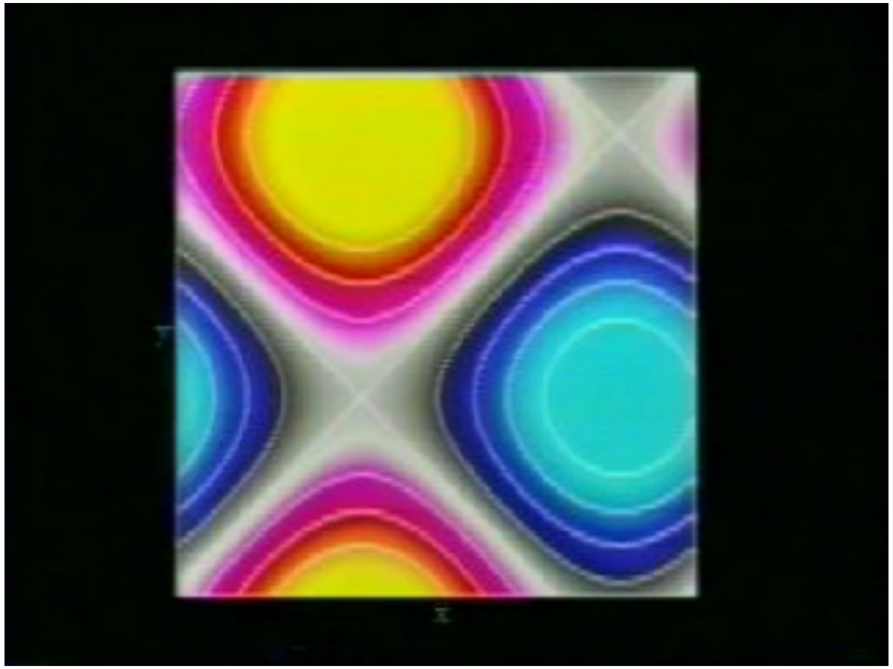
A brief introduction to dynamos



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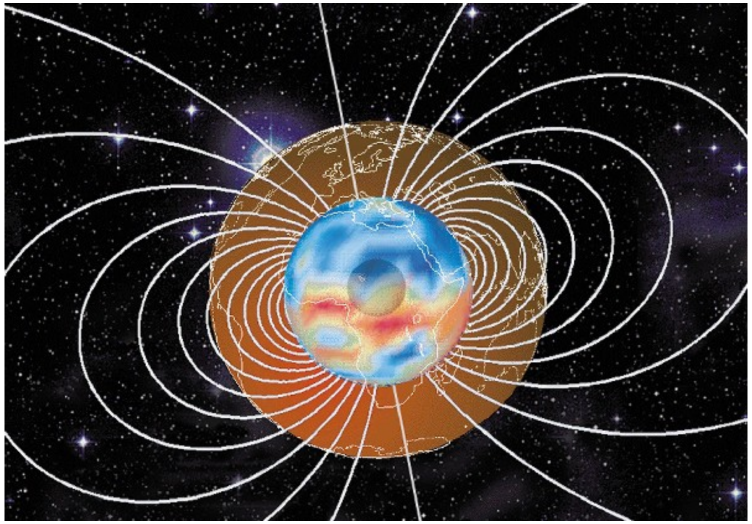
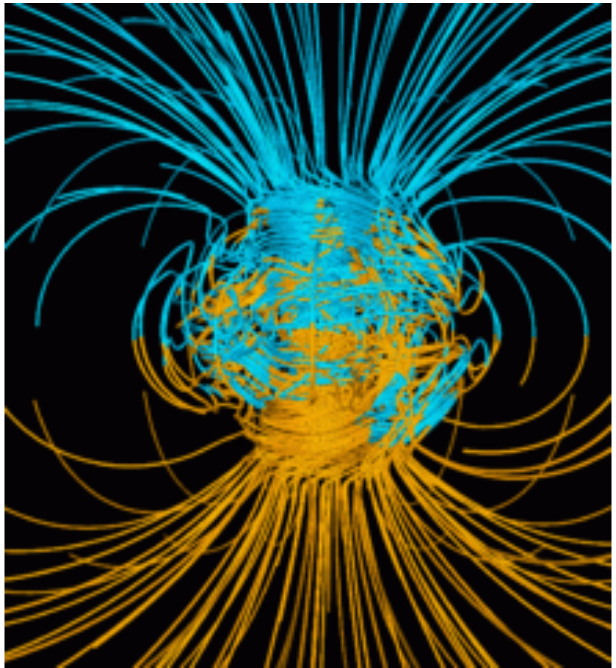
Astrophysical & Geophysical
Fluid Dynamics
DEPARTMENT OF APPLIED MATHEMATICS

UNIVERSITY OF LEEDS

Geophysical and Astrophysical Magnetism: The Earth

Earth has magnetic field!





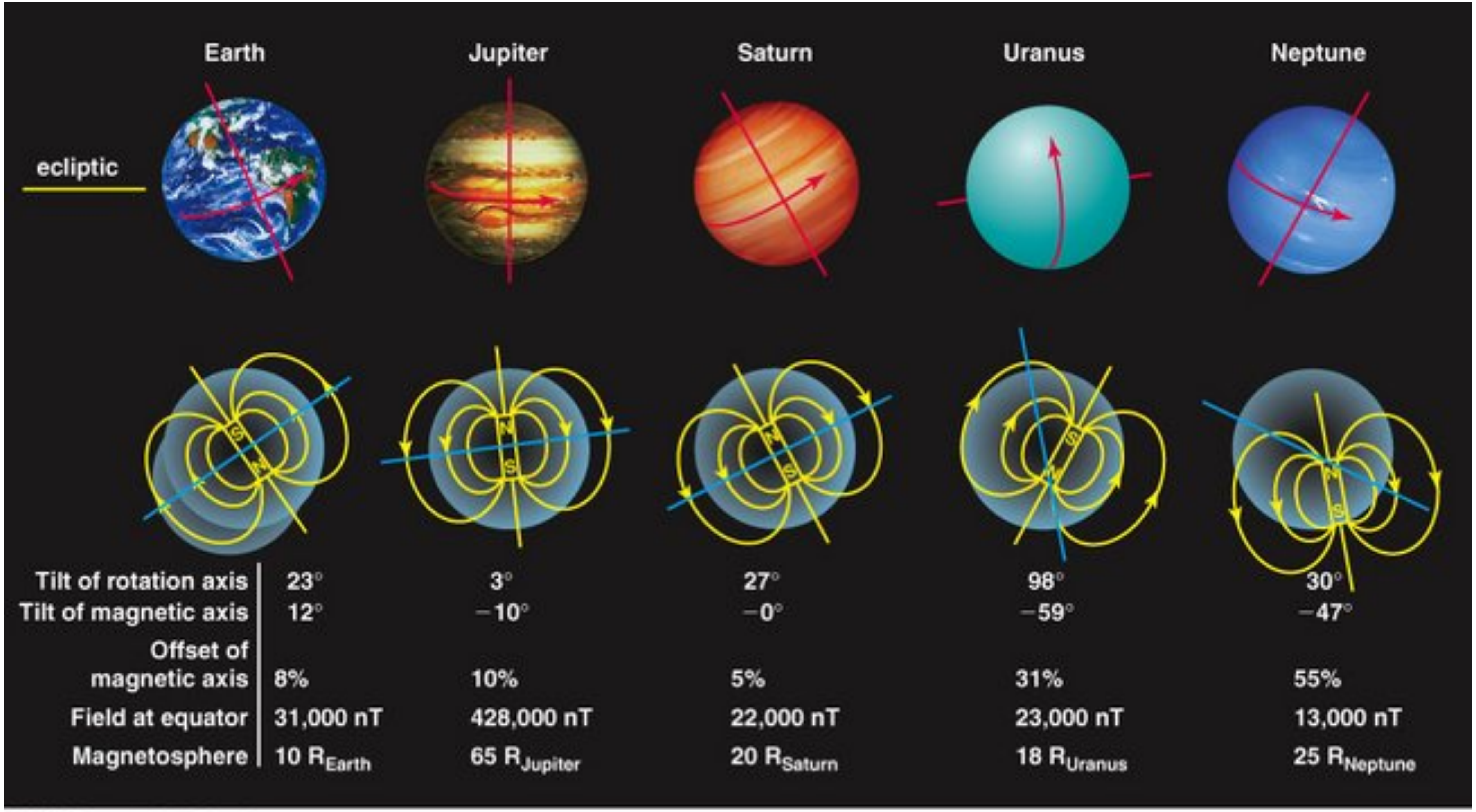
Paleomagnetism: around for long time

Largely Dipolar

Reverses every million or so years

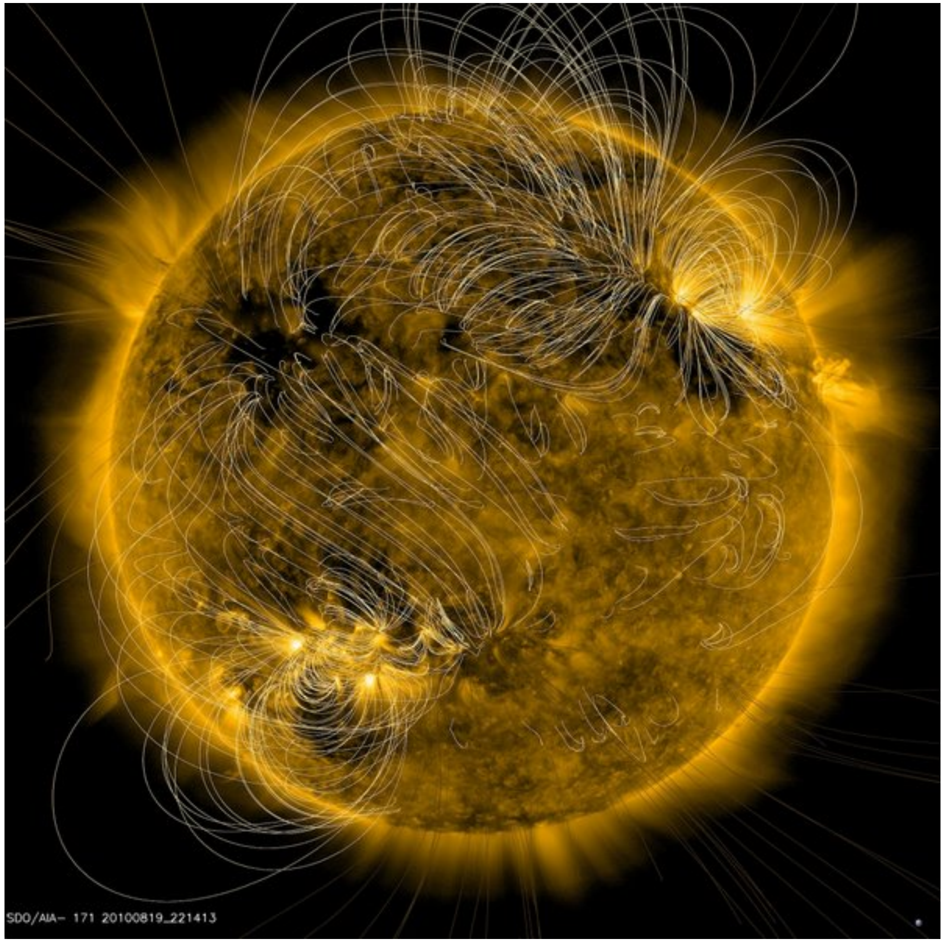
Geophysical and Astrophysical Magnetism: Solar System Planets

These tend to have magnetic fields



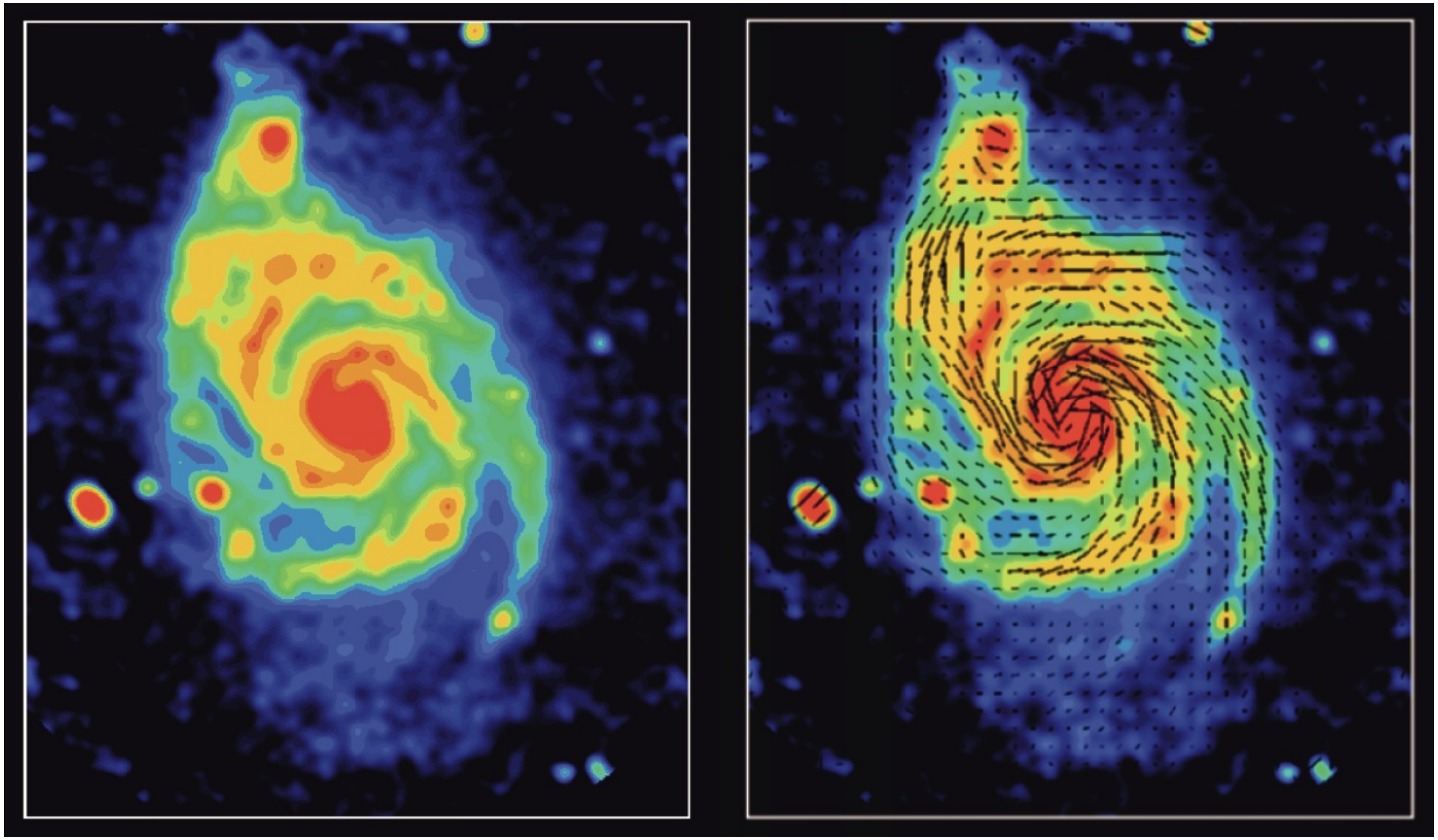
Geophysical and Astrophysical Magnetism: The Sun and Stars

These tend to have magnetic fields (more on this later)



Geophysical and Astrophysical Magnetism: Galaxies

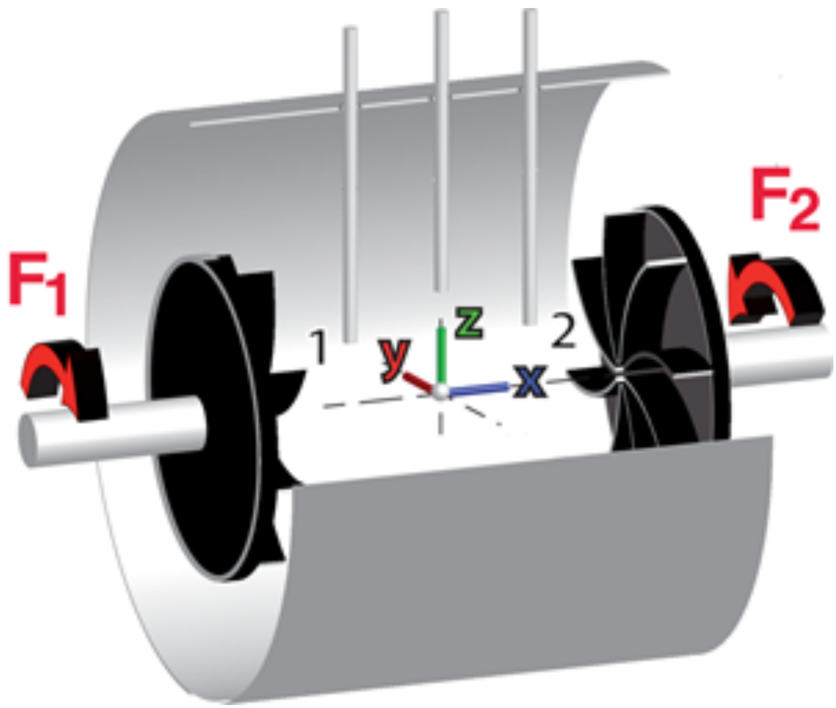
These tend to have magnetic fields

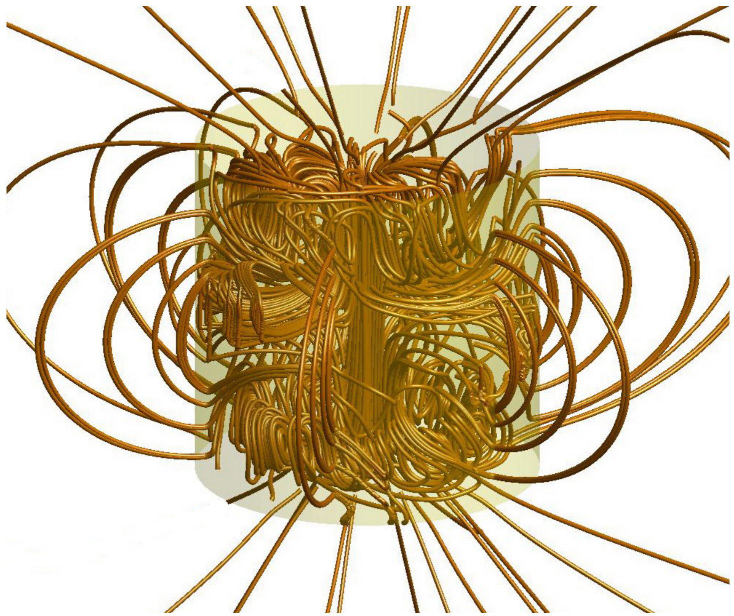


M51 Spiral Galaxy

Geophysical and Astrophysical Magnetism: Experiments

These tend ***not*** to have magnetic fields





VKS dynamo experiment

See Jean-Francois and Cary's talks

A brief introduction to dynamos

"In mathematics the art of proposing a question must be held of higher value than solving it"

Georg Cantor

"There are more questions than answers…"

Johnny Nash

A brief introduction to dynamos

Review of basics of dynamo theory:

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http://www-lgit.obs.ujf-grenoble.fr/houches/

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http://www1.maths.leeds.ac.uk/~smt/tcb_review13.pdf

The Induction Equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{Rm} \nabla^2 \mathbf{B}$$

$$Rm = \frac{T_d}{T_{turn}} = \frac{L^2/\eta}{L/U} = \frac{UL}{\eta}$$

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{B} = \underbrace{(\mathbf{B}.\nabla)\mathbf{u}}_{stretching} + \frac{1}{Rm} \nabla^2 \mathbf{B}$$

For what choices of u does B remain for large times? Clearly if u=0 then the field will decay away on diffusive timescale.

Magnetic Reynolds Number Rm

For geophysical and astrophysical flows Rm is large enough that stretching wins. In fact theoretical problems can arise because Rm is so large (see later)

Earth's Core	$10^2 - 10^3$
Jupiter	$\approx 10^5$
Solar Convection Zone	$\approx 10^8$
Galaxy	
Experiments	$\approx 10 - 10^2$

(though see Cary's talk for plasma Rm!)

Stretching vs Diffusion

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{B} = (\mathbf{B}.\nabla)\mathbf{u} + \frac{1}{Rm}\nabla^2\mathbf{B}$$

The induction equation is a linear problem in B.

For a given u solutions for the field either grow or decay exponentially (on average) with (average) growth-rate σ

Categorise flows into

those for which $\sigma > 0$ (kinematic dynamos)

Or... $\sigma < 0$ (non-dynamos)

… sounds easy.

Antidynamo Theorems

Cowling's Theorem (1934)

- An axisymmetric magnetic field can not be generated via dynamo action.
  - Field must be inherently 3D for $\sigma > 0$

Zel'dovich Theorem (1958)

- A planar velocity field is not capable of sustaining dynamo action

All calculations and numerics will have to be 3D.

Bounds on Rm

Backus (1958)

• Demonstrated that in a sphere that for dynamo action to occur

$$Rm = \frac{a^2 e_m}{\eta} > \pi^2$$

(Rm calculated on rate of strain)

Childress (1969) $$Rm = \frac{a^2 u_m}{\eta} > \pi$$

•Need enough stretching to overcome the diffusion

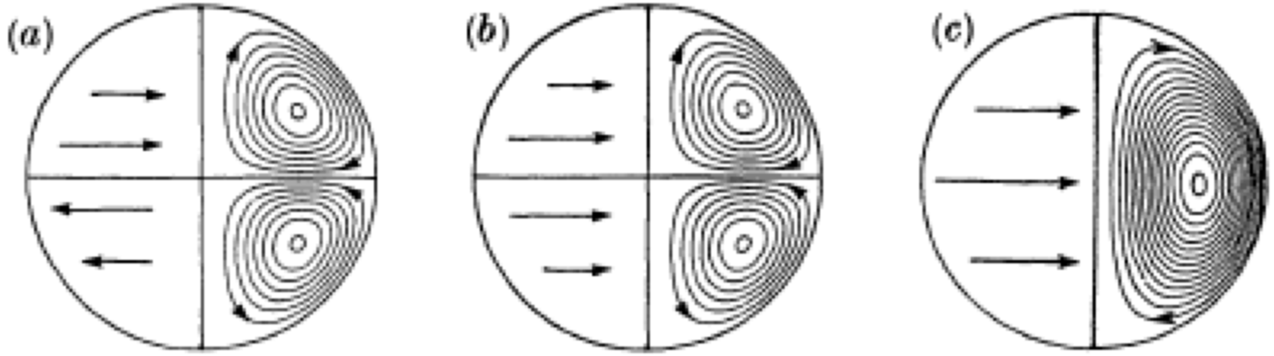
Numerical Spherical Dynamos

<u>Bullard & Gellman (1954)</u>

- Found numerical evidence of dynamos with a non-axisymmetric velocity
    - Higher resolution calculations proved these not to be dynamos
    - Under-resolving dynamos can seriously damage your health!
        - Give dynamos where none exists.

Spherical Dynamos

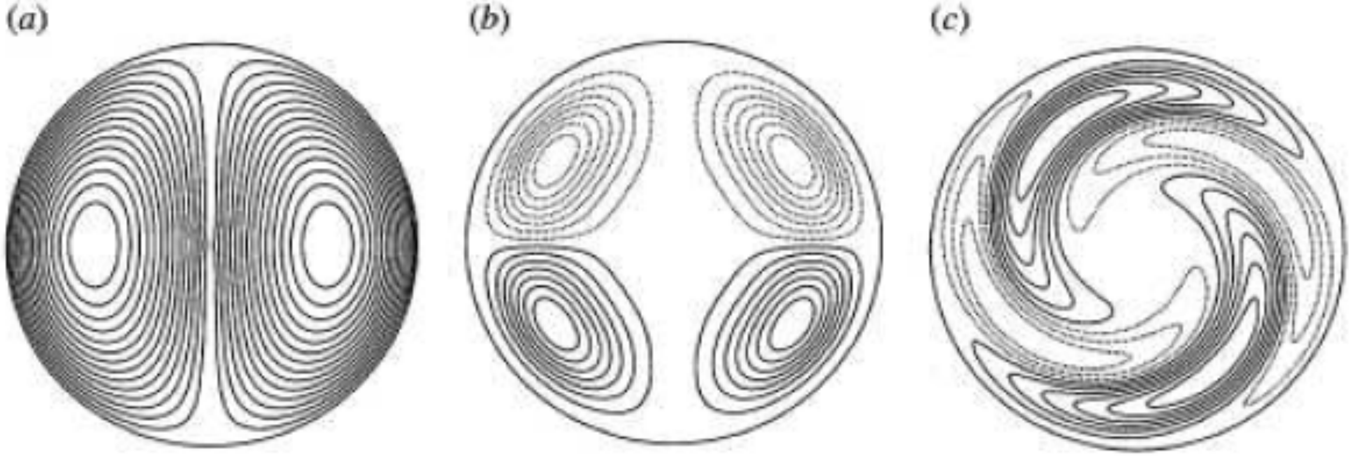
Dudley & James (1989)



$$\mathbf{u} = \mathbf{t}_2^0 + \epsilon \mathbf{s}_2^0 \ (a), \quad \mathbf{u} = \mathbf{t}_1^0 + \epsilon \mathbf{s}_2^0 \ (b), \quad \mathbf{u} = \mathbf{t}_1^0 + \epsilon \mathbf{s}_1^0 \ (c)$$

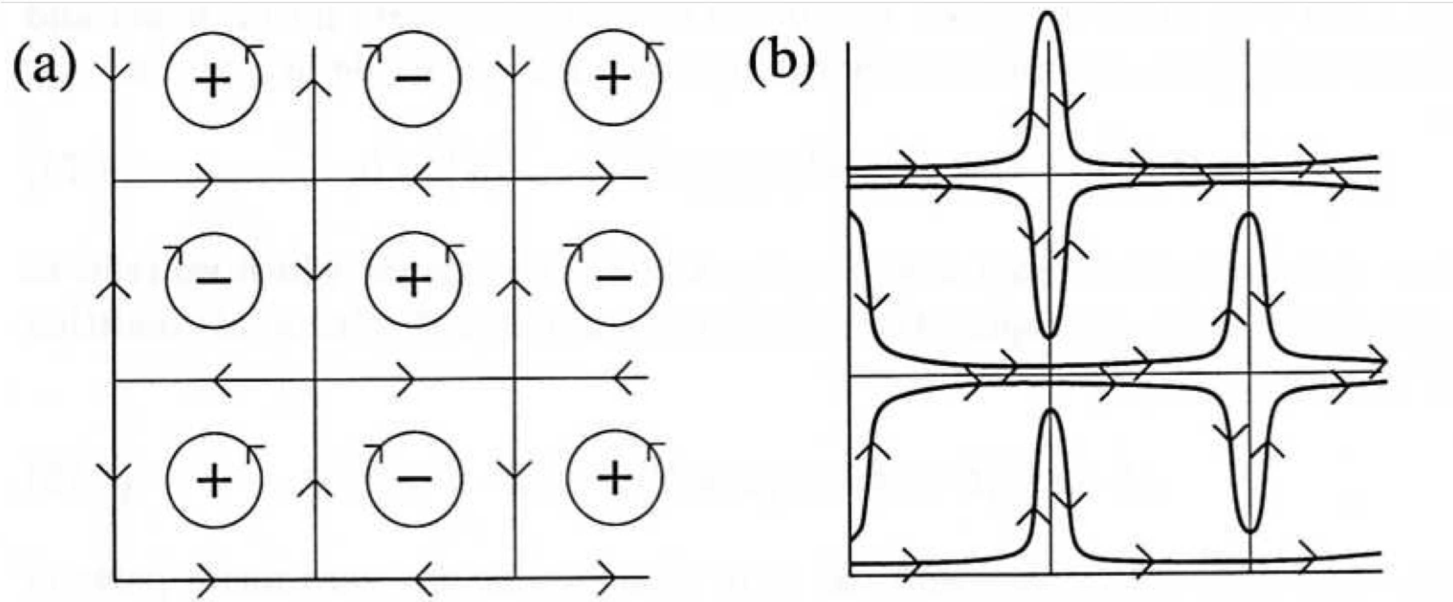
Kumar & Roberts

$$\mathbf{u} = \epsilon_0 \mathbf{t}_1^0 + \epsilon_1 \mathbf{s}_2^0 + \epsilon_2 \mathbf{s}_2^{2c} + \epsilon_3 \mathbf{s}_2^{2s}$$



Cellular Dynamos

G.O. Roberts(1970,1972)

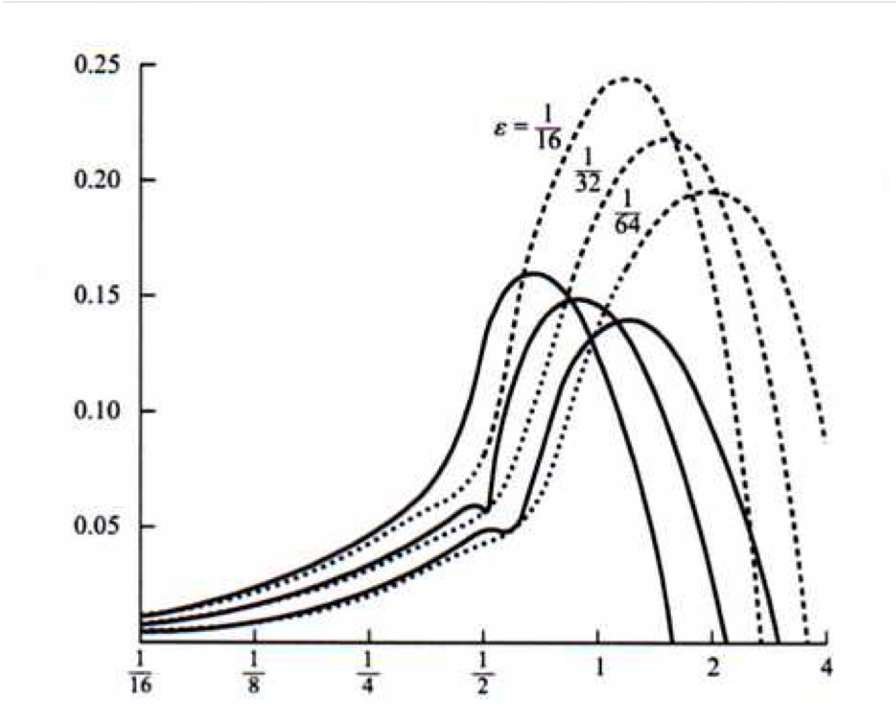


$$\mathbf{u} = (\psi_y, -\psi_x, \sqrt{2}\psi), \quad \psi = \sin x \sin y$$

2.5 dimensional integrable flow
Can solve for monochromatic solutions in z.
Essentially a 2D-problem

Cellular Dynamos

G.O. Roberts(1970)



Good dynamo at low to moderate Rm.

Fast Dynamos

Vainshtein & Zel'dovich (1978) Childress & Gilbert (1995)

- Easy to get dynamos now
- The "Fast Dynamo problem" is can flows be found for which

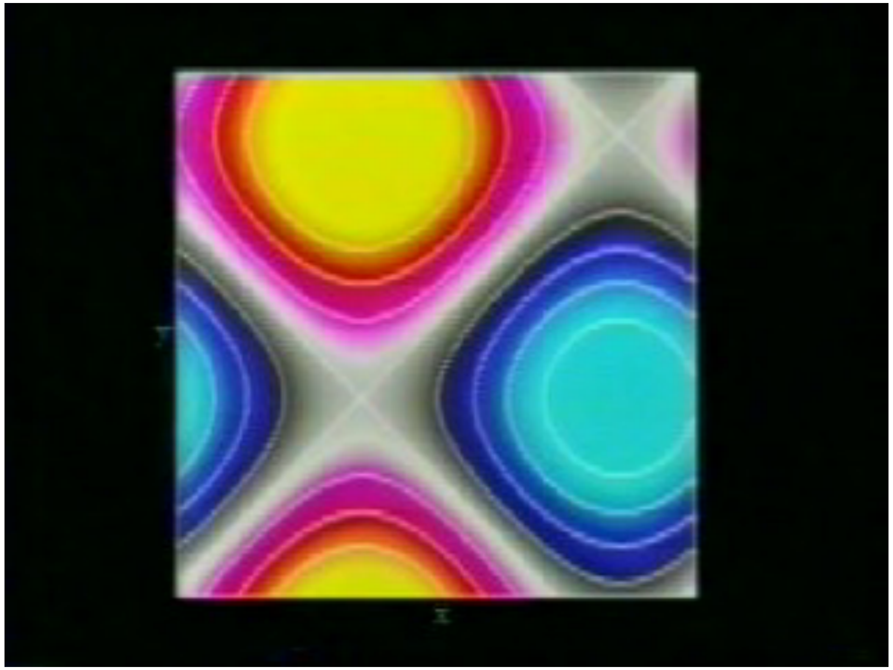
$$\sigma > 0 \text{ as } Rm \to \infty$$

- Growth-rate independent of diffusivity as the diffusivity becomes small.
- Clearly important for some astrophysical flows
- In order for this to occur the flow must have chaotic particle paths

Numerical Fast Dynamos

<u>Otani (1993) Galloway & Proctor (1992)</u>

- Can get chaos by making flows three-dimensional or time-dependent

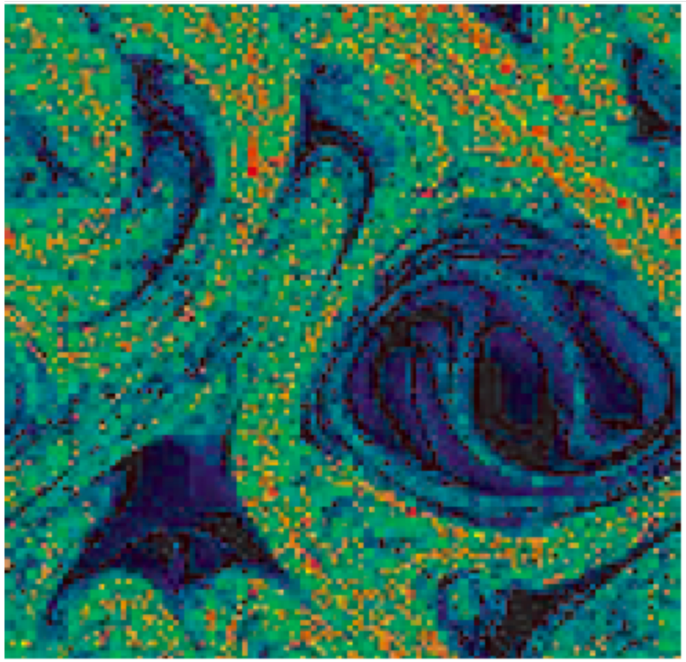


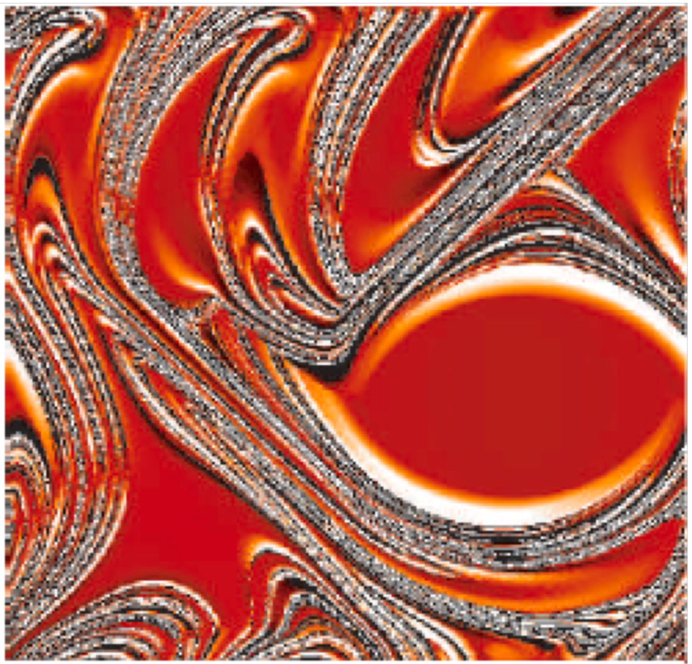
Numerical Fast Dynamos

<u>Galloway & Proctor (1992)</u>

- Can get chaos by making flows three-dimensional or time-dependent

Λ_1





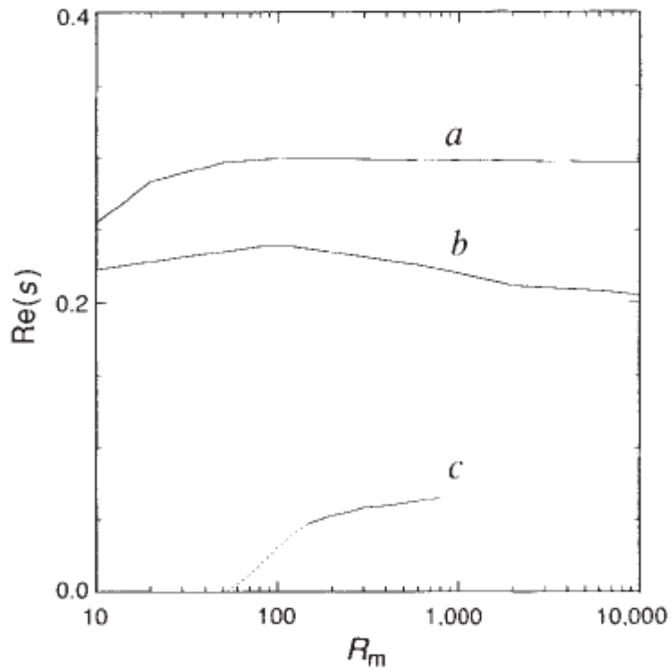
B_z

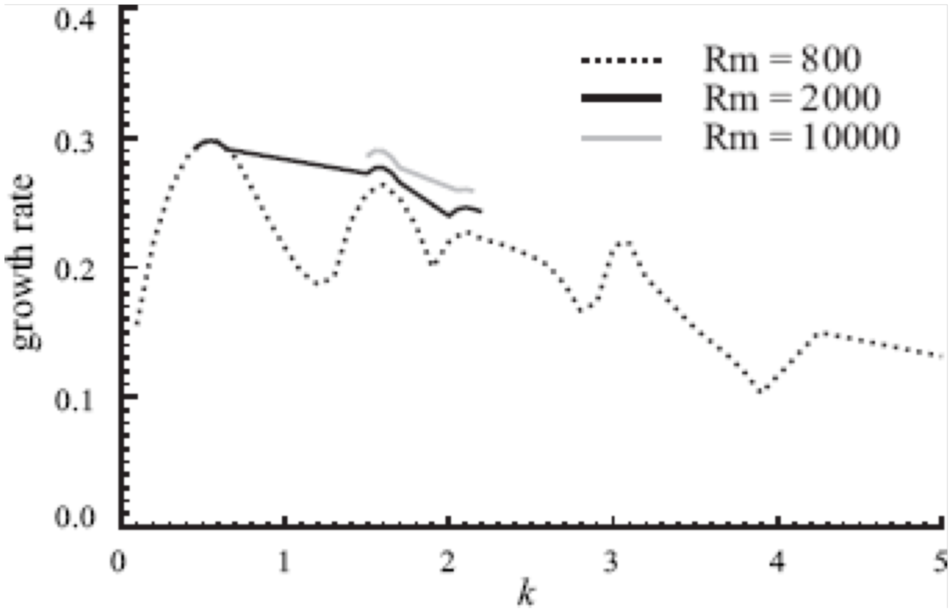
Here stretching beats the small diffusion and field is generated on a small scale $\propto Rm^{-1/2}$

Numerical Fast Dynamos

Galloway & Proctor (1992)

• Growth rate rapidly becomes independent of Rm





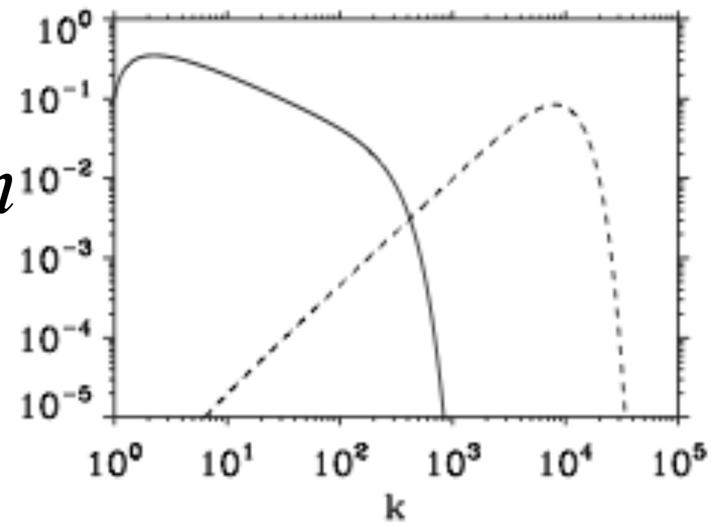
• and appears to stay independent of Rm as Rm gets large.

Turbulent Dynamos

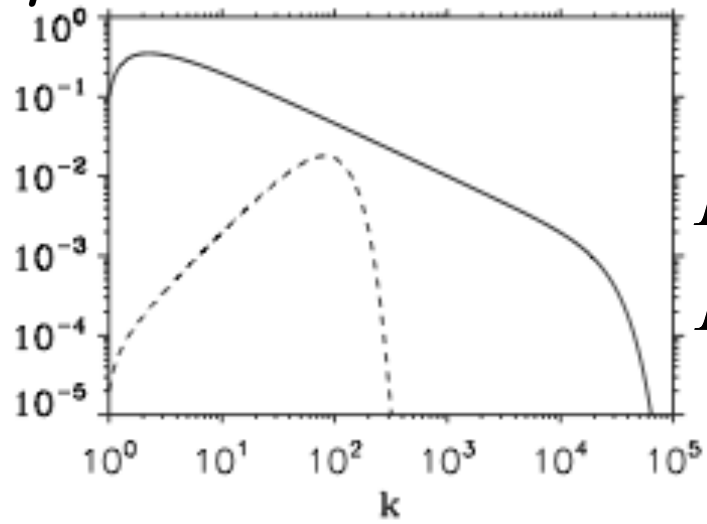
- In astrophysics, geophysics and experiments the flow is not usually at one scale!
- Usually high Re and turbulent. Many scales in the flow and the field.
- Two important cases: high and low Pm

$$Pm = \nu/\eta$$

High Pm



Galaxies/clusters



Low Pm

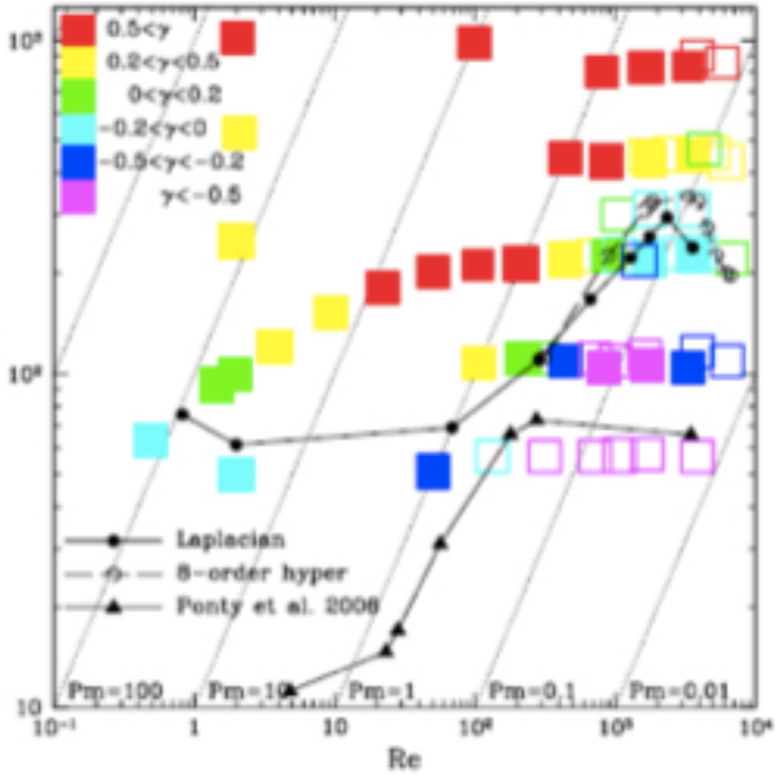
Planets/Stars/Metal Experiments

Low Pm Dynamos

- For high Pm dynamos there is no problem (except field is generated on very small scales)
- For low Pm dynamos the magnetic field dissipates in the inertial range of the turbulence.
- This may (and does) lead to an inhibition of dynamo action so the critical Rm goes up as Pm goes down.
- Eventually (though possibly not soon enough for the experiments) the critical Rm becomes independent of Pm.
  - Short correlation time (Kazantsev) flows (Boldyrev & Cattaneo 2004)

Low Pm Dynamos

- Numerical Demonstration



•*Schekochihin et al. (2007)*
•Also shown for turbulent dynamos with coherent structures by *Tobias & Cattaneo (2008)*

Dynamo Saturation

- Eventually the magnetic field will grow to a level where it is large enough to modify the flow that is generating it.
    - Good question is when this happens
- At this point the dynamo will saturate and the exponential growth will stop.
- How this is achieved is an open question and may be different in different situations

Dynamo Saturation

- Possible Mechanisms include
  - Scale-by scale suppression of the flows
  - Renormalisation of the flow to its marginal state
  - Modification of the chaotic properties of the flow
  - Back-reaction on the driving mechanisms
- None of these seem to work in all cases (see e.g. Cattaneo & Tobias 2009 who showed that the exponential growth only stops for a set of vector fields of measure zero.)

Systematic "Large-Scale Dynamos"

- So far…
  - can we get dynamos, growth of magnetic energy for a given flow.

- Astrophysics/Geophysics
  - We want to get a large-scale systematic dynamo, with a significant fraction of the energy on scales comparable with the size of the astrophysical object.
- Use as a paradigm the solar dynamo…

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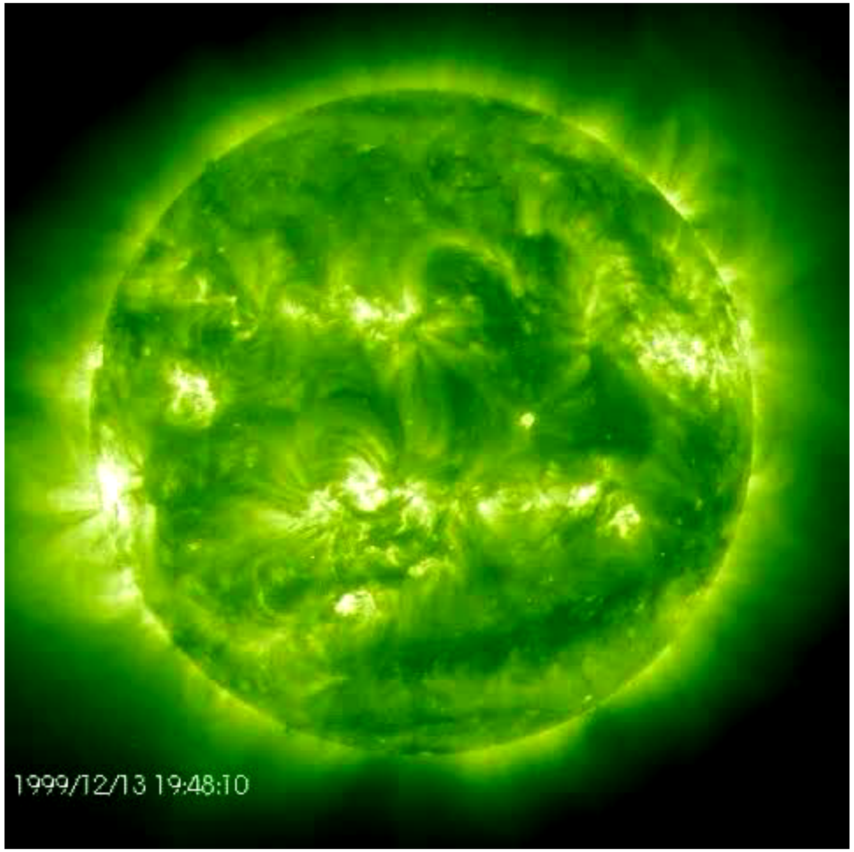
Solar and Stellar Dynamos



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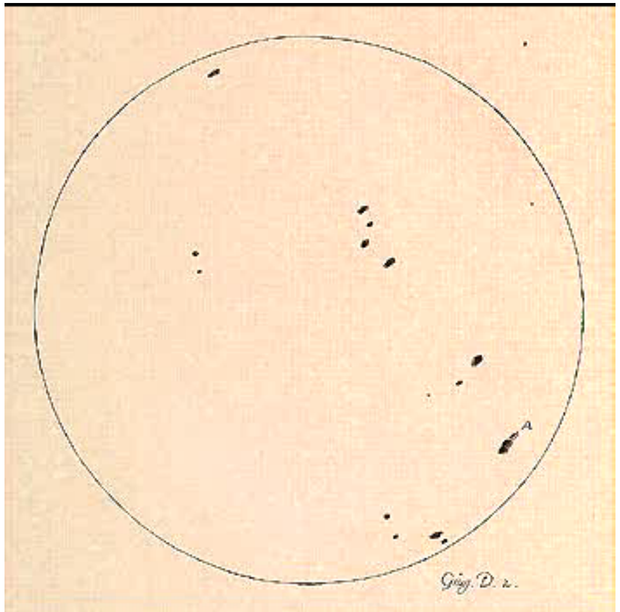
Talk Plan

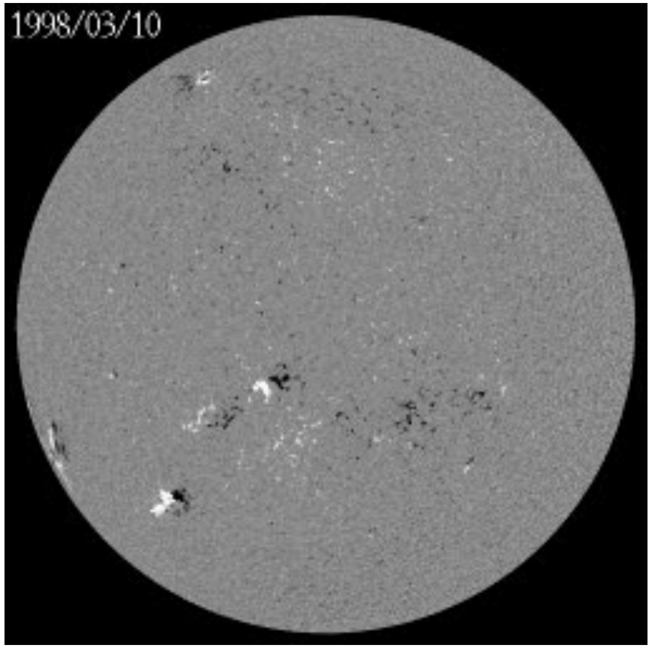
- Observations
- Modelling
- Mean-Field Electrodynamics
  - Basic theory
  - Problems
- Dynamo Scenarios and Computations

Reviews

- Ossendrijver (2003)
- Charbonneau (Living Reviews in Solar Physics)
- Tobias & Weiss (2007) in "Mathematical Aspects of Natural Dynamos",

Observations: Solar





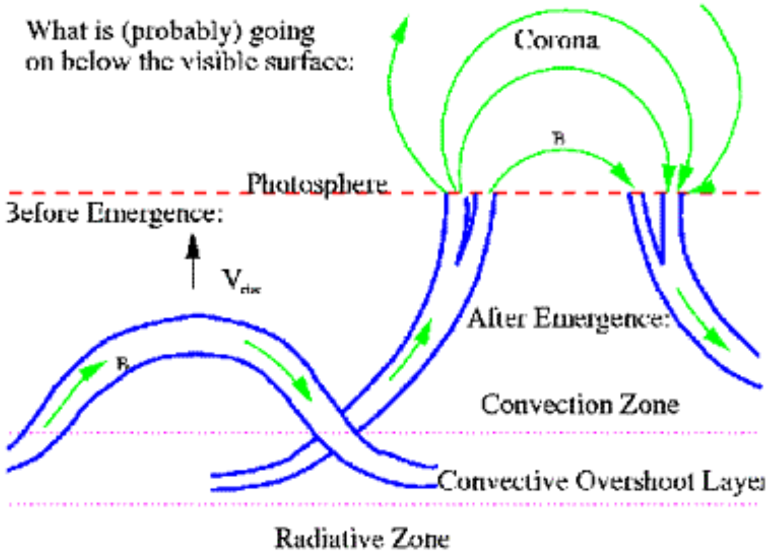
Magnetogram of solar surface shows radial component of the Sun's magnetic field.

Active regions: Sunspot pairs and sunspot groups.
Strong magnetic fields seen in an equatorial band (within 30^o of equator).

Rotate with sun differentially.

Each individual sunspot lives ~ 1 month.
As "cycle progresses" appear closer to the equator.

Observations Solar (a bit of theory)

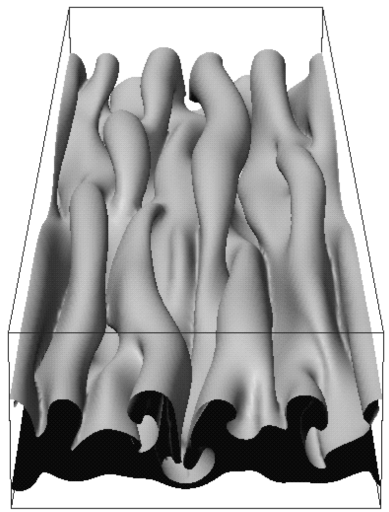


Sunspot pairs are believed to be formed by the instability of a magnetic field generated deep within the Sun.

Flux tube rises and breaks through the solar surface forming active regions.

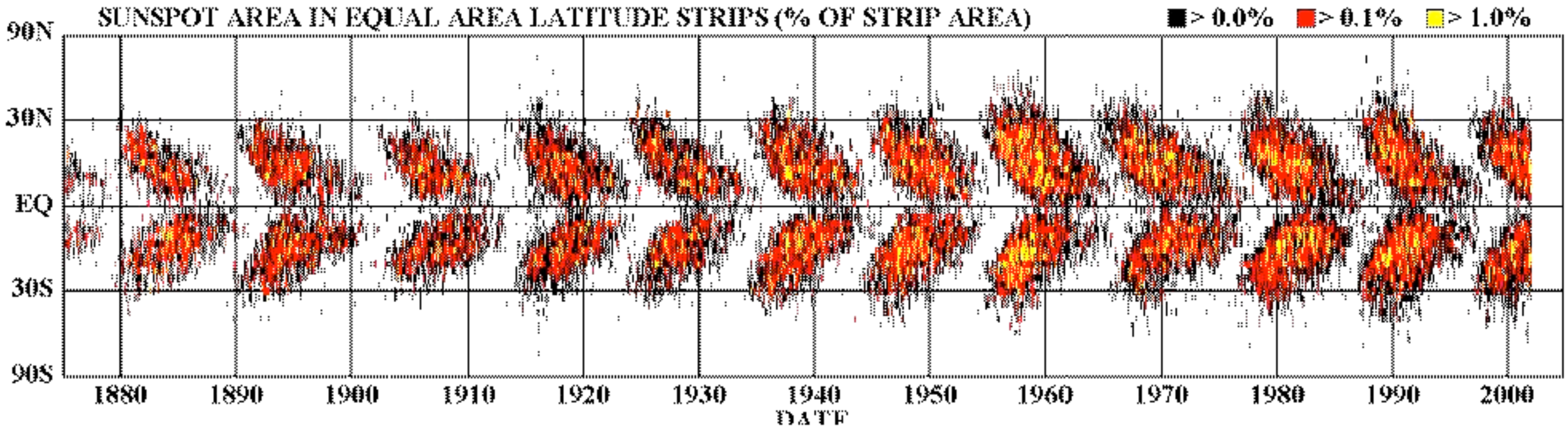
This instability is known as Magnetic Buoyancy.

It is also important in Galaxies and Accretion Disks and Other Stars.



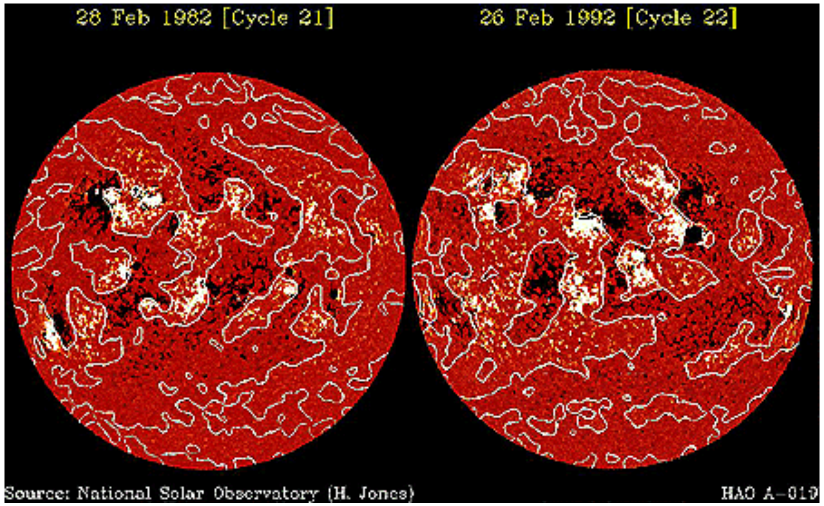
Wissink et al (2000)

Observations: Solar



BUTTERFLY DIAGRAM: last 130 years

Migration of dynamo activity from mid-latitudes to equator



Polarity of sunspots opposite in each hemisphere (Hale's polarity law).

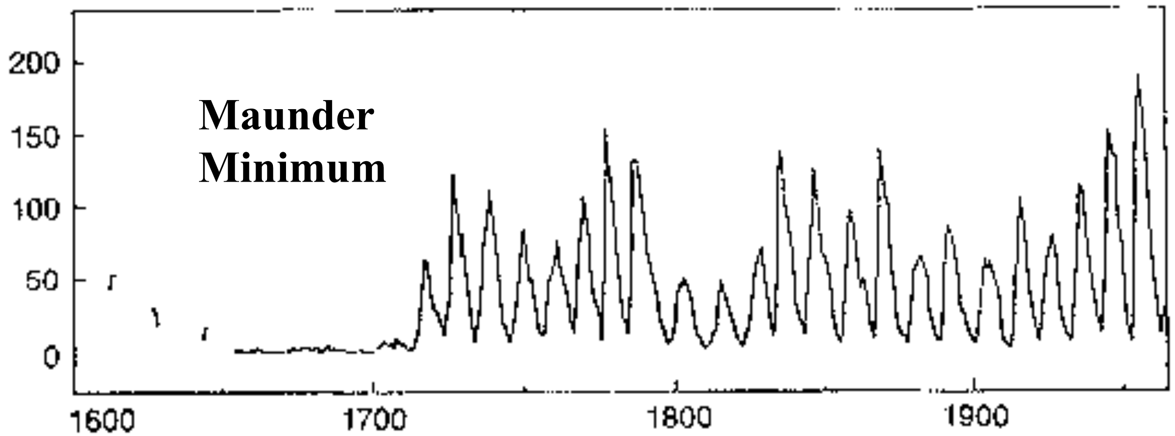
Tend to arise in "active longitudes"

DIPOLAR MAGNETIC FIELD

Polarity of magnetic field reverses every 11 years.

22 year magnetic cycle.

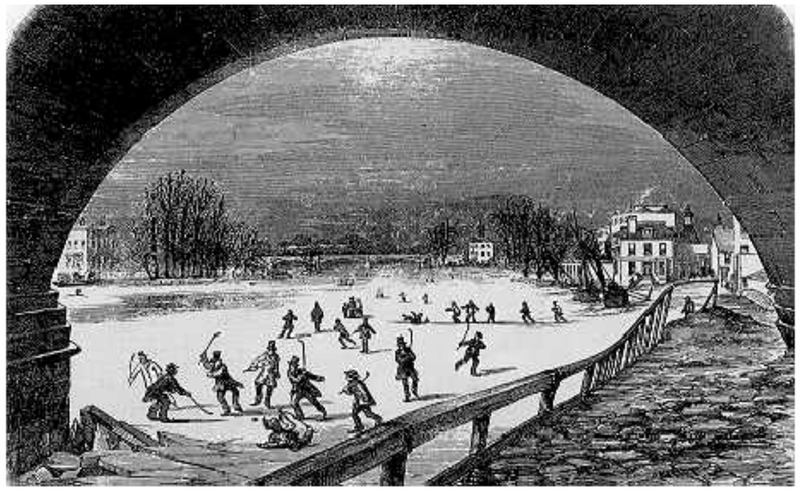
Observations: Solar



SUNSPOT NUMBER:
last 400 years

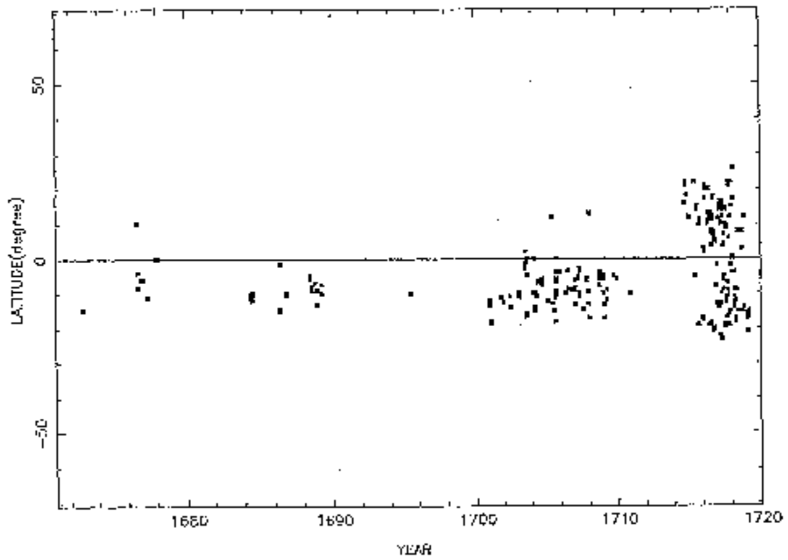
Modulation of basic cycle amplitude (some modulation of frequency)
Gleissberg Cycle: ~80 year modulation
MAUNDER MINIMUM: Very Few Spots , Lasted a few cycles
Coincided with little Ice Age on Earth

Abraham Hondius (1684)



Observations: Solar

RIBES & NESME-RIBES (1994)



BUTTERFLY DIAGRAM: as Sun emerged from minimum

Sunspots only seen in Southern Hemisphere

Asymmetry; Symmetry soon re-established.

No Longer Dipolar?

Hence: (Anti)-Symmetric modulation when field is STRONG

Asymmetric modulation when field is weak

Observations: Solar (Proxy)

PROXY DATA OF SOLAR MAGNETIC ACTIVITY AVAILABLE

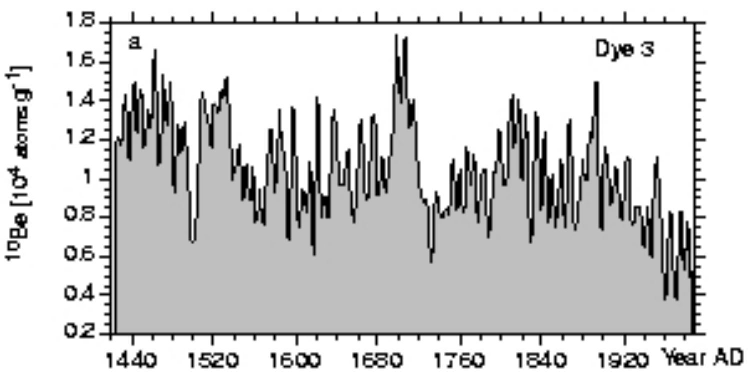
SOLAR MAGNETIC FIELD MODULATES AMOUNT OF COSMIC RAYS REACHING EARTH

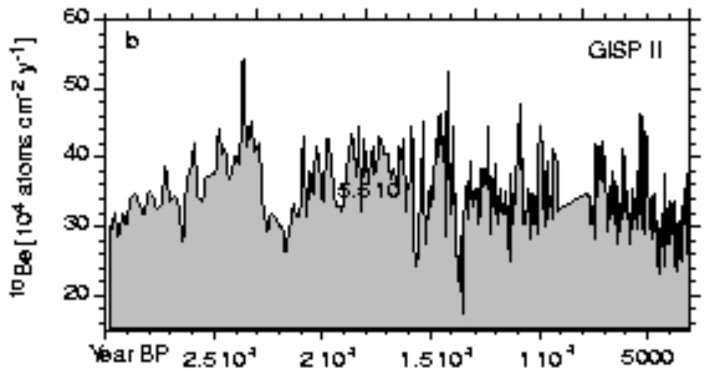
responsible for production of terrestrial isotopes

^{10}Be ^{14}C

^{10}Be : stored in ice cores after 2 years in atmosphere

^{14}C : stored in tree rings after ~30 yrs in atmosphere

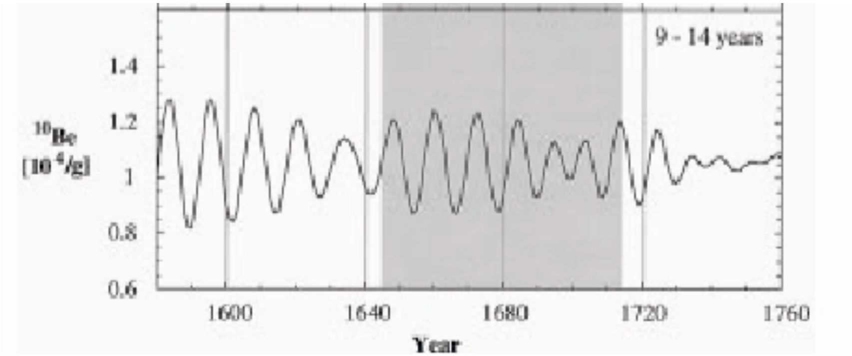




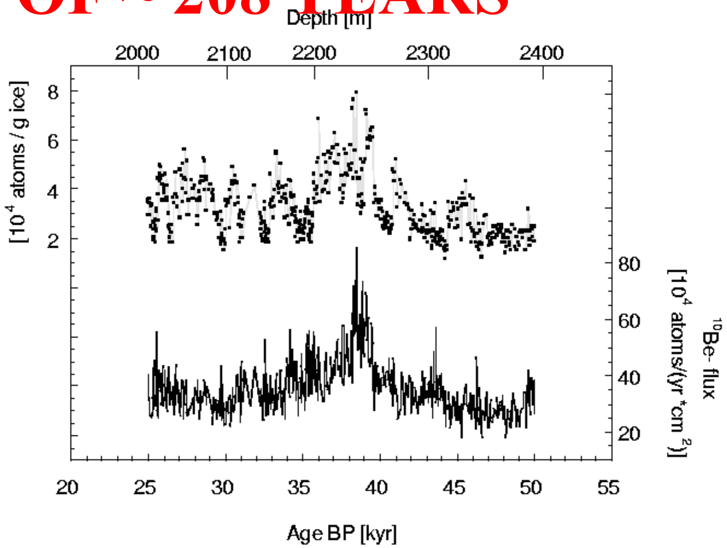
BEER (2000)

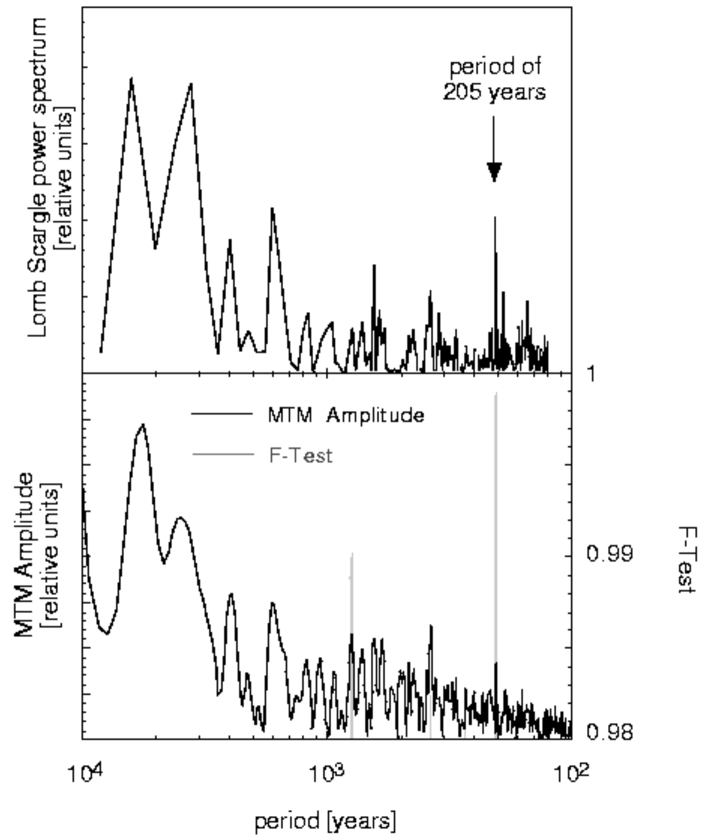
Observations: Solar (Proxy)

Cycle persists through Maunder Minimum (Beer et al 1998)



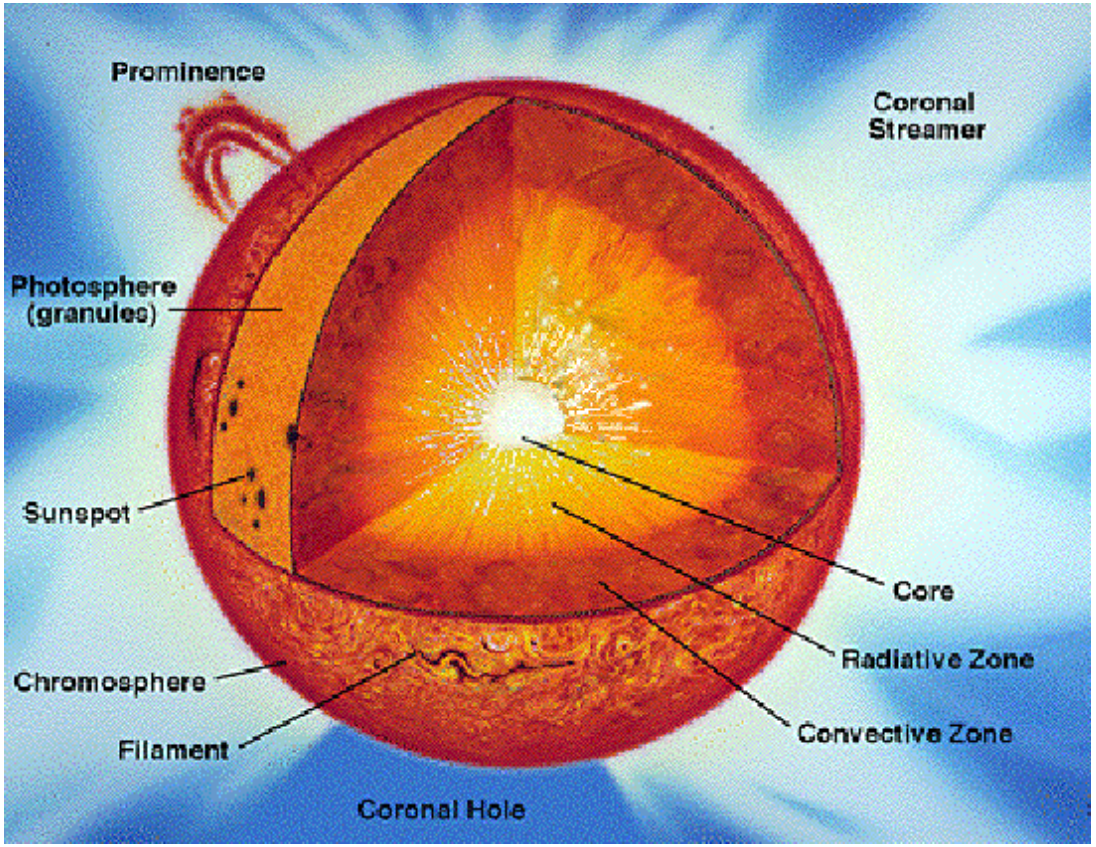
DATA SHOWS RECURRENT GRAND MINIMA WITH A WELL DEFINED PERIOD OF ~ 208 YEARS





Wagner et al (2001)

Solar Structure



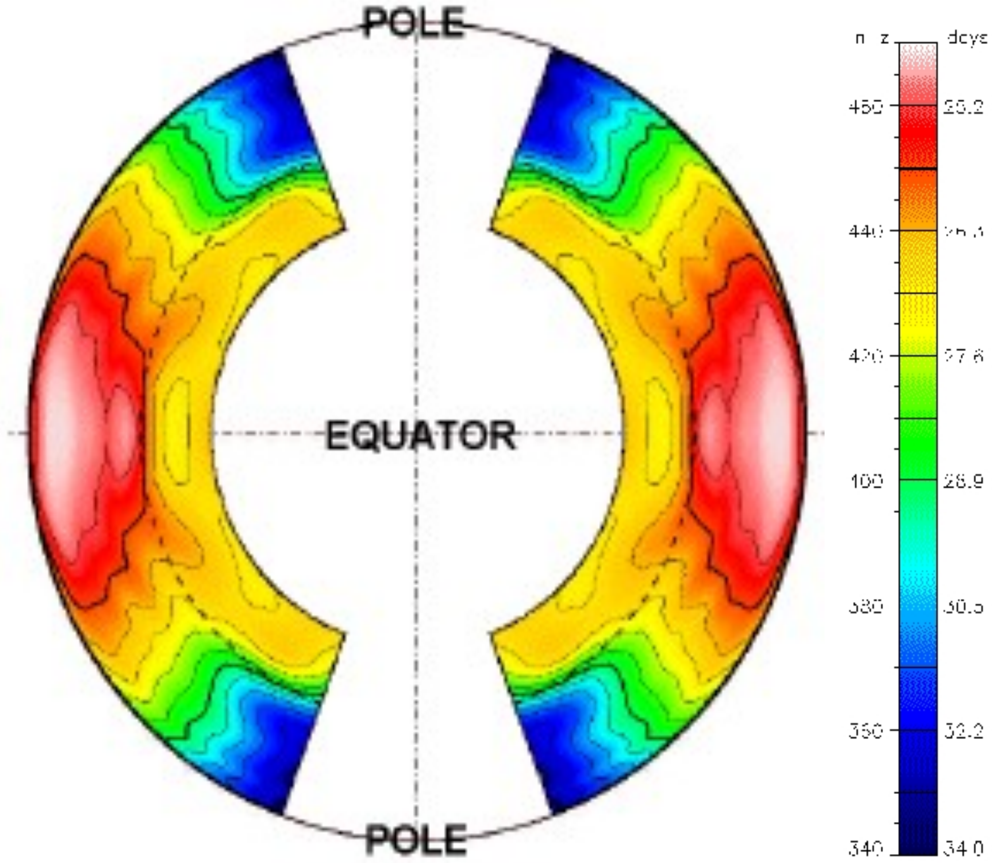
Solar Interior

1. Core
2. Radiative Interior
3. (Tachocline)
4. Convection Zone

Visible Sun

1. Photosphere
2. Chromosphere
3. Transition Region
4. Corona
5. (Solar Wind)

The Large-Scale Solar Dynamo



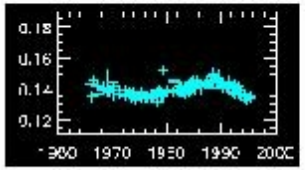
- Helioseismology shows the internal structure of the Sun.
- Surface Differential Rotation is maintained throughout the Convection zone
- Solid body rotation in the radiative interior
- Thin matching zone of shear known as the tachocline at the base of the solar convection zone (just in the stable region).

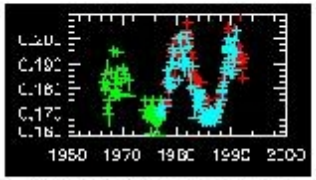
Observations: Stellar (Solar-Type Stars)

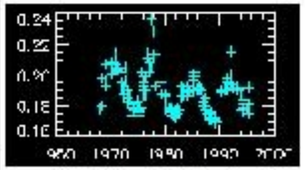
Stellar Magnetic Activity can be inferred by amount of Chromospheric Ca H and K emission

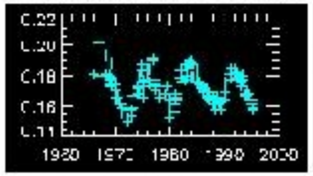
Mount Wilson Survey (see e.g. Baliunas)

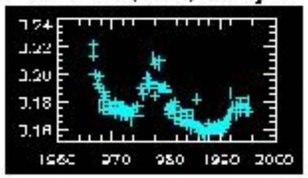
Solar-Type Stars show a variety of activity.

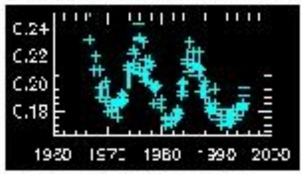

HD 136202 (F8IV-V) 23 yrs

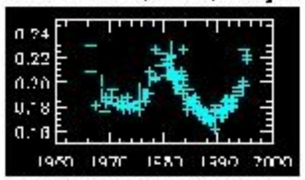

The Sun (G2V) 10.0 yrs

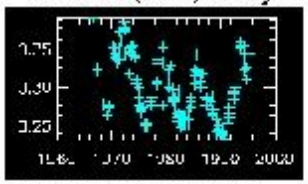

HD 103095 (G8VI) 7.3 yrs

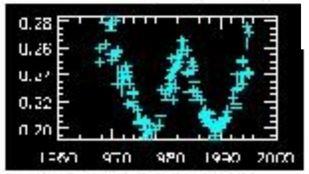

HD 81809 (K0V?) 8.2 yrs

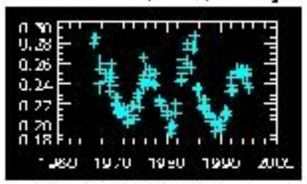

HD 3651 (K2V) 13.8 yrs

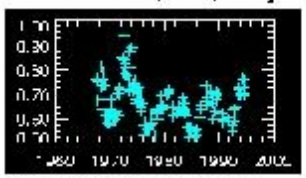

HD 10476 (K1V) 9.6 yrs

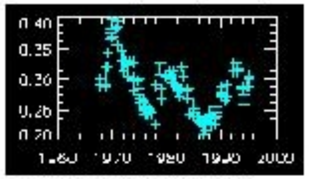

HD 166620 (K2V) 15.8 yrs

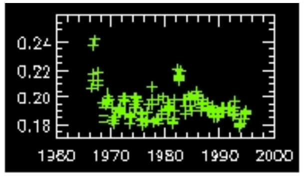

HD 160346 (K3V) 7.0 yrs

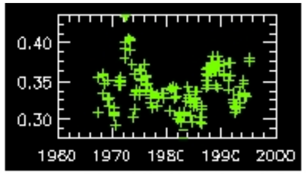

HD 16160 (K3V) 13.2 yrs

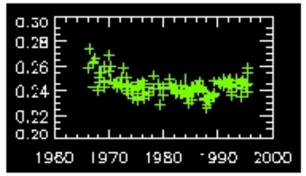

HD 4628 (K4V) 8.4 yrs

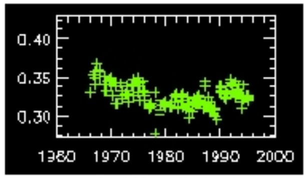

HD 201091 (K5V) 7.3 yrs

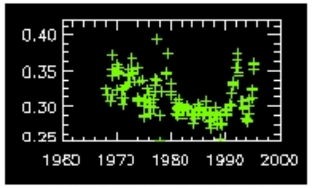

HD 32147 (K5V) 11.1 yrs

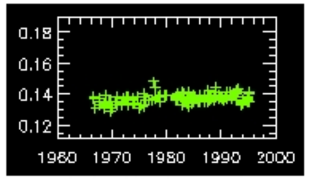

HD 190406 (G1V) 2.6 + 16.9 yrs


HD 149661 (K0V) 17.4 + 4.0 yrs


HD 114378 (F5V) Long


HD 39587 (G0V) Var


HD 101501 (G8V) Var


HD 9562 (G2V) Flat

Cyclic, Aperiodic, Modulated, Grand Minima

Observations: Stellar (Solar-Type Stars)

<u>Activity</u> is a function of spectral type/rotation rate of star

As rotation increases: activity increases

modulation increases

Activity measured by the relative Ca II HK flux density $\langle R'_{HK} \rangle$

$$\langle R'_{HK} \rangle \propto Ro^{-1}$$

(Noyes et al 1994)

But filling factor of magnetic fields also changes

$$F \propto Ro^{-0.9}$$

(Montesinos & Jordan 1993)

<u>Cycle period</u>

–Detected in old slowly-rotating G-K stars.

–2 branches (I and A) (Brandenburg et al 1998)

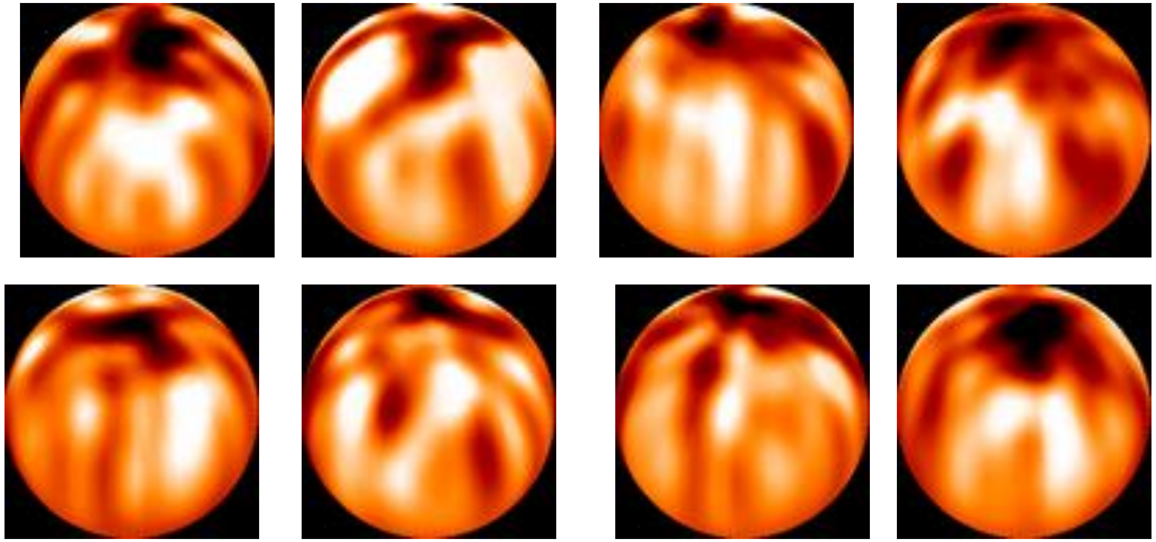
$\Omega_I \sim 6\ \Omega_A$ (including Sun)

$\Omega_{cyc}/\Omega_{rot} \sim Ro^{-0.5}$ (Saar & Brandenburg 1999)

Rapidly Rotating Stars

- Young stars are rapid-rotators (some have rotation rates ~6hrs)
- Magnetically active
- Usually have strong polar star-spots although spots may appear at latitudes >15degrees

AB Doradus pre main sequence Trinary star (Collier Cameron & Donati 1997)



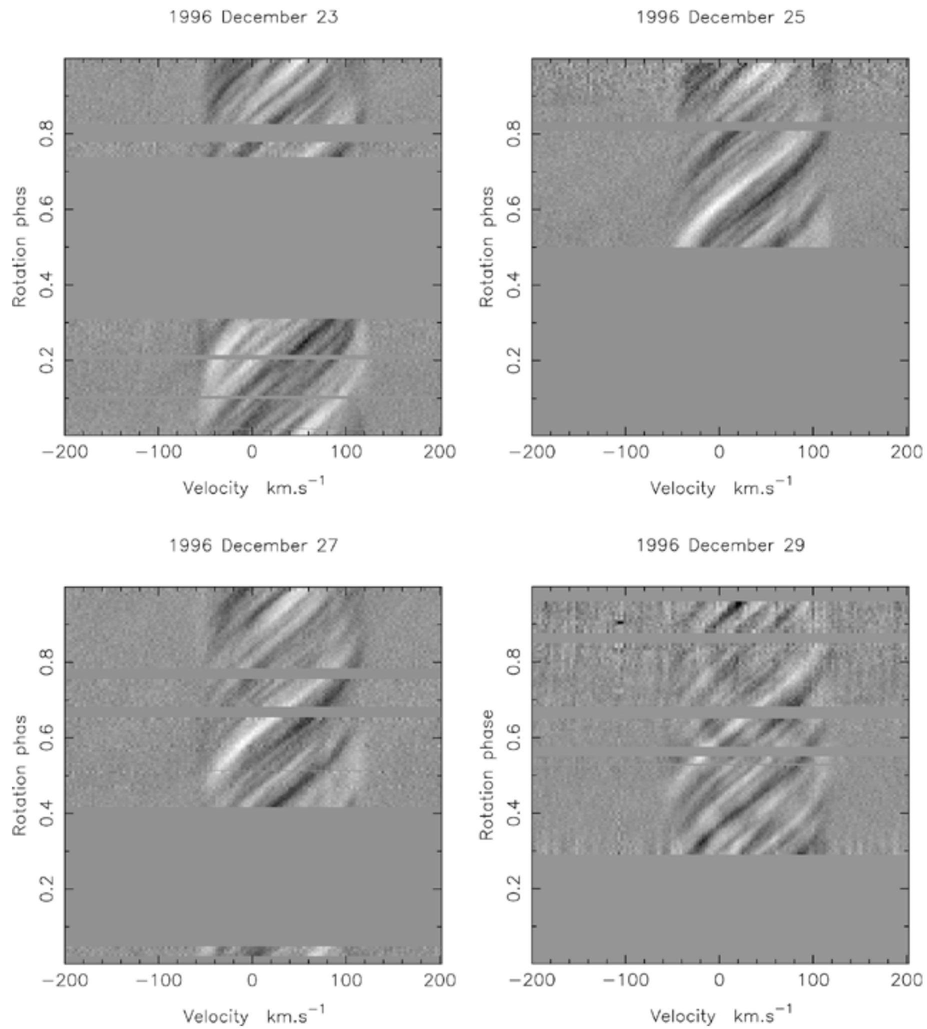
UZ Lin (binary)
Oláh,Strassmeier&Weber *2002*

Rapidly Rotating Stars

- Young stars are rapid-rotators (some have rotation rates ~6hrs)
- Can measure latitudinal differential rotation by tracking starspots

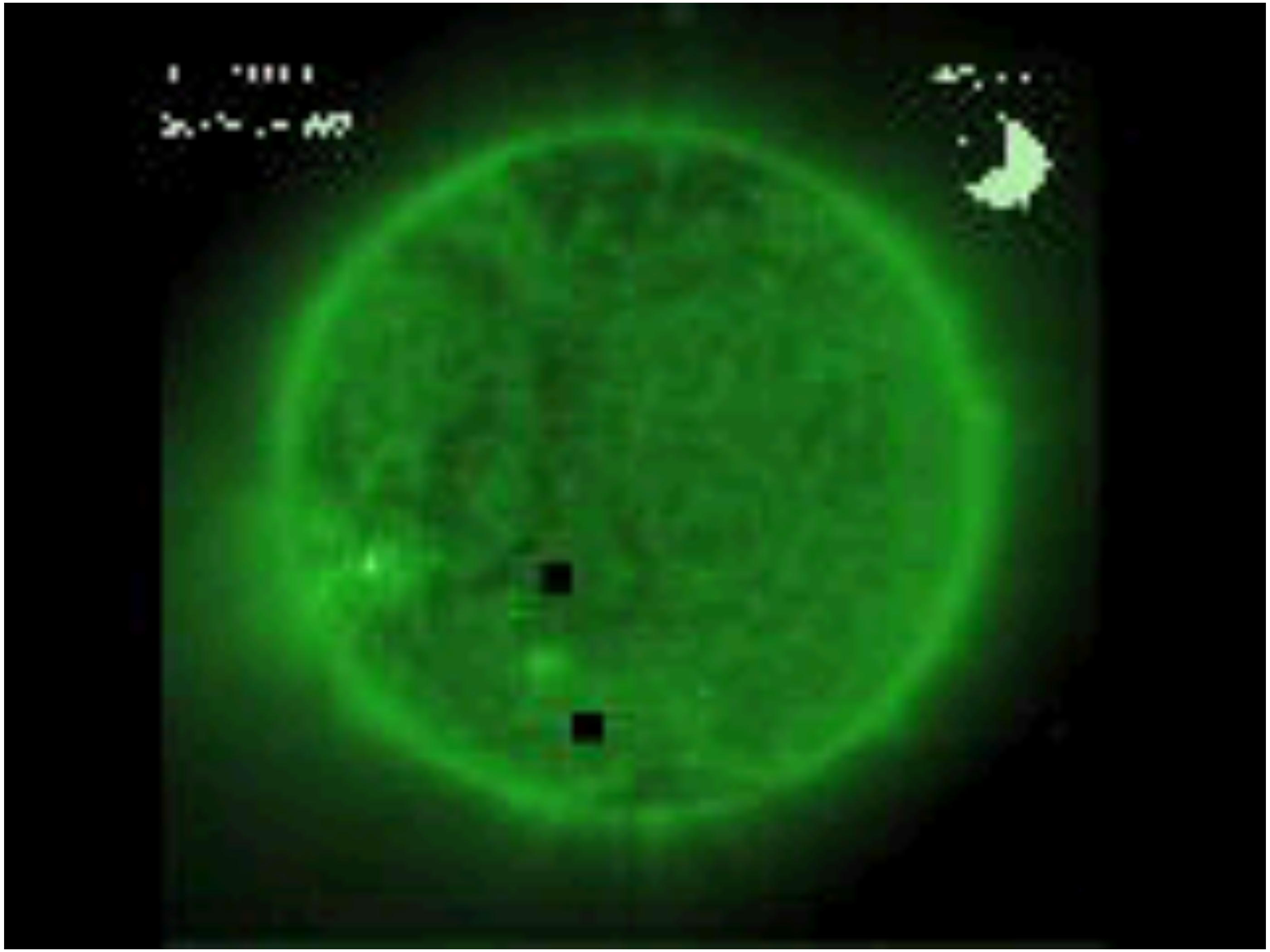
$$\Delta\Omega / \Omega \propto P$$

- Hence differential rotation only weakly dependent on rotation
- Equator faster than poles (e.g. AB Doradus) – but not much (1 part in 220)
- Some temporal evolution



Small-Scale dynamo action – the magnetic carpet

Small-Scale dynamo action – the magnetic carpet



Large and Small-scale dynamos

LARGE SCALE

- Sunspots
- Butterfly Diagram
- 11-yr activity cycle
- Coronal Poloidal Field
- Systematic reversals
- Periodicities

Field generation on scales $> L_{TURB}$

SMALL SCALE

- Magnetic Carpet
- Field Associated with granular and supergranular convection
- Magnetic network

Field generation on scales $\sim L_{TURB}$

Modelling for the Sun

Equations and Parameters

Basics for the Sun

Dynamics in the solar interior is governed by the following equations of MHD

INDUCTION

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \qquad (\nabla . \mathbf{B} = 0),$$

MOMENTUM

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} . \nabla \mathbf{u} \right) = -\nabla p + \mathbf{j} \times \mathbf{B} + \rho \mathbf{g} + \mathbf{F}_{viscous} + \mathbf{F}_{other},$$

CONTINUITY

$$\frac{\partial \rho}{\partial t} + \nabla . (\rho \mathbf{u}) = 0,$$

ENERGY

$$\frac{D(p \rho^{-\gamma})}{Dt} = \text{loss terms},$$

GAS LAW

$$p = R \rho T.$$

Basics for the Sun

	BASE OF CZ	PHOTOSPHERE
$Ra \equiv g\Delta\nabla d^4 / \nu\chi H_P$	10^{20}	10^{16}
$\mathrm{Re} \equiv UL/\nu$	10^{13}	10^{12}
$Rm \equiv UL/\eta$	10^{10}	10^{6}
$\mathrm{Pr} = \nu/\chi$	10^{-7}	10^{-7}
$\beta = 2\mu_0 p / B^2$	10^{5}	1
$Pm = \nu/\eta$	10^{-3}	10^{-6}
$M = U/c_s$	10^{-4}	1
$Ro = U/2\Omega L$	$0.1-1$	$10^{-3}-0.4$

(Ossendrijver 2003)

Modelling Approaches

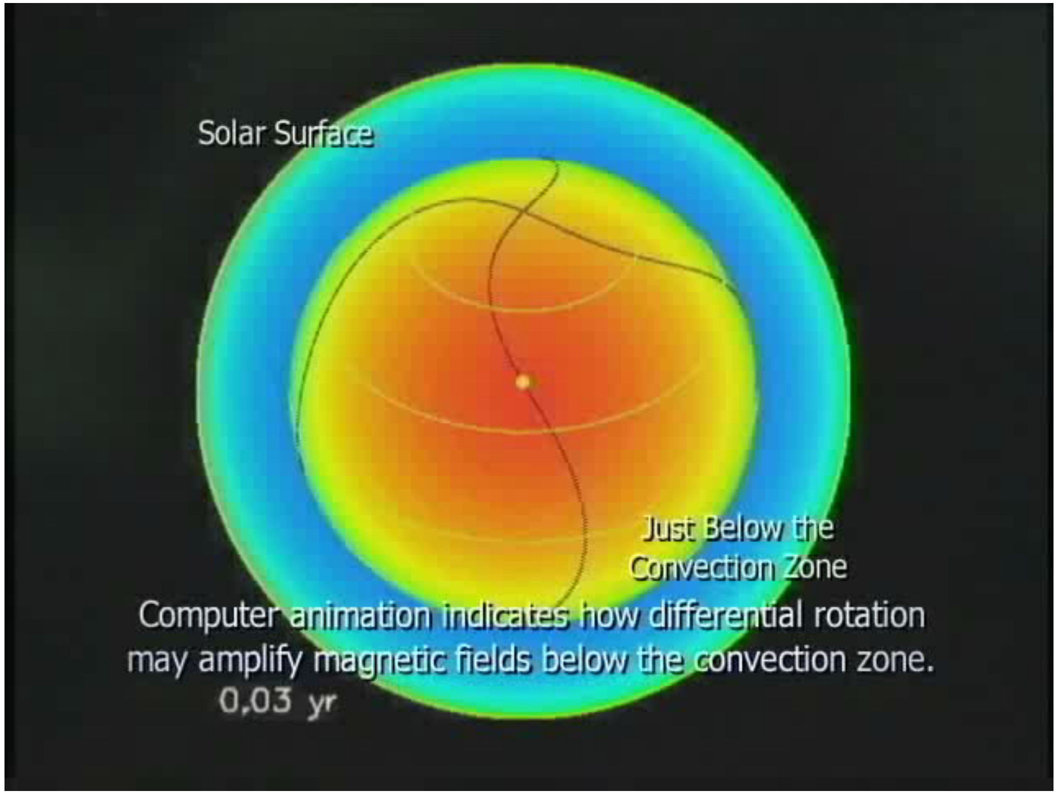
- Because of the extreme nature of the parameters in the Sun and other stars there is no obvious way to proceed.
- Modelling of dynamo processes has typically used
  - Mean Field Models (~50%)
    - Derive equations for the evolution of the mean magnetic field (and perhaps velocity field) by parametrising the effects of the small scale motions.
    - The role of the small-scales can be investigated by employing local computational models
  - Global Computations DNS (~45%)
    - Solve the relevant equations on a massively-parallel machine.
    - Either accept that we are at the wrong parameter values or claim that parameters invoked are representative of their turbulent values.
  - Low-order models (~4.9%)
    - Try to understand the basic properties of the equations with reference to simpler systems (cf Lorenz equations and weather prediction)
  - Direct Statistical Simulation
    - Integrate equations for the statistics of the flow directly
    - So far only used for model instability problems

Mean-field electrodynamics

Some basics

Mean-field electrodynamics

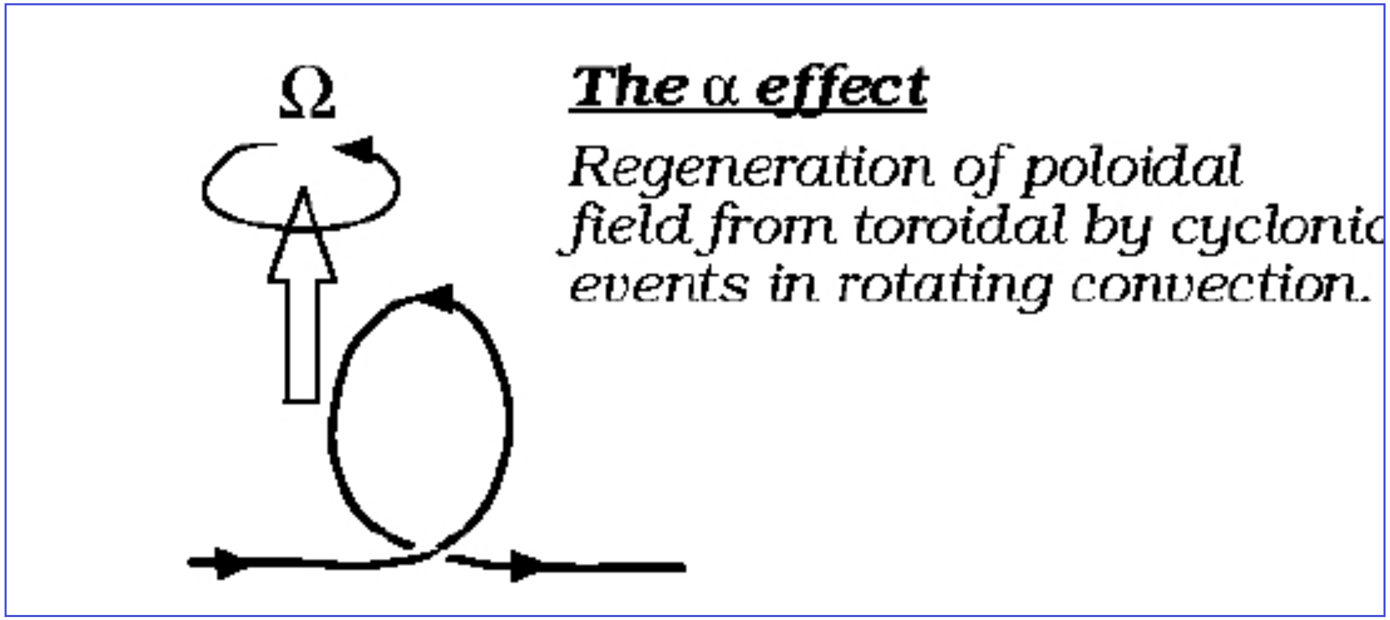
A basic physical picture



Ω-effect – poloidal → toroidal

Mean-field electrodynamics

A basic physical picture



α-effect – toroidal → poloidal

poloidal → toroidal

Mean-field theory

Derivation of the mean-field equations

Kinematic Mean Field Theory

Starting point is the magnetic induction equation of MHD:

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B},}$$

where **B** is the magnetic field, **u** is the fluid velocity and η is the magnetic diffusivity (assumed constant for simplicity).

Assume scale separation between large- and small-scale field and flow:

$$\boxed{\mathbf{B} = \mathbf{B}_0 + \mathbf{b}, \quad \mathbf{U} = \mathbf{U}_0 + \mathbf{u},}$$

where **B** and **U** vary on some large length scale *L*, and **u** and **b** vary on a much smaller scale *l*.

$$\boxed{\langle \mathbf{B} \rangle = \mathbf{B}_0, \quad \langle \mathbf{U} \rangle = \mathbf{U}_0,}$$

where averages are taken over some intermediate scale *l* « *a* « *L*.

For simplicity, ignore large-scale flow, for the moment.

Induction equation for mean field:

$$\frac{\partial \mathbf{B}_0}{\partial t} = \nabla \times \mathcal{E} + \eta \nabla^2 \mathbf{B}_0$$

where mean emf is $\mathcal{E} = \langle \mathbf{u} \times \mathbf{b} \rangle$

This equation is exact, but is only useful if we can relate $\mathbf{E}$ to $\mathbf{B}_0$.

Consider the induction equation for the fluctuating field:

$$\boxed{\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}_0) + \nabla \times \mathbf{G} + \eta \nabla^2 \mathbf{b},}$$

Where $\mathbf{G} = \mathbf{u} \times \mathbf{b} - \langle \mathbf{u} \times \mathbf{b} \rangle$. "pain in the neck term"

Traditional approach is to assume that the fluctuating field is driven solely by the large-scale magnetic field.

i.e. in the absence of $\boldsymbol{B}_0$ the fluctuating field decays.

i.e. No small-scale dynamo (not really appropriate for high Rm turbulent fluids)

Under this assumption, the relation between $\mathbf{b}$ and $\mathbf{B}_0$ (and hence between $\mathcal{E}$ and $\mathbf{B}_0$) is linear and homogeneous.

Postulate an expansion of the form:

$$\mathcal{E}_i = \alpha_{ij} B_{0j} + \beta_{ijk} \frac{\partial B_{0j}}{\partial x_k} + \ldots$$

where α_{ij} and β_{ijk} are *pseudo*-tensors, determined by the statistics of the turbulence.

Simplest case is that of isotropic turbulence, for which $\alpha_{ij} = \alpha\delta_{ij}$ and $\beta_{ijk} = \beta\varepsilon_{ijk}$.
Then mean induction equation becomes:

$$\boxed{\frac{\partial \mathbf{B}_0}{\partial t} = \nabla \times (\alpha \mathbf{B}_0) + (\eta + \beta)\nabla^2 \mathbf{B}_0.}$$

α: regenerative term, responsible for large-scale dynamo action.
Since $\mathbf{E}$ is a polar vector whereas ***B*** is an axial vector then α **can be non-zero only for turbulence lacking reflexional symmetry** (i.e. possessing handedness).

β: turbulent diffusivity.

BUT WHAT ARE α and β ? MORE LATER

Mean-field theory

Revision: Basic properties of the kinematic mean field equations

BASIC PROPERTIES OF THE MEAN FIELD EQUATIONS

Add back in the mean flow U_0 and the mean field equation becomes

$$\frac{\partial \mathbf{B}_0}{\partial t} = \nabla \times (\alpha \mathbf{B}_0 + \mathbf{U}_0 \times \mathbf{B}_0) + (\eta + \beta) \nabla^2 \mathbf{B}_0 .$$

Now consider simplest case where $\alpha = \alpha_0 \cos\theta$ and $U_0 = U_0 \sin\theta \, e_\phi$

In contrast to the induction equation, this can be solved for axisymmetric mean fields of the form

$$\mathbf{B}_0 = B_{0t}\mathbf{e}_\varphi + \nabla \times (A_{0P}\mathbf{e}_\varphi)$$

BASIC PROPERTIES OF THE MEAN FIELD EQUATIONS

- Linear growth-rate of $\mathbf{B}_0$ depends on dimensionless combination of parameters.
- Critical parameter given by

$$\boxed{D = R_\alpha R_\Omega \equiv \frac{\alpha L}{\eta}\frac{\Delta\Omega L^2}{\eta}}$$

- If $|D| > D_c$ then exponentially growing solutions are found – dynamo action.
- Estimates suggest $|D_\alpha| \sim 2$, $|D_\Omega| \sim 10^3$ for the Sun and hence one can make the $\alpha\Omega$-approximation where the α-effect is ignored in generating the toroidal field.
- Can also have $\alpha^2\Omega$ and α^2 dynamos – may be of relevance for fully convective or more rapidly rotating stars.

BASIC PROPERTIES OF THE MEAN FIELD EQUATIONS

- In general $\mathbf{B}_0$ takes the form of an exponentially growing dynamo wave that propagates.
- Direction of propagation depends on sign of dynamo number D.
  - If $D > 0$ waves propagate towards the poles,
  - If $D < 0$ waves propagate towards the equator.
- In this linear regime the frequency of the magnetic cycle Ω_{cyc} is proportional to $|D|^{1/2}$
- Solutions can be either

  dipolar or quadrupolar

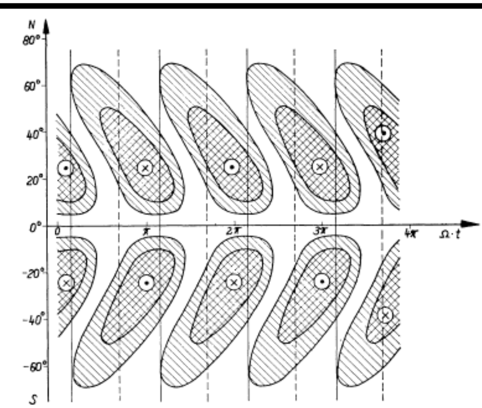


Abb. 9. Schmetterlingsdiagramm, Modell 1. Schraffiert sind die Gebiete $B \geqq 1/3\ B_{max}$ und doppelt schraffiert die Gebiete $B \geqq 2/3\ B_{max}$ gekennzeichnet. Die durchgezogenen Linien kennzeichnen die Phasenlage des Umpolens an den Polkappen, die gestrichelten Linien die Phasenlage des Umpolens des Dipolanteiles. Zur Phasenlage $\Omega t = 0$ erreicht das toroidale Feld seinen Maximalwert.

Mean-field theory

Some fundamental and rather nasty questions

Crucial questions

Mean field electrodynamics therefore seems to work very well - but there are some very obvious questions to ask

1. How can we calculate α and β? What will these be in the Sun. Can we relate them to the properties of the flow in the kinematic regime?

2. Even if we know how α and β behave kinematically, what is the role of the Lorentz force on the transport coefficients α and β?

3. How weak must the large-scale field be in order for it to be dynamically insignificant? Dependence on *Rm*?

4. Even if we can get all of this to work, does the large-scale dynamo ever win out over the dynamo's natural instinct to generate field on small scales at high Rm?

1. How can we calculate α and β? Can we relate them to the properties of the flow in the kinematic regime?

- Of course α and β can only really be calculated by determining

$$\boxed{\mathbf{E} = \langle \mathbf{u} \times \mathbf{b} \rangle.}$$

- But we can only know **b** if we solve the fluctuating field equation.
- Analytic progress can be made *<u>by making one of two approximations</u>*
- Either Rm or the correlation time of the turbulence τ_{corr} is small.
- Then can ignore "pain in the neck" G term in fluctuating field equation.
- Get famous results that α is related to the helicity of the flow with a constant of proportionality given by the small parameter e.g.

$$\alpha = -\frac{\tau_{corr}}{3} \langle \mathbf{u} . \boldsymbol{\omega} \rangle$$

- Note we have parameterised correlations between **u** and **b** by correlations between **u** and **ω** -- is this sensible, or even helpful?

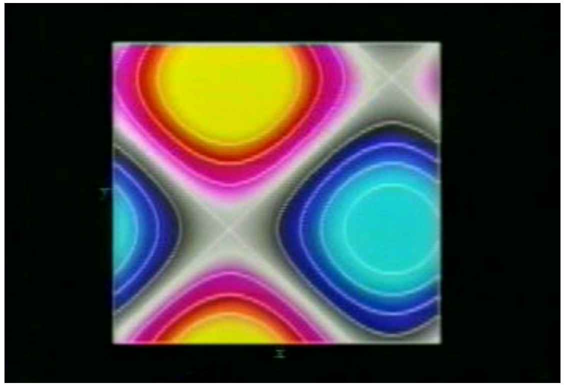
1. How can we calculate α and β? Can we relate them to the properties of the flow in the kinematic regime?

- We could do some numerical experiments and simply measure α
- The best way to do this is to impose a ***known weak mean field*** **B** ***and then calculate numerically*** $\mathbf{E} = \langle \mathbf{u} \times \mathbf{b} \rangle$.
- Example: Choose a flow with Rm not small and not at short correlation time and simply evaluate α.
- So we solve the kinematic induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + Rm^{-1} \nabla^2 \mathbf{B}, \quad \nabla . \mathbf{B} = 0.$$

With an applied mean field to calculate E.

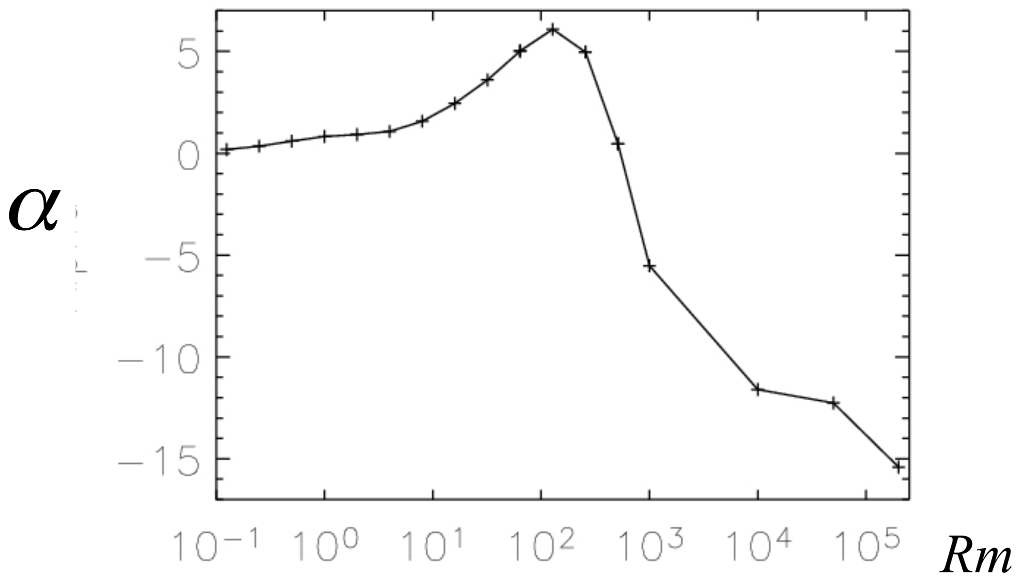
Here we choose **u** to be the famous G-P flow



$$\mathbf{u} = (\psi_y, -\psi_x, \psi); \qquad \psi = \sqrt{\frac{3}{2}} \left(\cos(x + \cos(t)) + \sin(y + \sin(t)) \right)$$

1. How can we calculate α and β? Problems with Rm

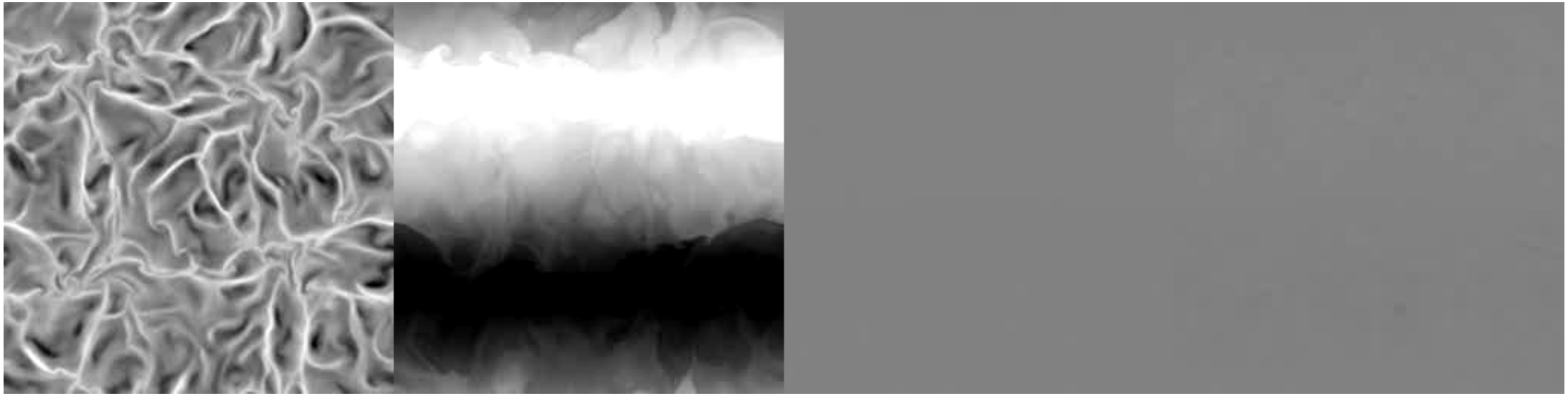
- For this flow the α term is a tensor.
- The α−effect is a very sensitive function of Rm.
- It even changes sign.
- It can in no way be related in a simple manner to the helicity of the flow (bit of a strange flow as it has infinite correlation time)
- Neither of the approximations work very well at high Rm



Courvoisier et al 2006

1. How can we calculate α and β? Calculating β

- Calculating the turbulent diffusivity is more challenging.
- In numerical experiments either use
- The Test Field Method (Schrinner et al 2005)
  - Impose a fictitious "test field" that grows exponentially
  - Relate the emf generated by this growing solution to the imposed field.
- The Turbulent Ångström method or The Method of Oscillatory Sines (Tobias & Cattaneo 2012, C&T 2013)
  - Use variations of the method proposed by Ångström (1843) to measure thermal conductivity of a metal rod.
  - Impose an oscillatory magnetic field and measure response



1. How can we calculate α and β? Calculating β

- In a computation very hard to get to high Rm
- Get your favourite plasma experimentalist to do it for you…

2. Large-scale vs Small-Scale

- Ingredients for large-scale mean field dynamo
  - Turbulence with lack of reflexional symmetry
  - Low/moderate Rm?
- Ingredients for small-scale dynamo
  - Chaotic stretching, not too much cancellation
- Problem is that small-scale dynamos seem to win as Rm is increased…
- Possible Solutions
  - Boost large-scale EMF by adding in a systematic large-scale flow (e.g. shear; see Yousef et al 2008, Kapyla & Brandenburg 2009, Sridhar & Singh 2010, Hughes & Proctor 2012)
  - Somehow suppress the small-scale dynamo (see Tobias & Cattaneo 2008, 2013; Courvoisier & Kim 2009)
    - Decreasing stretching or increasing cancellation?

2. How are α and β modified by the mean field in the Nonlinear Regime?

- This is a CRUCIAL question.
- Assume kinematic theory is OK (hmm)
- The mean field <**B**> will act back on the turbulence so as to switch off the generation mechanism via the Lorentz Force.
- When does this happen?
- Traditional argument…
  - This occurs when mean field reaches equipartition with the turbulence so

$$\boxed{\langle \rho u^2 \rangle \approx \langle \mathbf{B} \rangle^2.}$$

2. How are α and β modified by the mean field in the Nonlinear Regime?

- But…
- It is the small scale magnetic field that will act back on the small-scale turbulence.
- The dynamo will switch off when the small-scale magnetic energy becomes comparable with the small-scale kinetic energy of the flow.
- There are many different possibilities, but it seems clear that due to amplification by the turbulence the small scale magnetic field is much bigger than the mean magnetic field

From a simple scaling it follows that: $\boxed{\langle B^2 \rangle \approx Rm^p \langle \mathbf{B} \rangle^2.}$

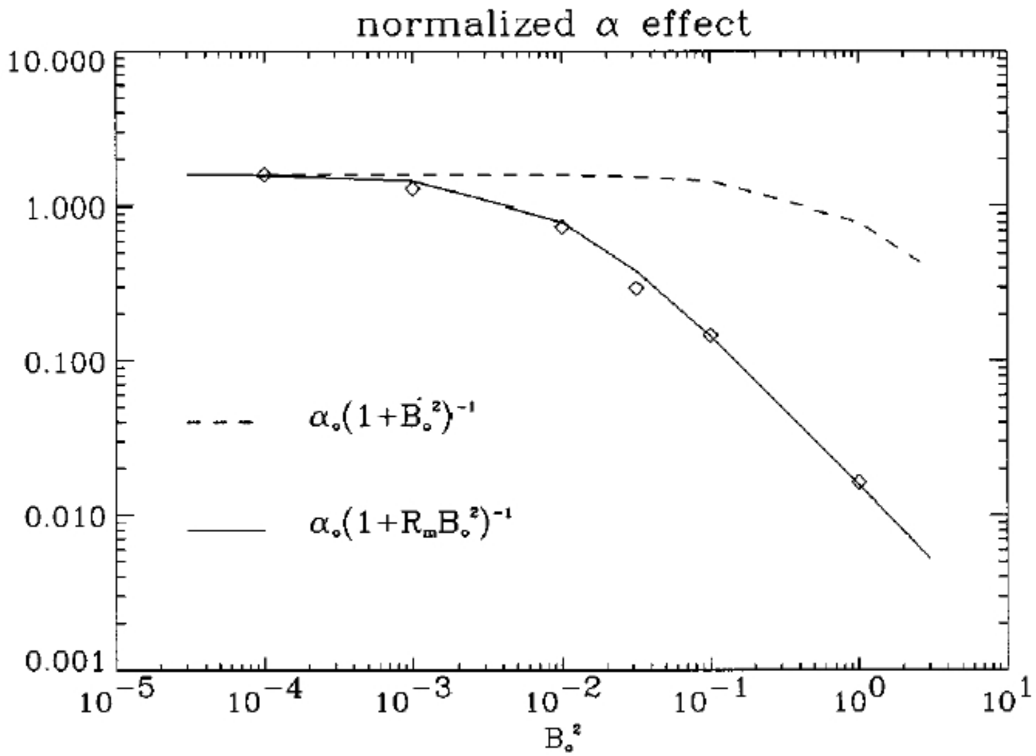
where p is a flow and geometry dependent coefficient (p>0)

2. How are α and β modified by the mean field in the Nonlinear Regime?

- This poses a major problem for mean field theory (see Proctor 2003;Diamond et al 2004 for an erudite discussion)
- If true then this implies that the α-effect (and probably the β-effect) is switched off when the mean magnetic field is small (i.e. when

$$\boxed{\langle \mathbf{B} \rangle^2 \approx \langle \rho u^2 \rangle / Rm^p}$$

- Hence the source term (α) will be *catastrophically quenched* when the mean field is very small.
- Is this correct?
- Two ways of checking
  – Analytical results based on approximations
  – Numerical results at moderate Rm



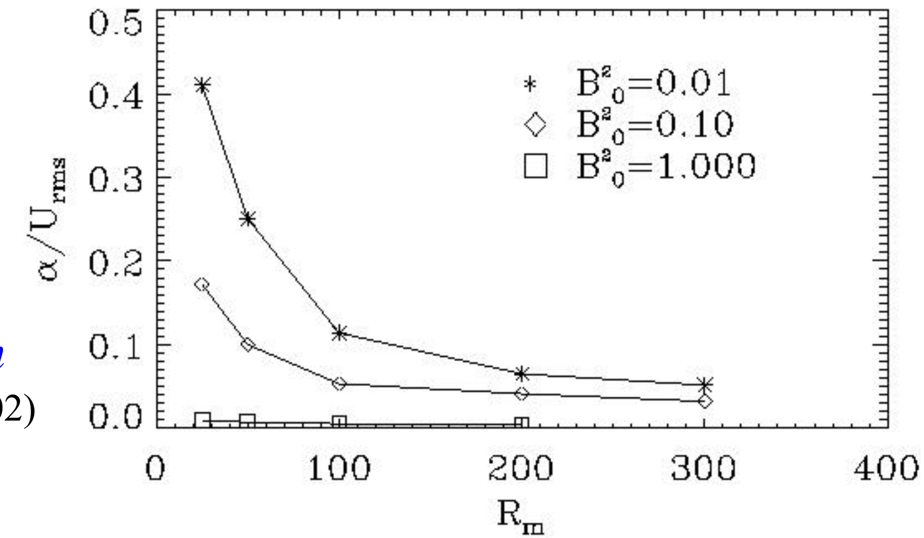
α versus B_0^2

(Cattaneo & Hughes 1996)

Suggestive of the formula:

$$\boxed{\alpha = \frac{\alpha_0}{1 + Rm^{\gamma} B_0^2}}$$

for γ = O(1).



α versus *Rm*

(C, H & Thelen 2002)

Other Possibilities

Given the problems with mean-field theory, what other mechanisms are there for producing poloidal field?

Other Possible Mechanisms for Producing Poloidal Field

- In addition to the conventional turbulent driven α-effect, there have been other mechanisms suggested for generating a large scale poloidal field
- Most of these are dynamic and rely on the presence of a large-scale toroidal field.

Other Possible Mechanisms for Producing Poloidal Field

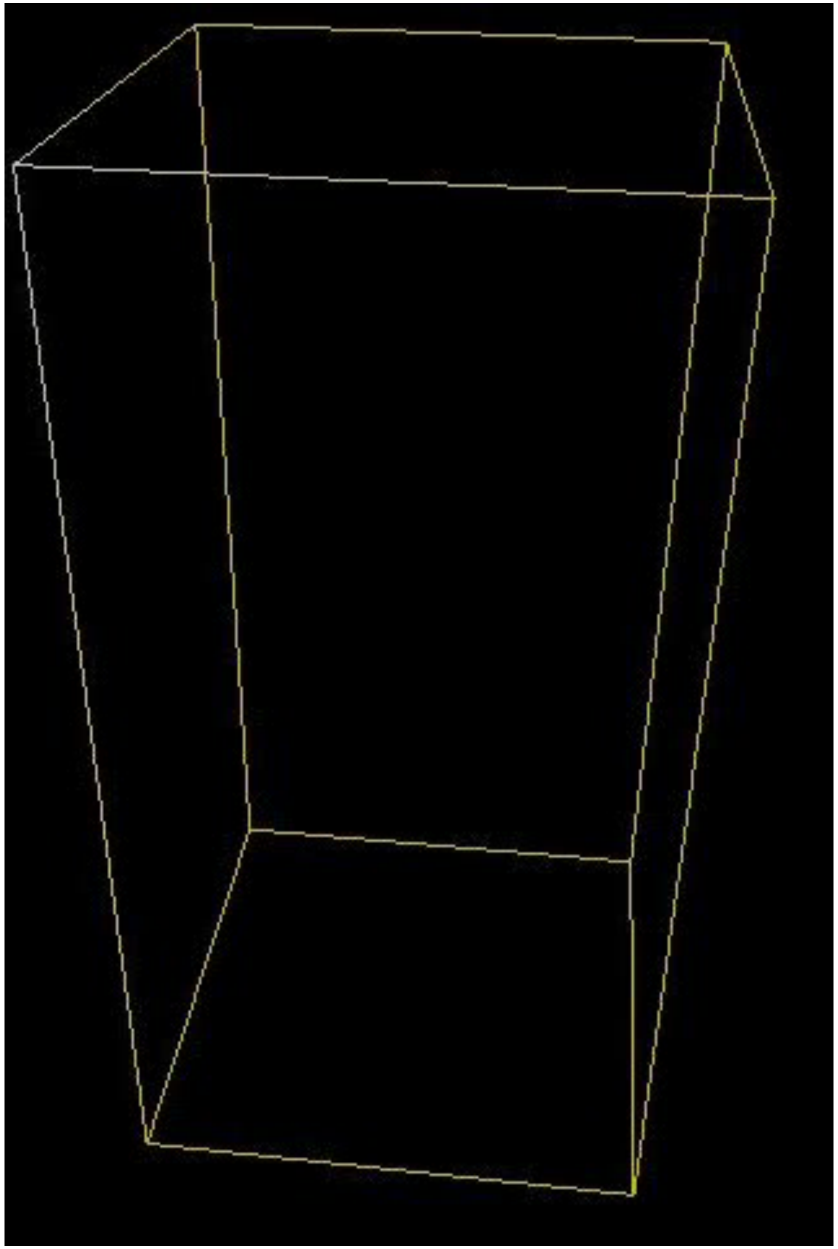
- Poloidal field generated by magnetic buoyancy instability in connection with rotation or shear
  - Either the instability of (thin) magnetic flux tubes
  - Or more likely the instability of a layer of magnetic field
  - Joint Instability of field and differential rotation in the tachocline (Gilman, Dikpati etc)
  - Produces a mean flow with a net helicity
- Decay and dispersion of tilted active regions at the solar surface (Babcock-Leighton mechanism)

Buoyancy and rotation

- Strong toroidal magnetic fields may be unstable to a magnetic buoyancy instability (contribution of magnetic field to total pressure reduces density).
- The field becomes buoyant and can rise to form coherent magnetic structures (see e.g. Kersalé et al 2007).
- These structures can rise and interact with the rotation, twisting to form a poloidal field.

- In its most extreme form only those fields that make it to the surface contribute to the dynamo (Babcock-Leighton mechanism)

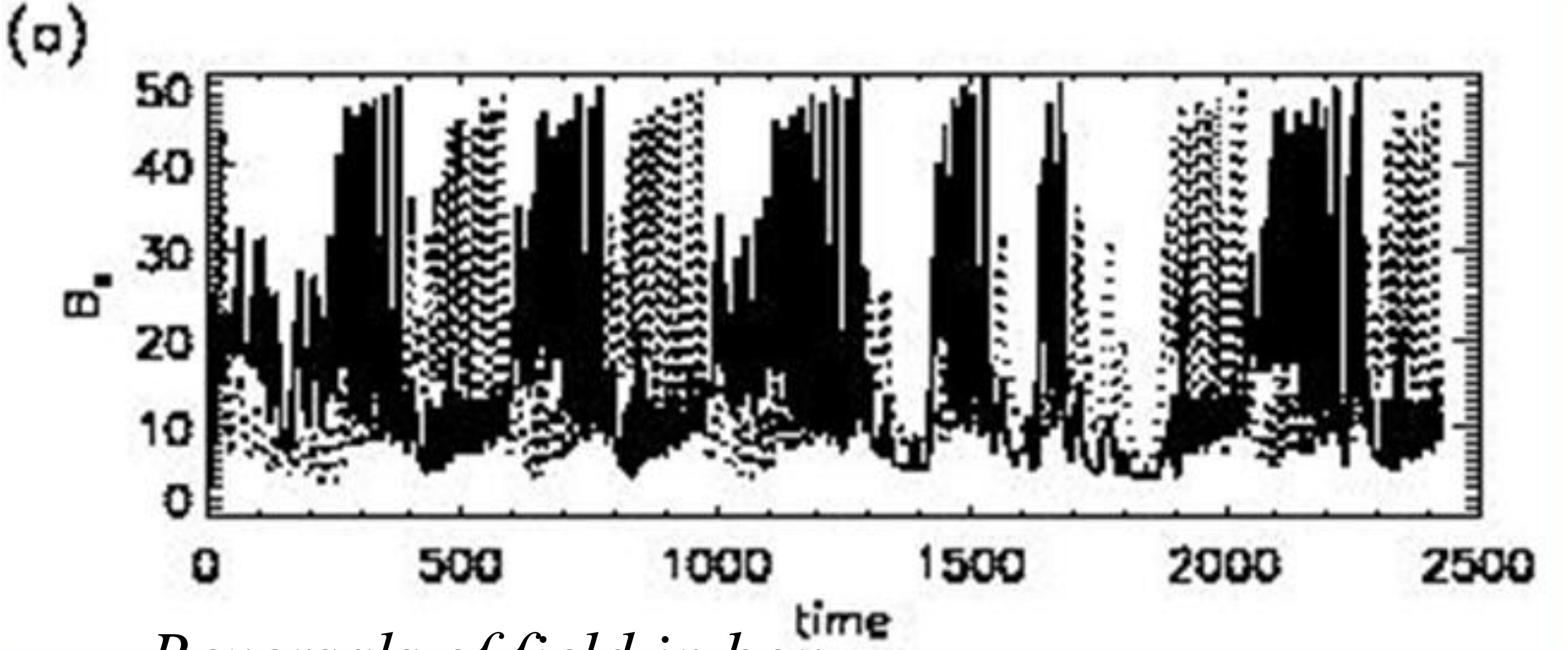
A buoyancy driven dynamo



Shear +Magnetic Buoyancy
No turbulence
No alpha effect
No problem…

Kline et al 2005

A buoyancy driven dynamo



Reversals of field in box
Periods of reduced activity

Kline et al 2005

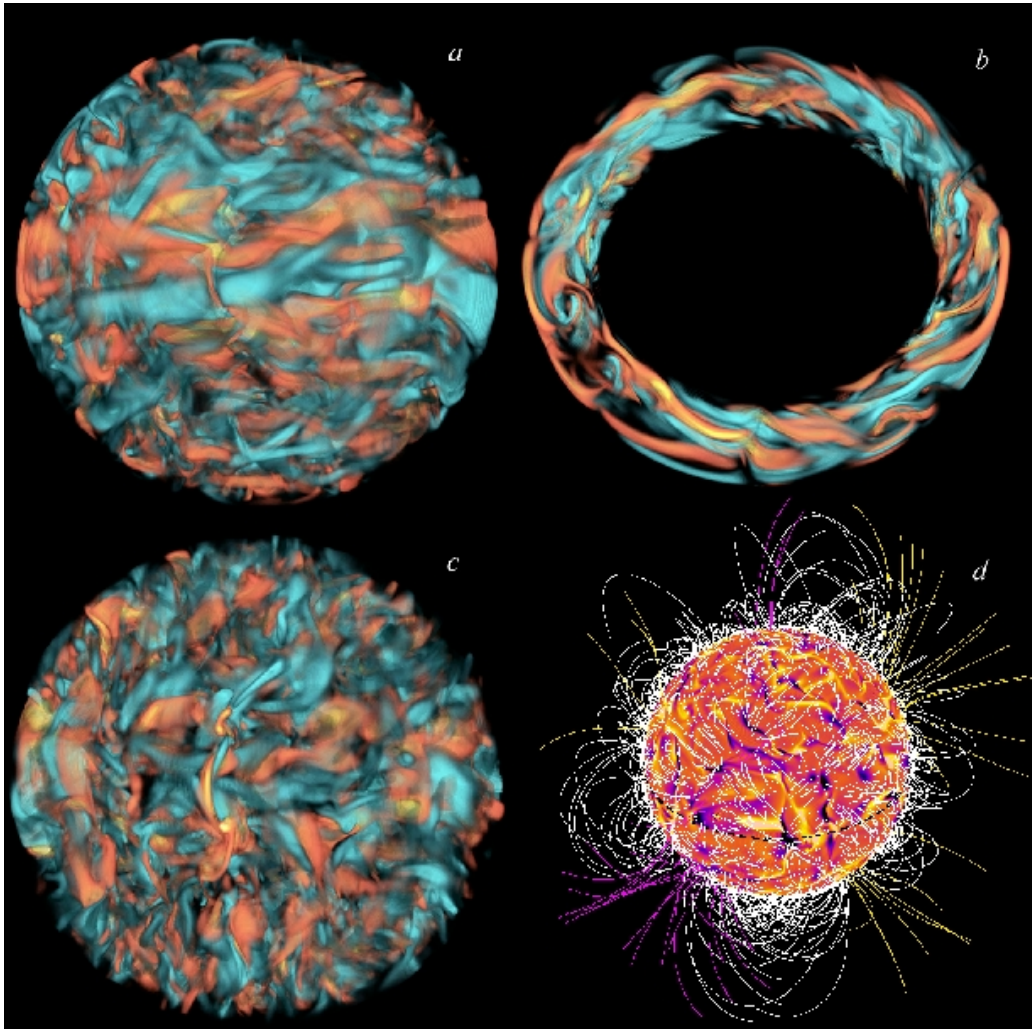
Questions: how does this work at higher Rm?
Can this survive the presence of turbulence?

Some solar dynamo scenarios

Distributed, Deep-seated, Flux Transport, Interface, Near-Surface.

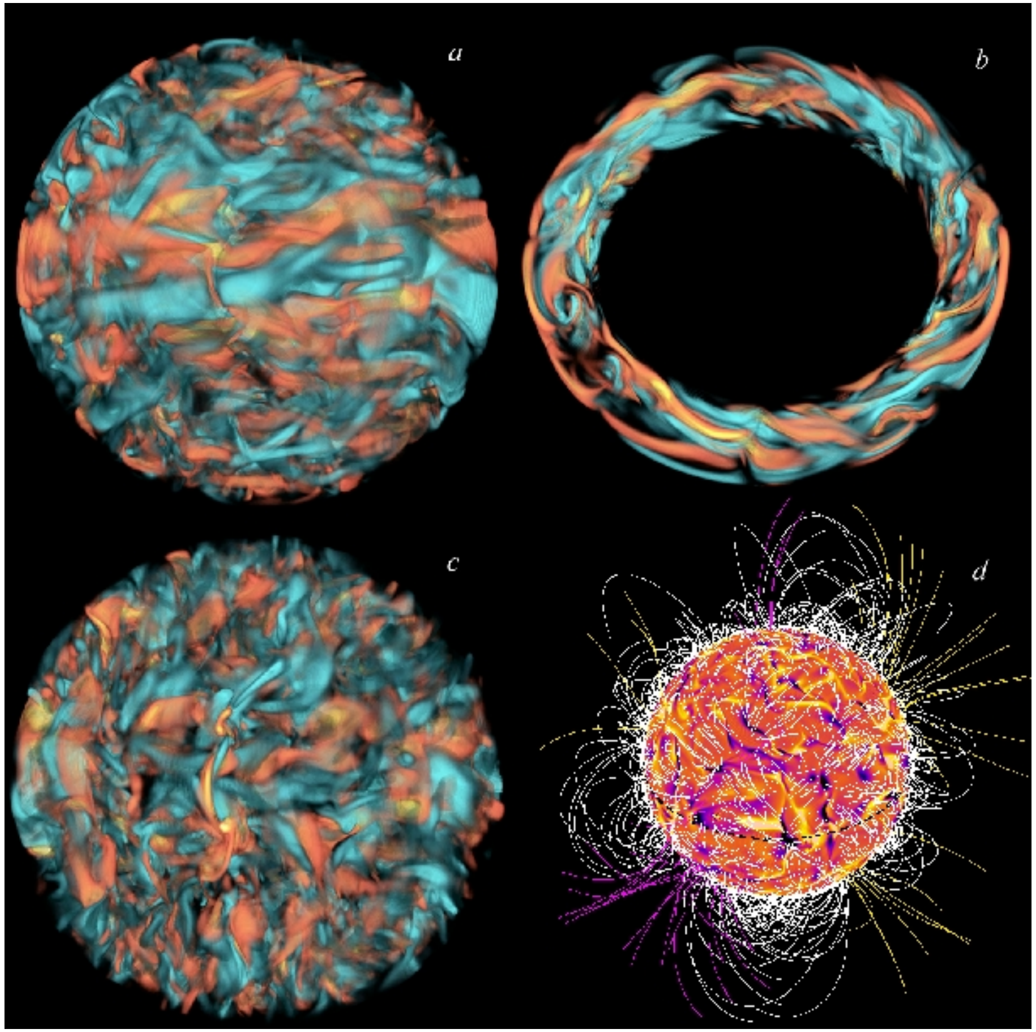
This is simply a matter of choosing plausible profiles for α and β depending on your prejudices or how many of the objections to mean field theory you take seriously!

Distributed Dynamo Scenario



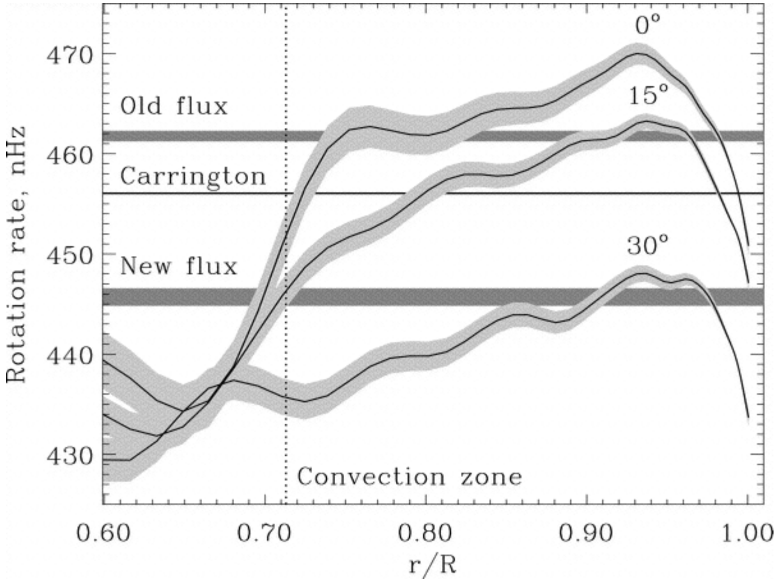
- **Here the poloidal field is generated throughout the convection zone by the action of cyclonic turbulence.**
- **Toroidal field is generated by the latitudinal distribution of differential rotation.**
- **No role is envisaged for the tachocline**
- **Angular momentum transport would presumably be most effective by Reynolds and Maxwell stresses**

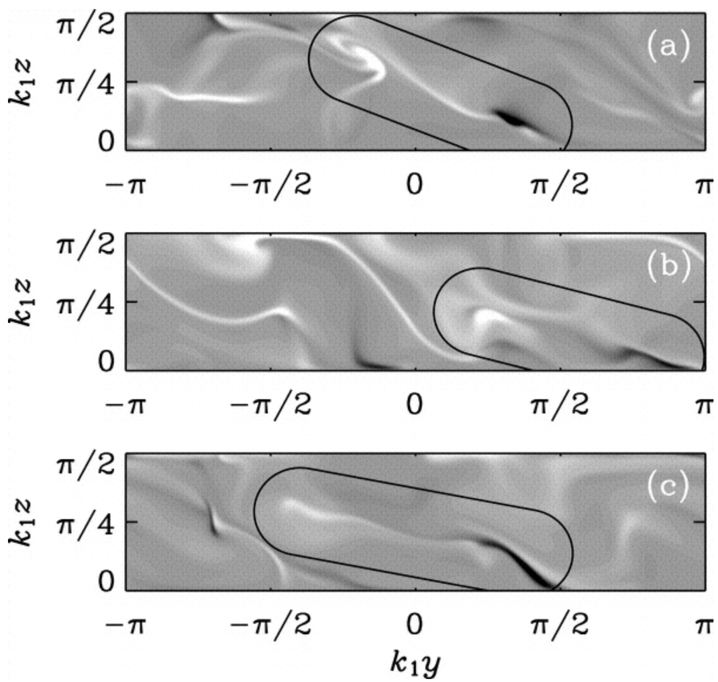
Distributed Dynamo Scenario



- **PROS**
  - **Scenario is "possible" wherever convection and rotation take place together**
- **CONS**
  - **Computations show that it is hard to get a large-scale field**
  - **Mean-field theory shows that it is hard to get a large-scale field (catastrophic α-quenching)**
  - **Buoyancy removes field before it can get too large**

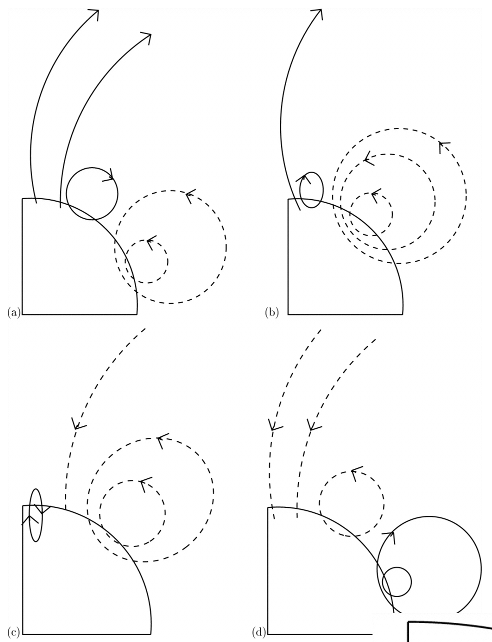
Near-surface Dynamo Scenario

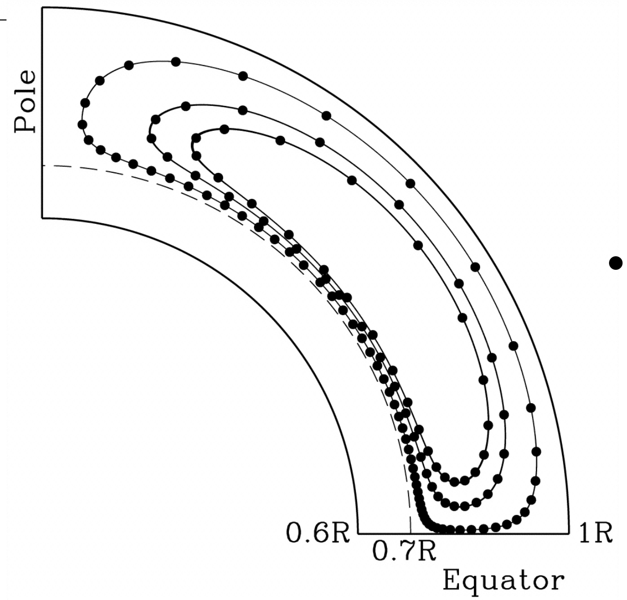




- **This is essentially a distributed dynamo scenario.**
- **The near-surface radial shear plays a key role.**
- **Magnetic features tend to move with rotation rate at the bottom of the near surface shear layer.**
- **Same pros and cons as before.**
- **Brandenburg (2006)**

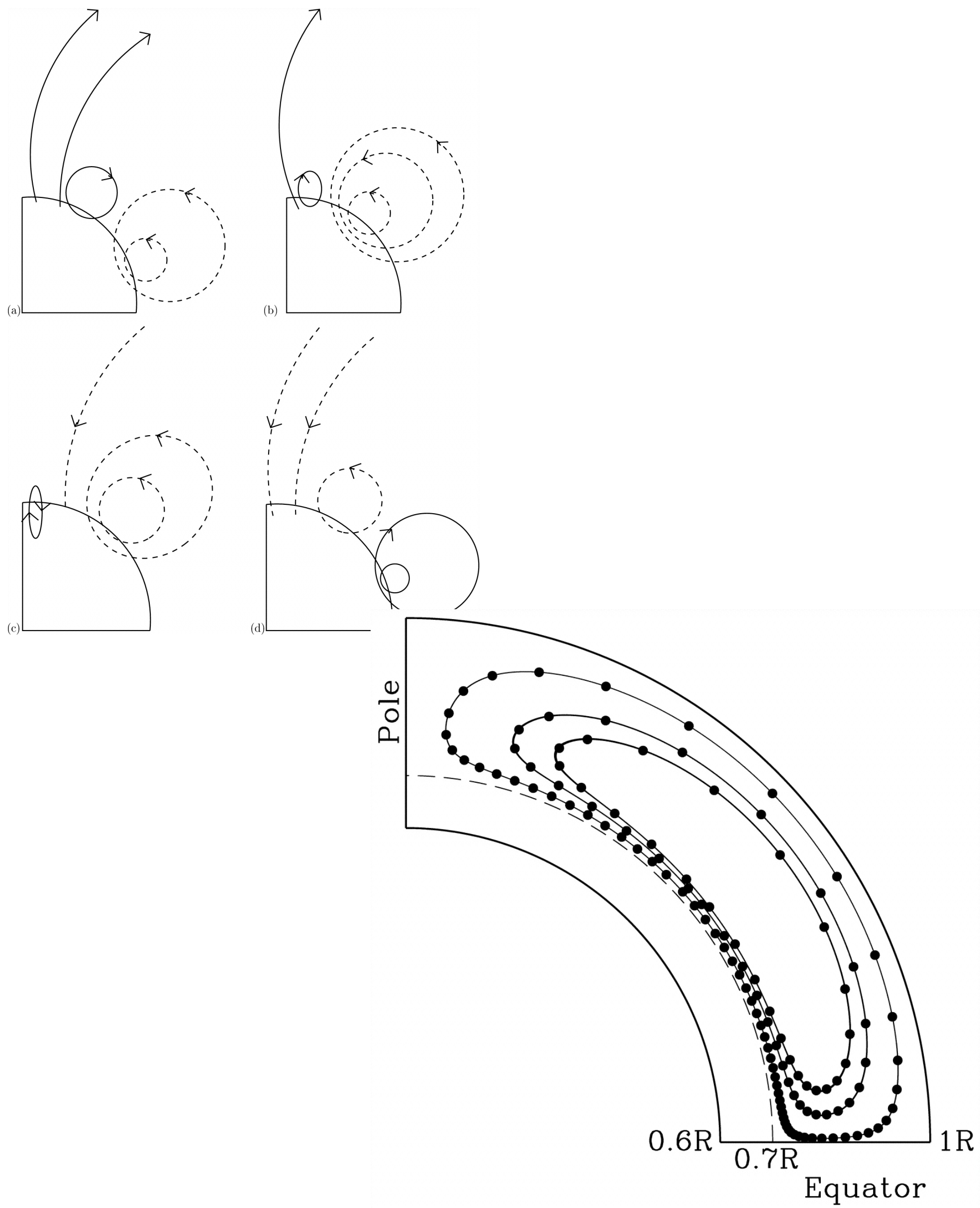
Flux Transport Scenario





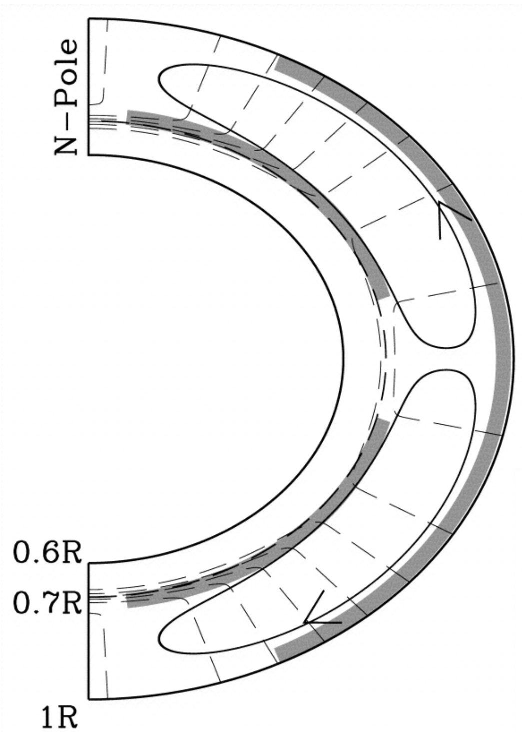
- **Here the poloidal field is generated at the surface of the Sun via the decay of active regions with a systematic tilt (Babcock-Leighton Scenario) and transported towards the poles by the observed meridional flow**
- **The flux is then transported by a conveyor belt meridional flow to the tachocline where it is sheared into the sunspot toroidal field**
- **No role is envisaged for the turbulent convection in the bulk of the convection zone.**

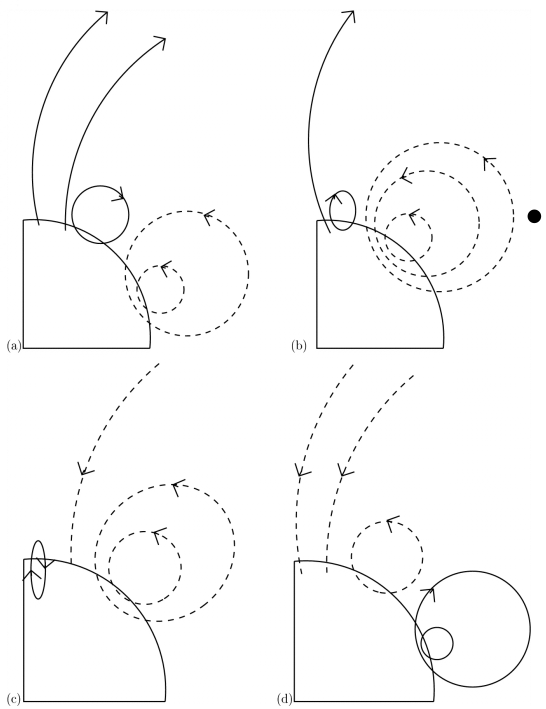
Flux Transport Scenario



- **PROS**
  - **Does not rely on turbulent α-effect therefore all the problems of α-quenching are not a problem**
  - **Sunspot field is intimately linked to polar field immediately before.**
- **CONS**
  - **Requires strong meridional flow at base of CZ of exactly the right form**
  - **Ignores all poloidal flux returned to tachocline via the convection**
  - **Effect will probably be swamped by "α-effects" closer to the tachocline**
  - **Relies on existence of sunspots for dynamo to work (cf Maunder Minimum)**

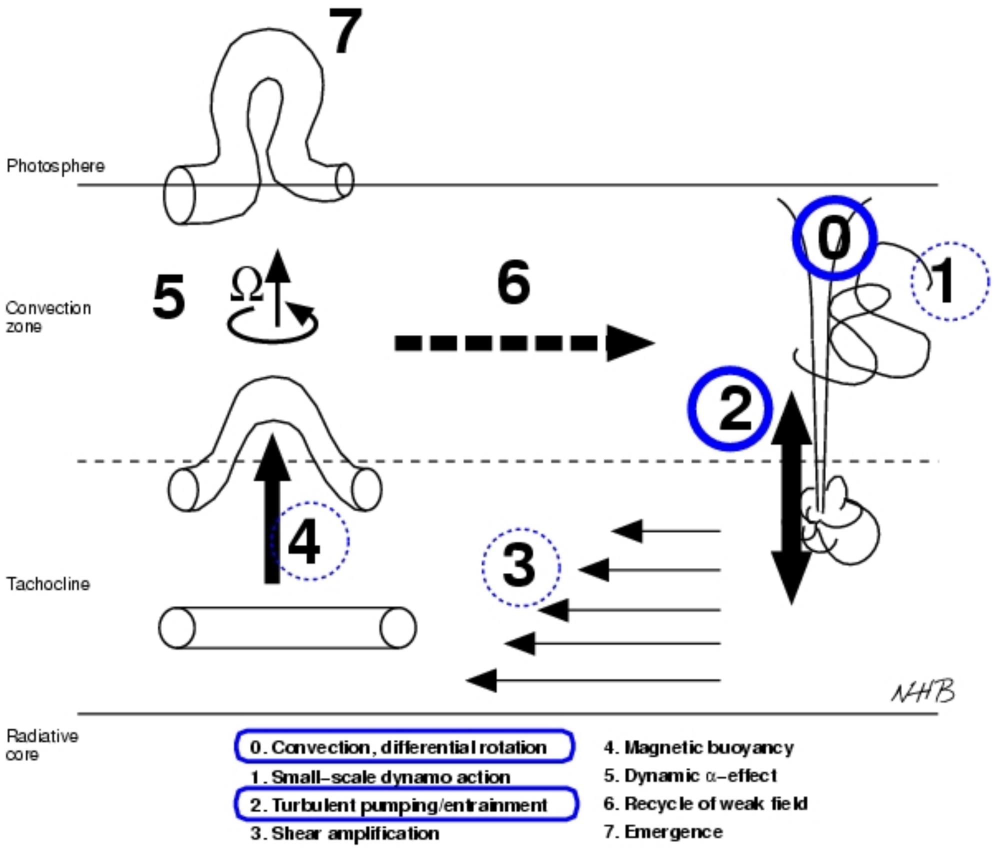
Modified Flux Transport Scenario





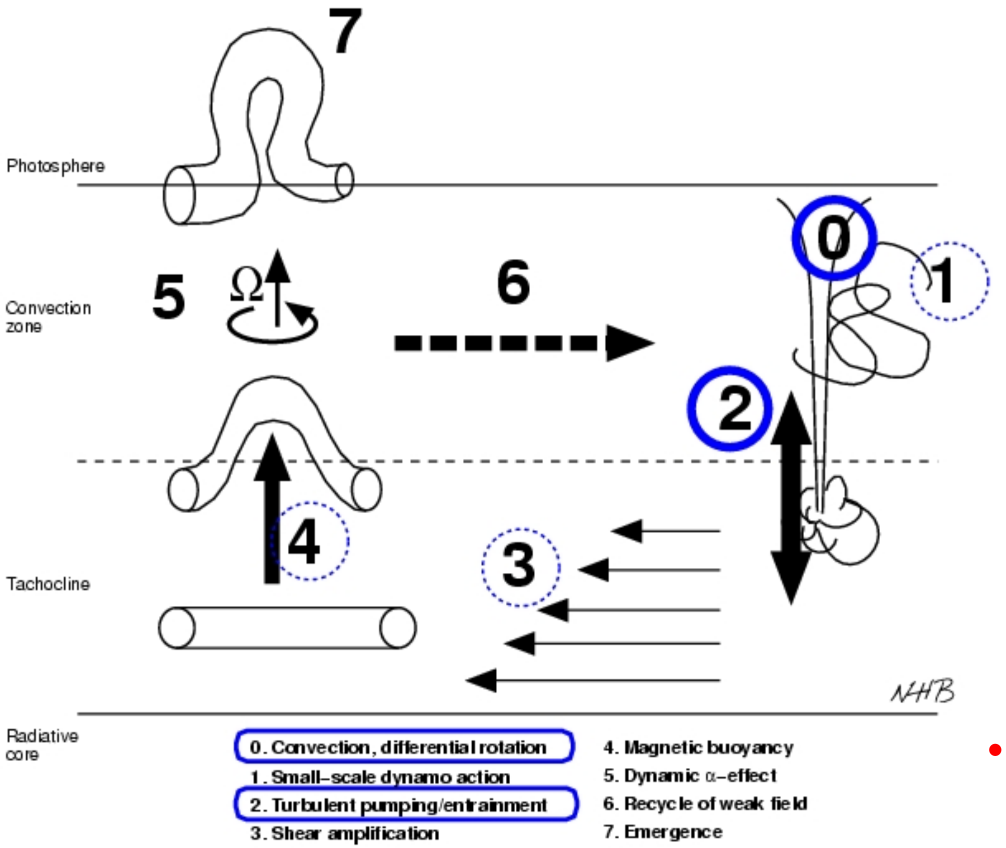
- **In addition to the poloidal flux generated at the surface, poloidal field is also generated in the tachocline due to an MHD instability.**
- **No role is envisaged for the turbulent convection in the bulk of the convection zone in generating field**
- **Turbulent diffusion still acts throughout the convection zone.**

Interface/Deep-Seated Dynamo



- The dynamo is thought to work at the interface of the convection zone and the tachocline.
- The mean toroidal (sunspot field) is created by the radial diffential rotation and stored in the tachocline.
- And the mean poloidal field (coronal field) is created by turbulence (or perhaps by a dynamic α-effect) in the lower reaches of the convection zone

Interface/Deep-Seated Dynamo



- **PROS**
  - **The radial shear provides a natural mechanism for generating a strong toroidal field**
  - **The stable stratification enables the field to be stored and stretched to a large value.**
  - **As the mean magnetic field is stored away from the convection zone, the α-effect is not suppressed**
  - **Separation of large and small-scale magnetic helicity**
- **CONS**
  - **Relies on transport of flux to and from tachocline – how is this achieved?**
  - **Delicate balance between turbulent transport and fields.**
  - **"Painting ourselves into a corner"**

Conclusions/Speculations and Annoying Questions

- Why does mean-field theory work so well?
  - Input parameters need to be constrained
    - Requires a full understanding of MHD turbulence
      - Turbulent α-effect
      - Turbulent diffusion
    - Measurement of mean flows.
- What can serious(?) computations teach us
  - Small scale (parts of the jigsaw)
  - Large scale (global dynamics)
- We can learn a lot from the mathematical structure of the equations.