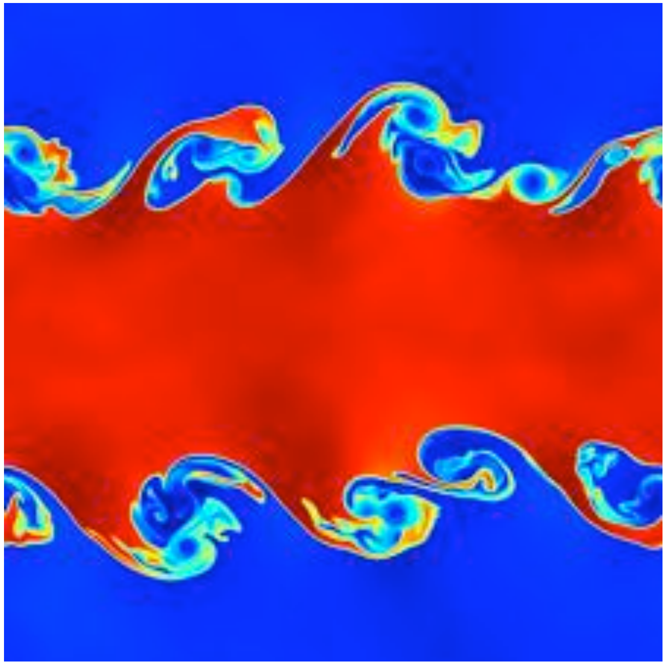
$$M_J = \left(\frac{5kT}{Gm}\right)^{3/2}\left(\frac{3}{4\pi\rho}\right)^{1/2}$$

$$\text{if } \mathrm{M_{cloud}} > M_J \rightarrow \text{collapse!}$$

V.1 Hydrodynamic instabilities:
Kelvin-Helmholtz (KH) instability

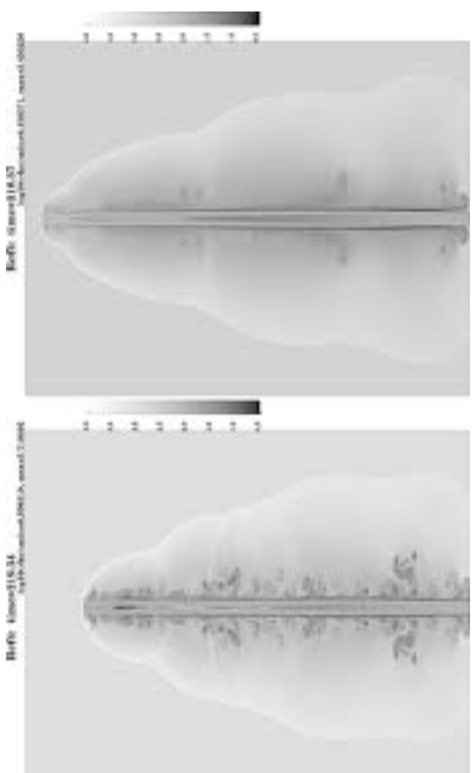
Velocity shear instability

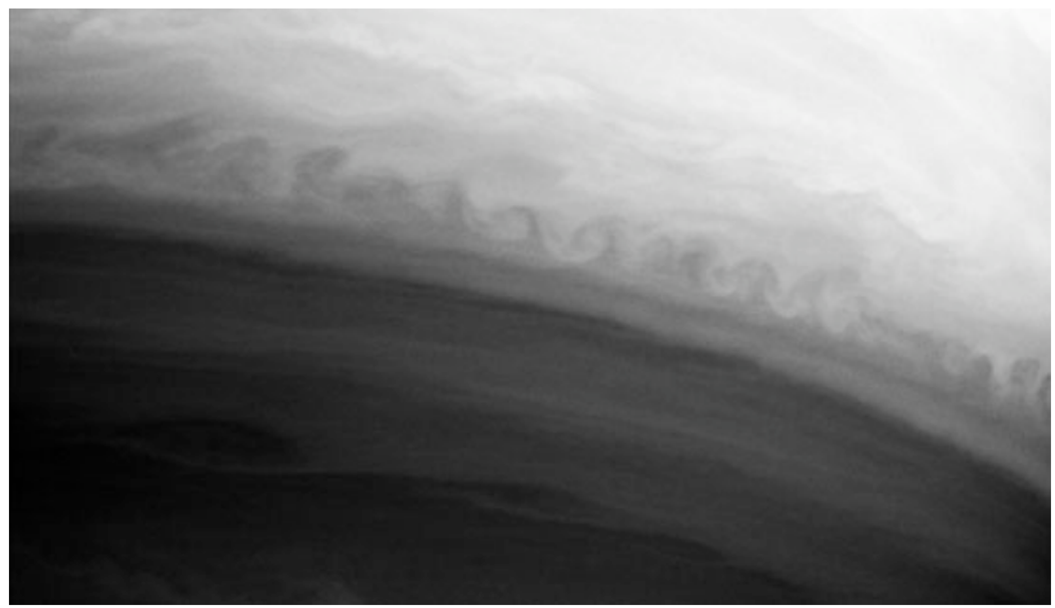




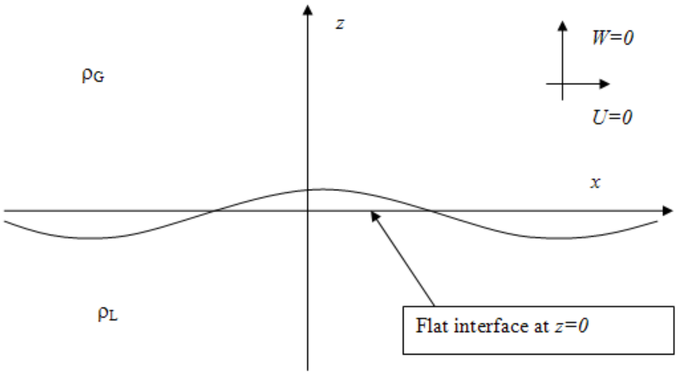
V.1 Hydrodynamic instabilities:
Kelvin-Helmholtz (KH) instability

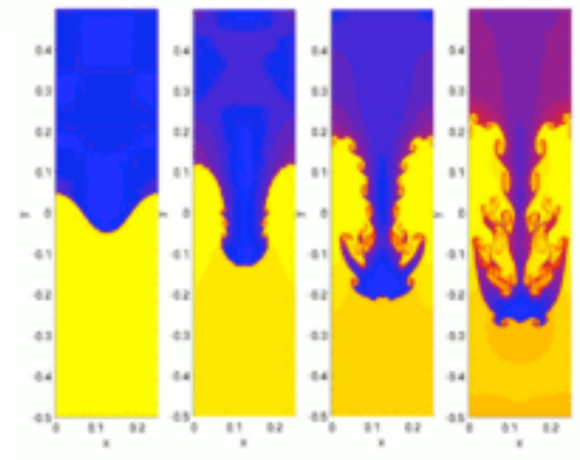
Velocity shear instability

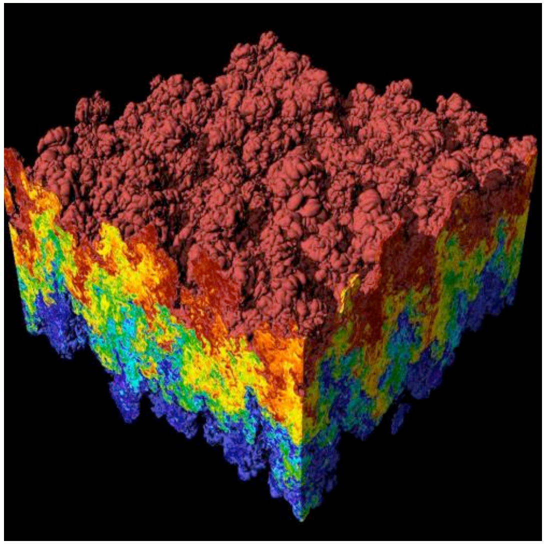




V.1 Hydrodynamic instabilities:
Rayleigh-Taylor (RT) instability



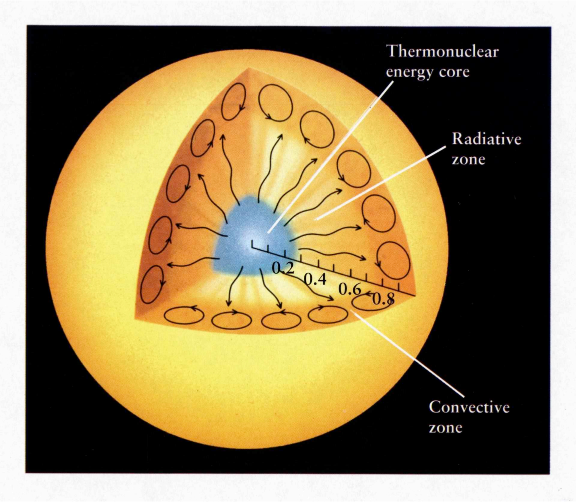




V.1 Hydrodynamic instabilities:
Convective Stability in Stars

Stellar structure is governed by a combination of nuclear-reactions heating, hydrostatic pressure balance with gravity, and radial heat transport:

- Core: heat is produced by nuclear reactions.
- Radiative zone: heat is transported out by radiation diffusion.
- Convective zone: heat is transported outward by convection.

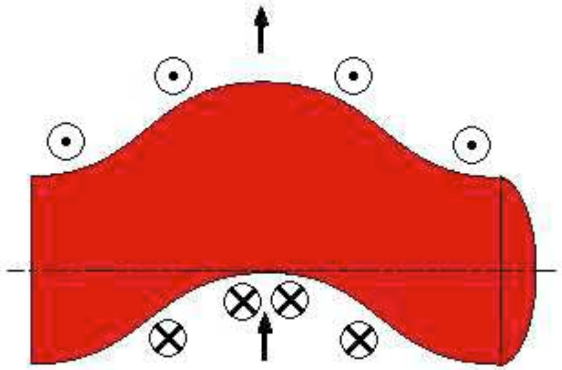


Schwarzschild instability criterion:

$$\left|\frac{\mathrm{d}T}{\mathrm{d}z}\right| > \left|\frac{\mathrm{d}T}{\mathrm{d}z}\right|_{\mathrm{adia}} \quad \text{with} \quad \left|\frac{\mathrm{d}T}{\mathrm{d}z}\right|_{\mathrm{adia}} = \frac{g}{c_p}.$$

$\Rightarrow$ $dS/dz < 0$

Macro- (MHD) Instabilities I: Kink



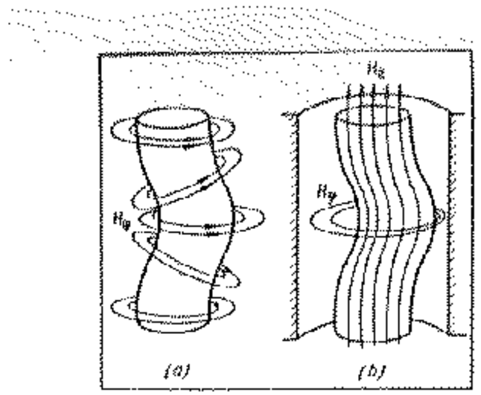
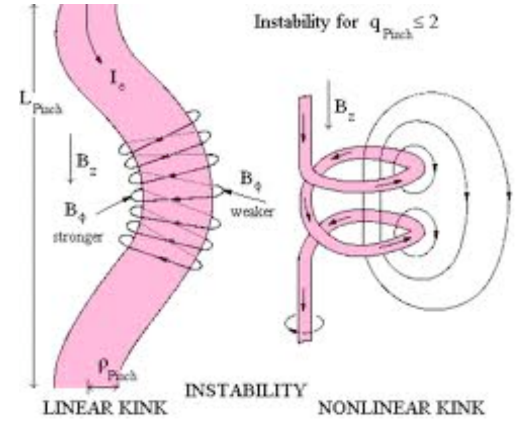
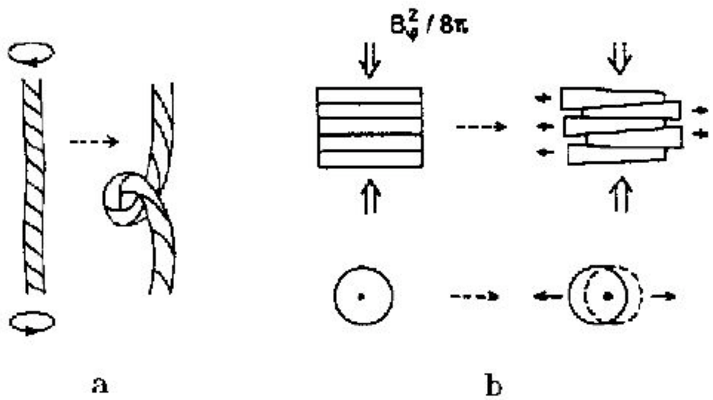
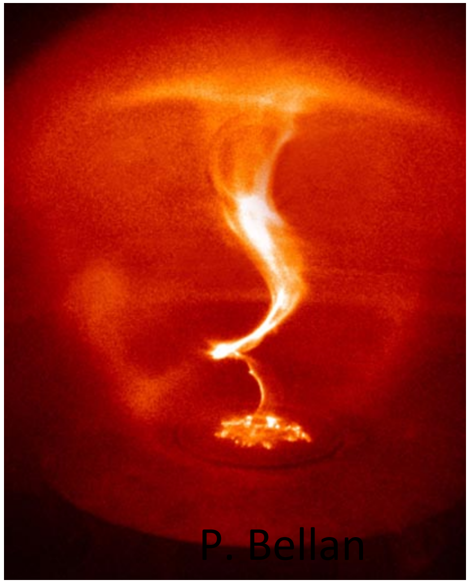


Figure 9: Mode=1 kink instability — From [6] Golant







Macro- (MHD) Instabilities I: Kink

Source of energy: magnetic

Examples:

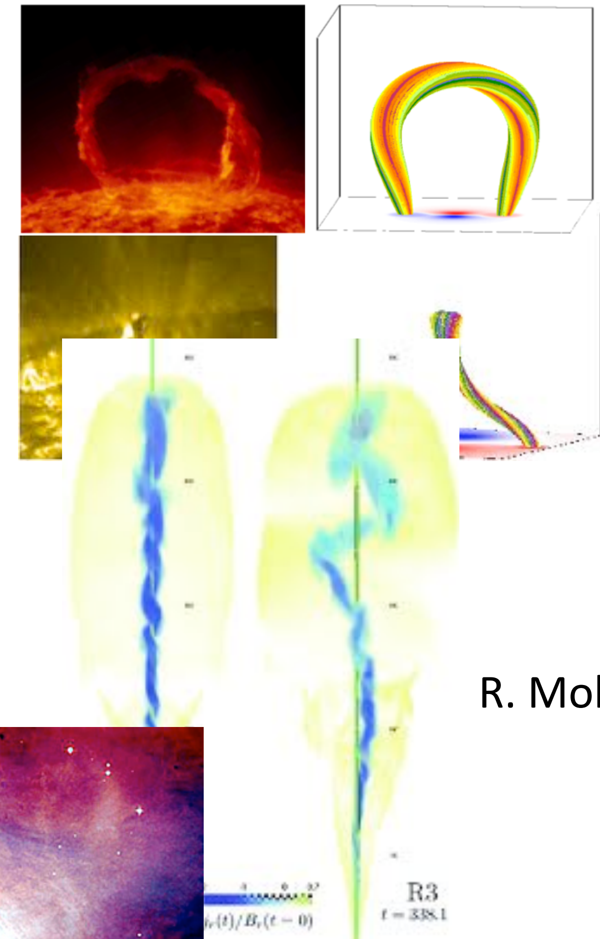
- solar eruptions?

- Astrophysical jets (AGN, GRB):

mild --- does not destroy the jet, but visible.

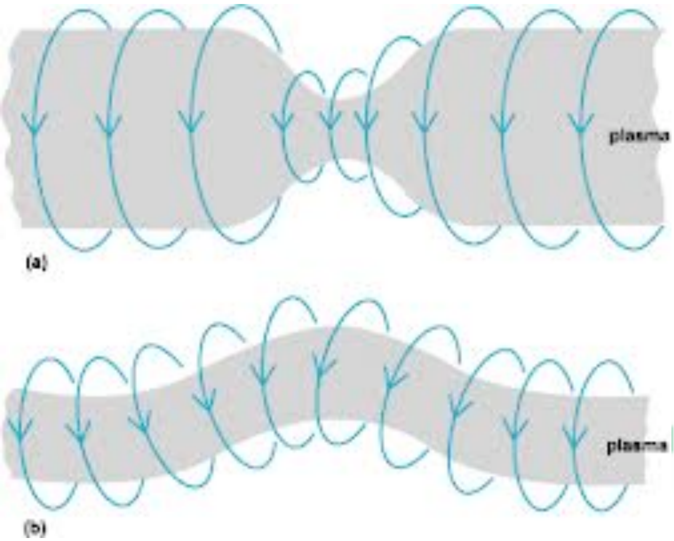
- Pulsar Wind Nebula (PWN):

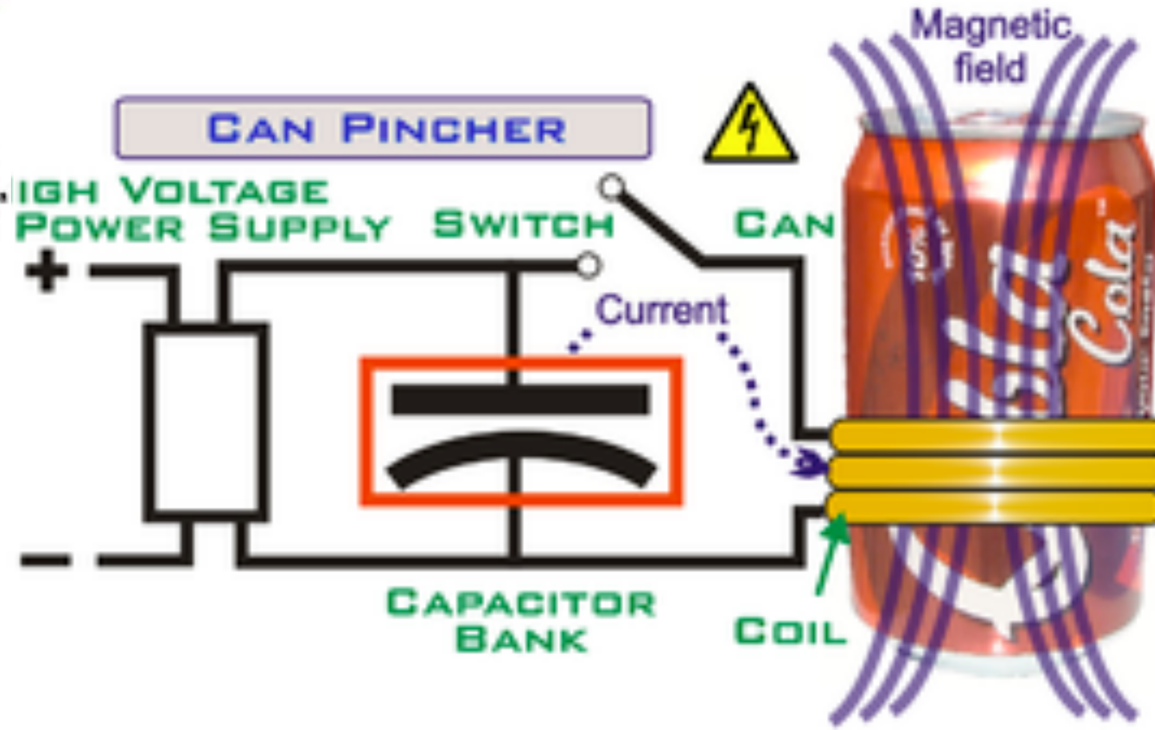
(e.g., Begelman 1998, Mizuno et al.2011)
analogy with Z-pinch
powers X-ray emission in the Crab jet.



R. Moll

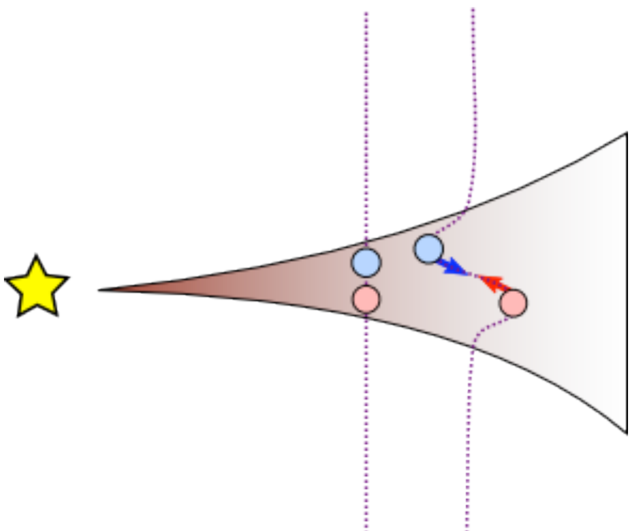
MHD instabilities: Sausage/pinch instability

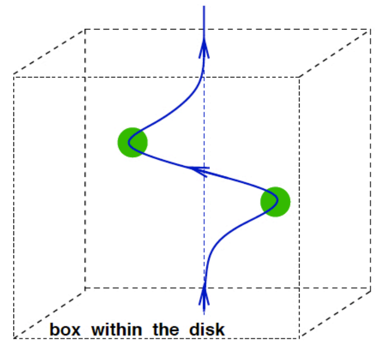


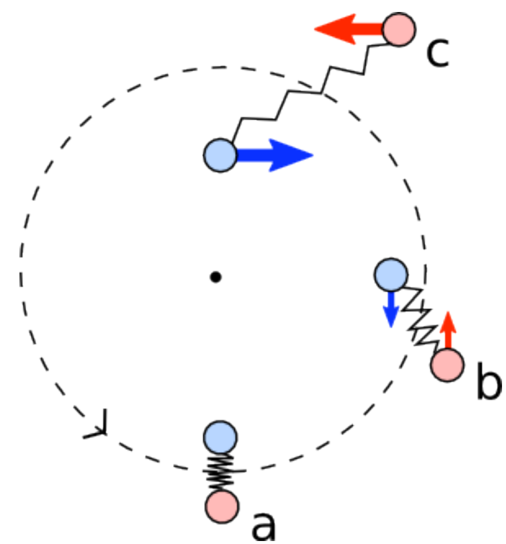


MHD instabilities: Sausage/pinch instability

Macro- (MHD) Plasma Instabilities II: Magneto-Rotational Instability (MRI)



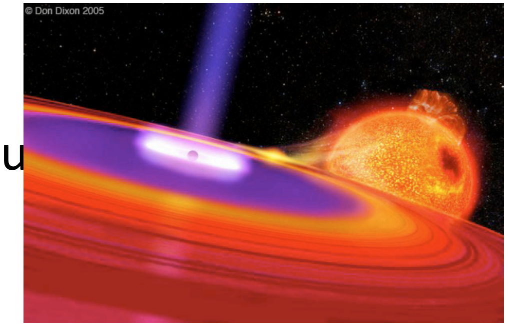


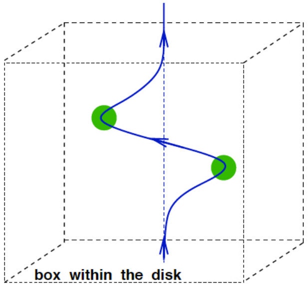


See talk by Lesur

Macro- (MHD) Plasma Instabilities II: Magneto-Rotational Instability (MRI)

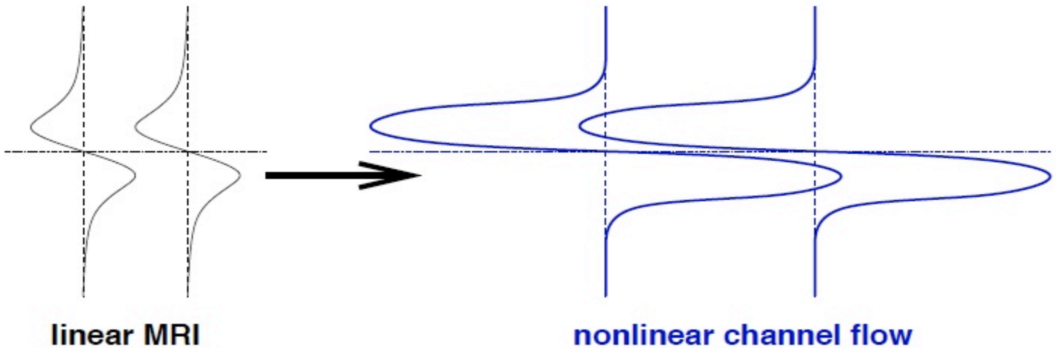
- History:
  - Velikhov, Chandrasekhar (~1960) with no applications
  - rediscovered by Balbus & Hawley in 1991 – to explain tu in accretion disks;
  - accretion disk: gas slowly spiraling onto central object.
  - to accrete, gas has to loose angular momentum.
  - molecular viscosity too small
  - Shakura-Sunyaev (1973) invoked turbulent viscosity.
  - but what causes turbulence? Need an instability.
  - in fluid dynamics Keplerian disks should be stable.
  - Balbus & Hawley (1991) lead to a revolution in our understanding of accretion disks, and in High-Energy Astrophysics in general --- many papers, extensive numerical studies of the MRI-driven turbulence.
  - MRI lab experiments.





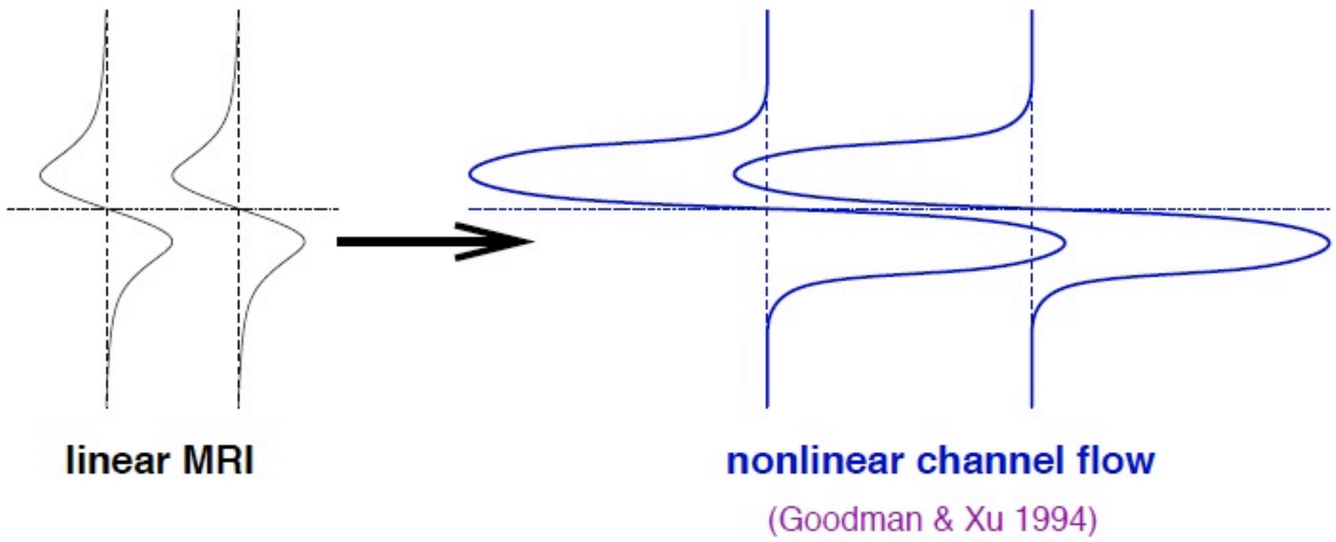
MRI, Channel Modes, and Parasites

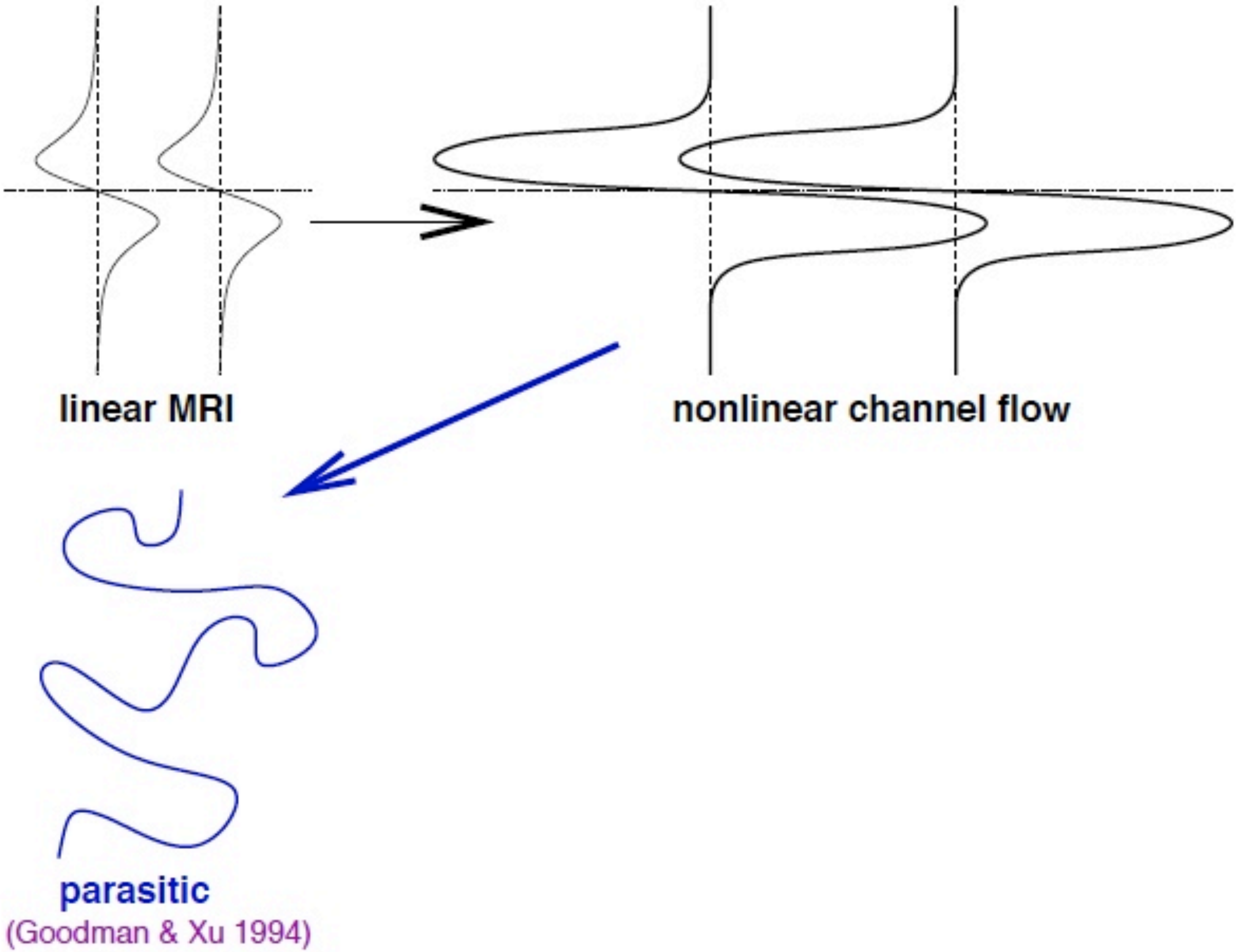
- MRI mode is an Alfven wave gone mad.
- Exact ***nonlinear*** solution of MHD equations *(Goodman & Xu 1994)*!
- Leads to ***channel modes***:



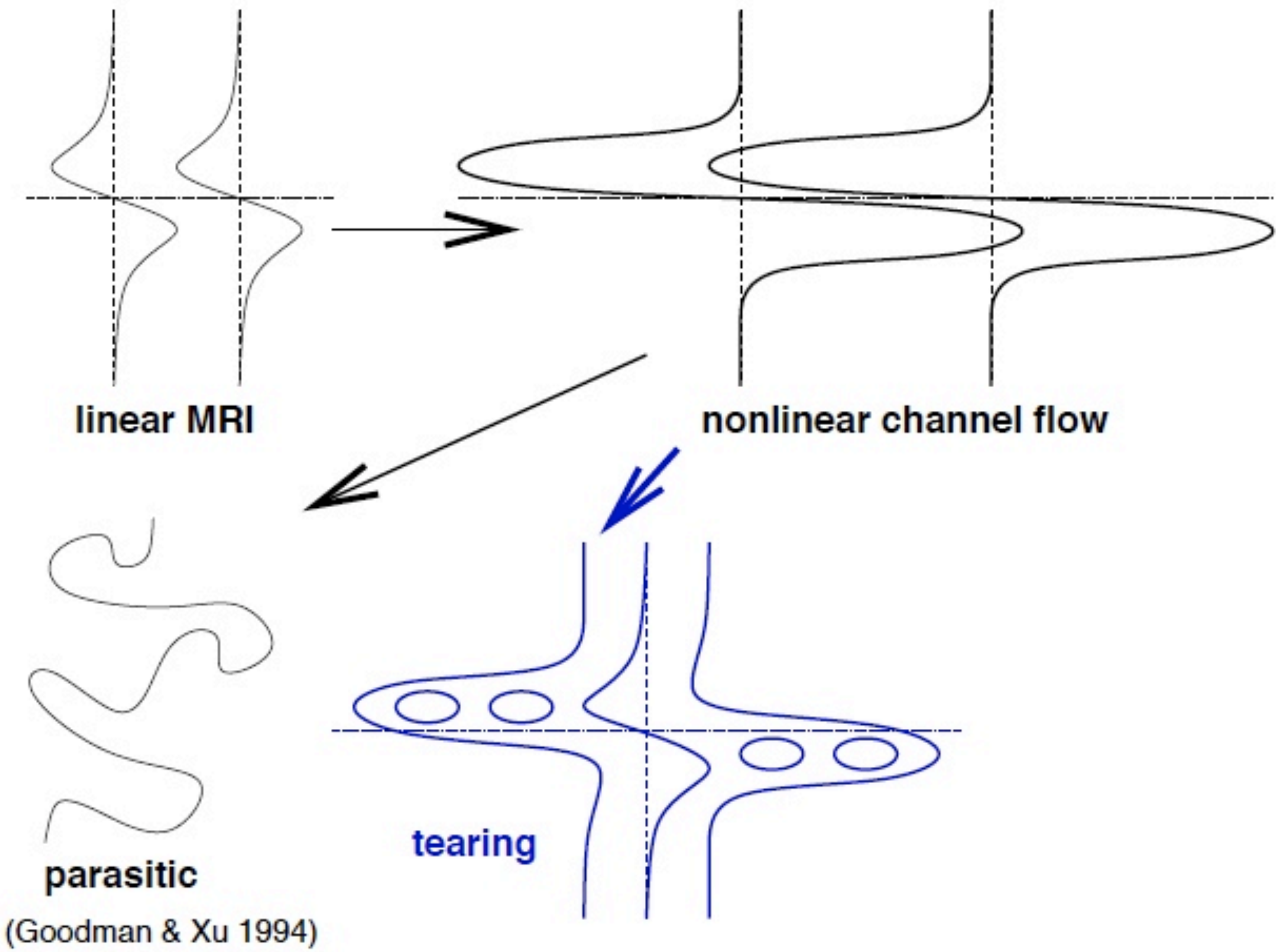
- Channel modes themselves become unstable to other, so-called ***secondary, parasitic instabilities*** *(Goodman & Xu 1994)*. Whereas MRI is powered by rotation with shear, these parasitic instabilities are powered by the kinetic bulk motion and magnetic energy of the velocity and magnetic fields generated by the MRI.
- Examples: GX94, KH, tearing, and, in stratified disks, Parker.

MRI, Channel Modes, and Parasites.



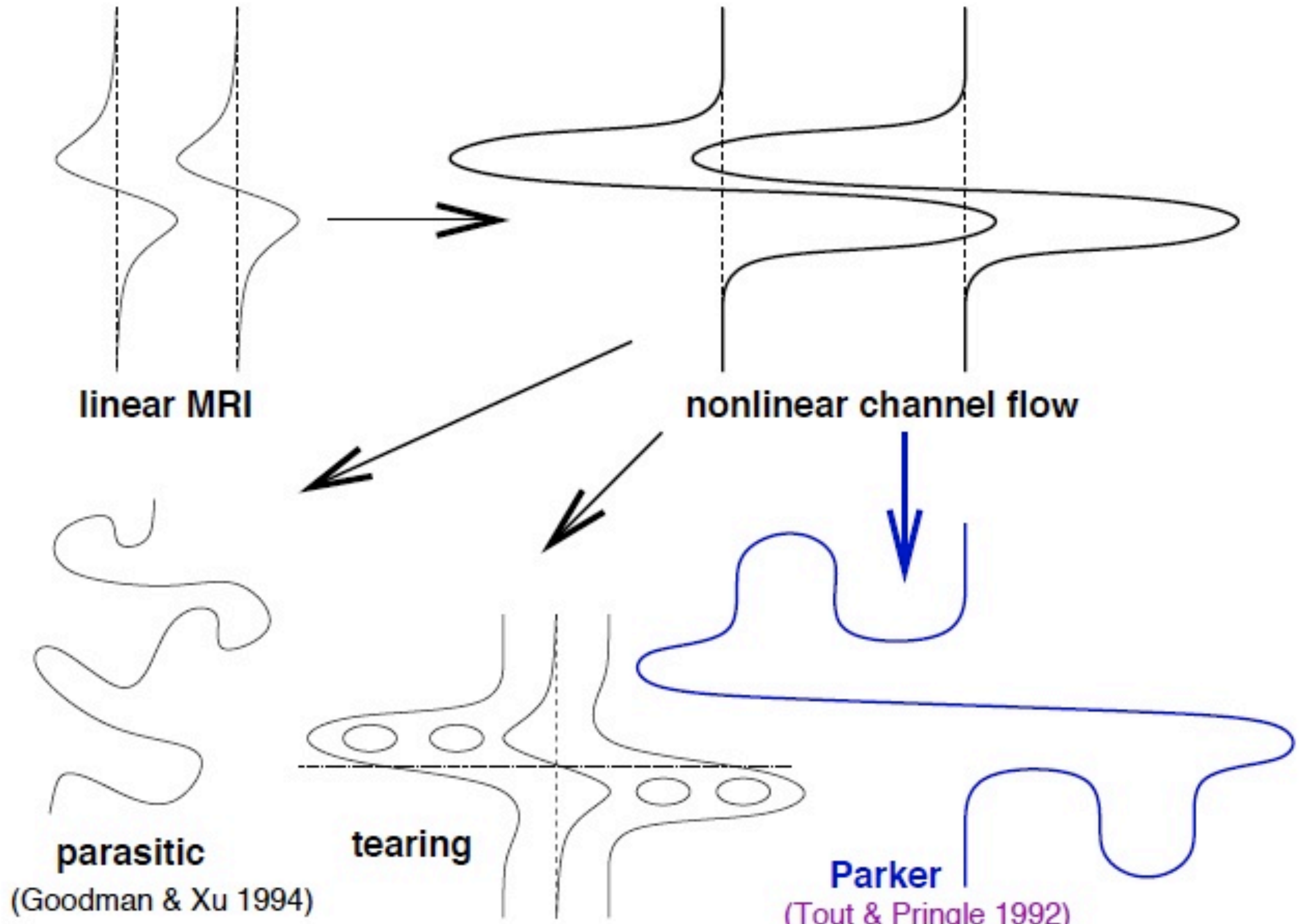

linear MRI
nonlinear channel flow
parasitic
(Goodman & Xu 1994)

MRI, Channel Modes, and Parasites.



(Pessah & Goodman)

MRI, Channel Modes, and Parasites.

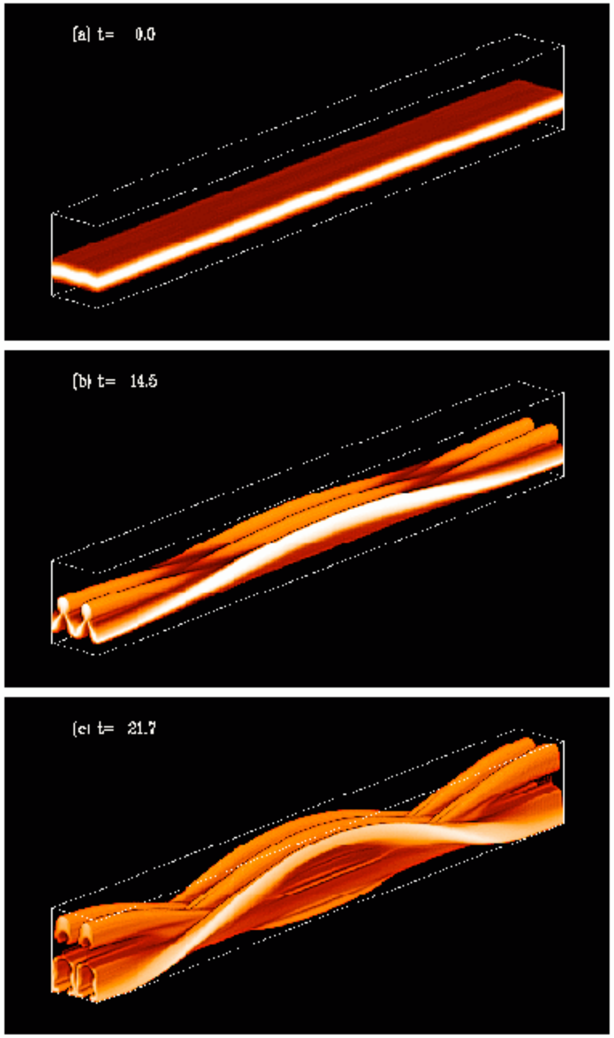


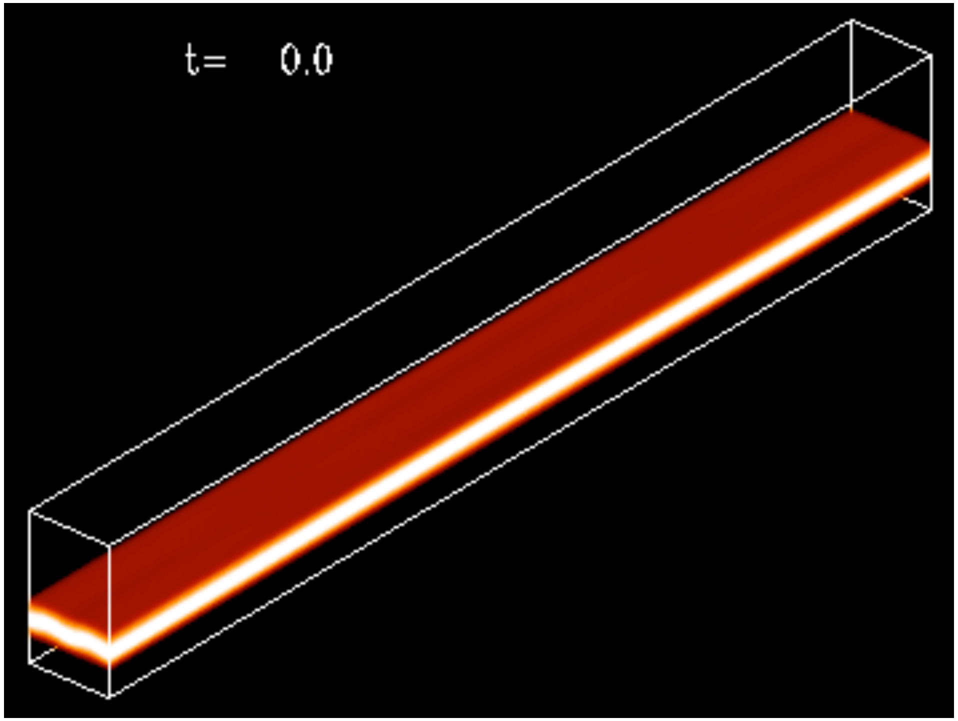
(Pessah & Goodman)

Parker instability

- Parker (1967): in context of galactic magnetic fields, but it is also important in many other situations, e.g., formation of the coronae of stars and accretion disks.
- Gravitationally stratified, relatively cold dense magnetized medium (solar conv. zone or accretion disk).
- The medium may be turbulent (e.g., due to MRI or buoyancy), magnetoactive.
- Horizontal magnetic fields are Parker unstable --- this is undulation/ballooning mode of the magneto-buoyancy instability.
- Ω-loops emerge into the overlying region, bring magnetic energy with them that then dissipates via other mechanisms (e.g, reconnection), and heats the corona.

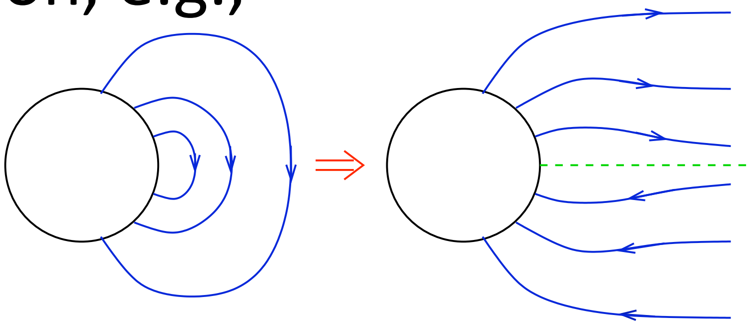
Parker Instability (cont'd)



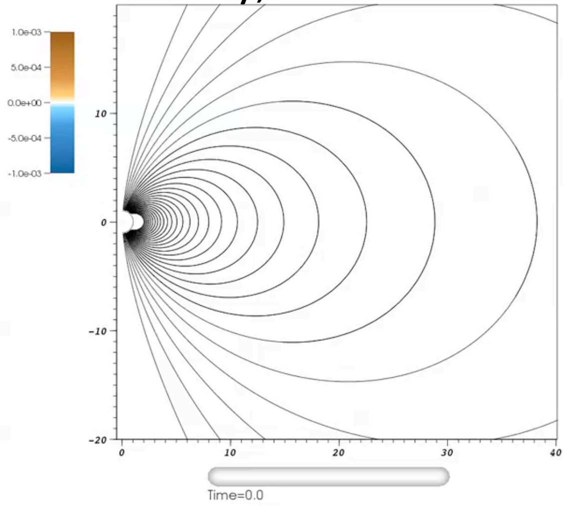


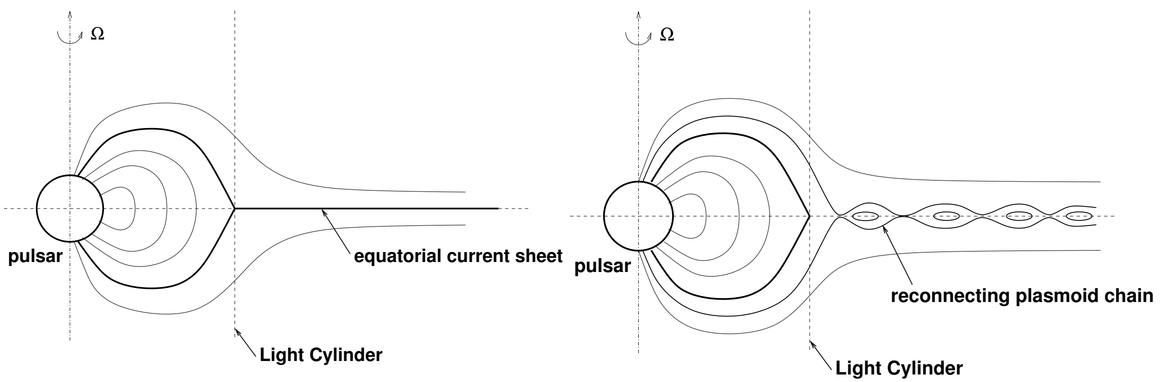
Tearing/reconnection

- Examples: current sheet formation, e.g., via field line opening:
- YSO star-disk interaction,
- Pulsar magnetosphere.
- there is a lot of free energy lying around in this field.



Parfrey, Beloborodov



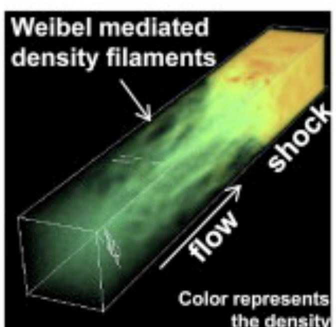


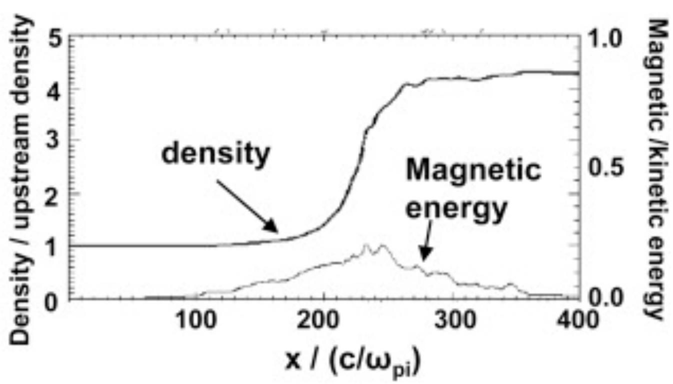
see talk by Loureiro

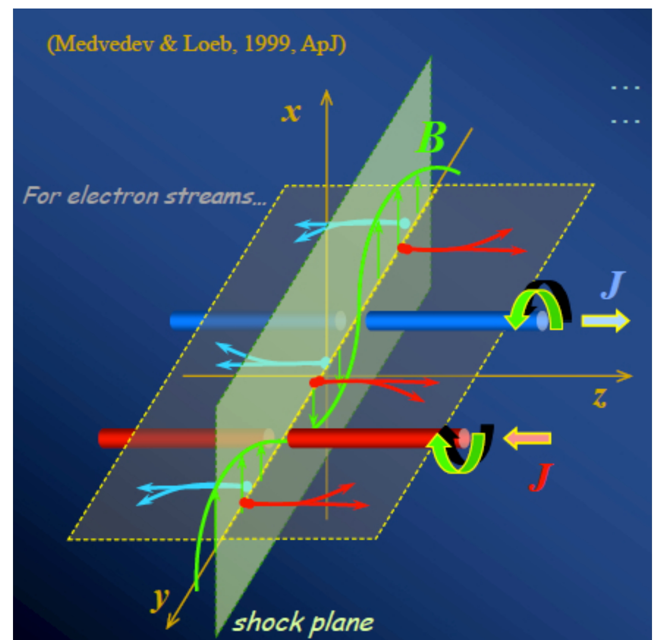
Microscopic kinetic instabilities: Weibel instability

- Collisionless shocks:
- shock: bulk kinetic energy → heat, entropy production.
- How is a shock mediated in the absence of collisions?
- Some "effective collisions": kinetic micro-turbulence.
- What generates this turbulence?
- Weibel (1959) Instability:

(electromagnetic streaming instability)

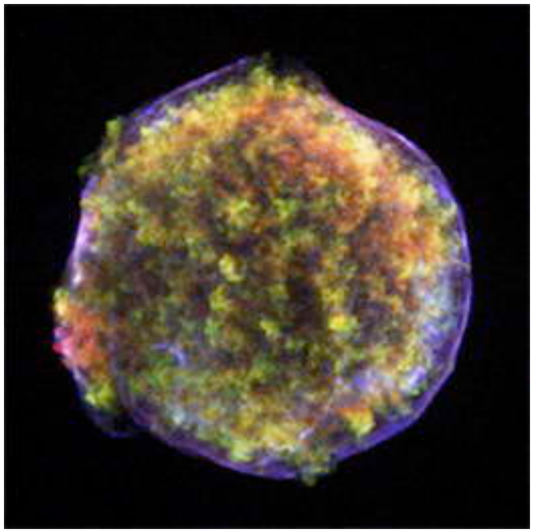




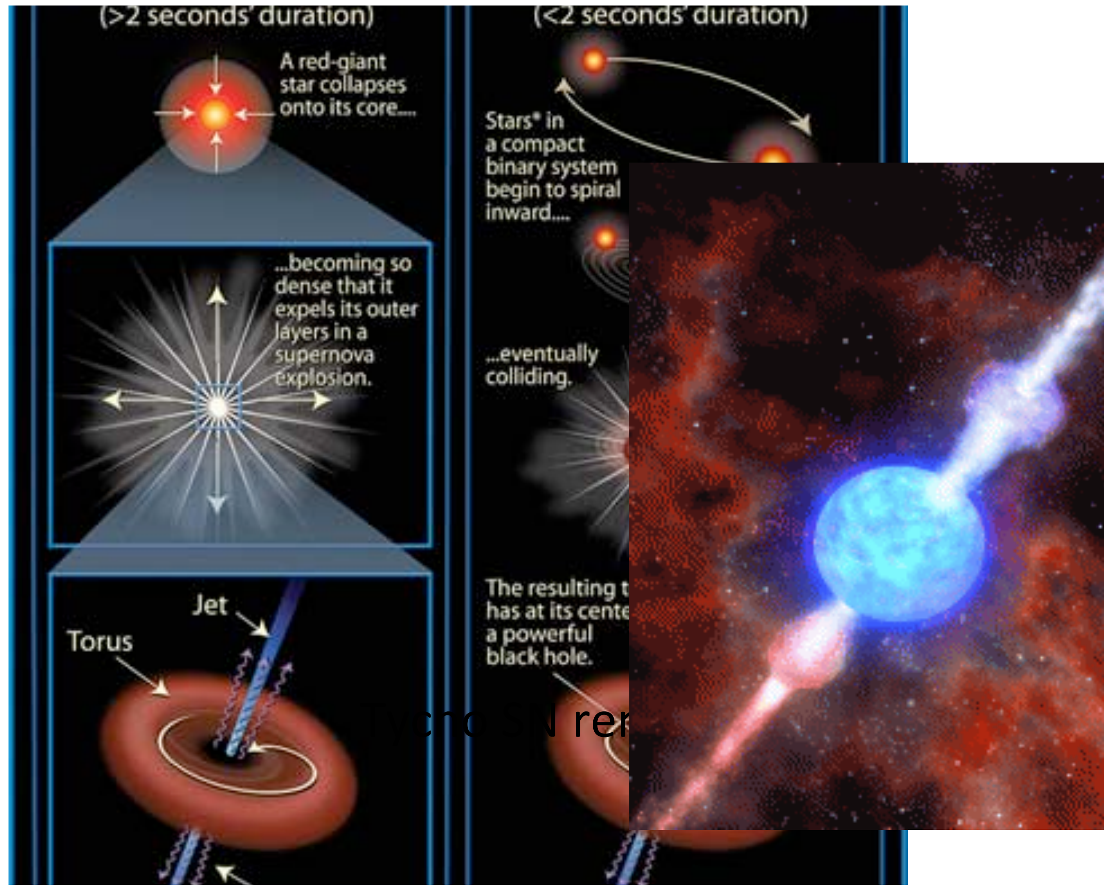


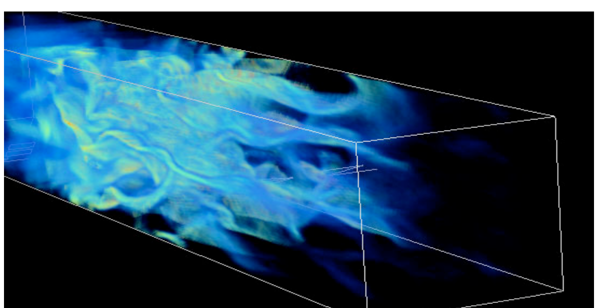
Astrophysical Collisionless Shocks

Supernova shocks (Cosmic Ray Acceleration)



Gamma-Ray Bursts (GRB)



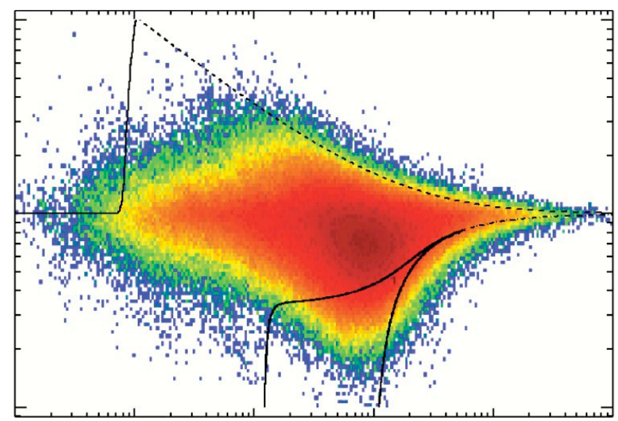


PIC simulation (Spitkovsky'08) of relativistic collisionless shock

Microscopic kinetic instabilities: Pressure-anisotropy-driven instabilities

- firehose/mirror:

$$\frac{\Delta P}{P} = \frac{P_\perp - P_\parallel}{P}$$



Summary

- Cosmic Plasmas are very complex, have many components…
- Plasma processes describe energy exchange among the components.
- Instabilities often initiate these processes.
- Linear stage: real systems are rarely set up in a highly unstable state, usually they cross the stability boundary gradually →

  $\delta x \sim \exp(at^2)$

- Nonlinear evolution -- many possibilities:
  - transition to another equilibrium (saturation(
  - transition to turbulence (e.g., via parasitic instabilities)
  - system disruption
- Many astrophysical examples….