





Cascade and dissipation of solar wind turbulence from MHD to electron scales

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Outline

- 1. Solar wind turbulence vs heating
- 2. The problem of measuring spatial properties of space plasma turbulence
- 3. 3D spatial spectra and anisotropies of MHD turbulence in the solar wind (Cluster data)
- 4. Kinetic scales in the SW: Some hotly debated question vs Cluster observations
 - > Cascade or dissipation below ρ_i ?
 - > The scaling: power-law? Exponential? Others?
 - > The nature of the cascade: KAW? whistler? Others?
 - > The nature of the dissipation: wave-particle interactions? Current sheets/Reconnection?
 - > Weak vs strong turbulence? Monofractality vs multifractality?
- 5. Conclusions & perspectives (turbulence & the future space missions)

Turbulence in the Univers



Performances limited

1	N~10 ⁶ cm ⁻³						
	$T_i \sim 10^{12} K$						
	B~ 1	10 ⁶ nT					
M100 galaxy 10 ²³ m							
Eagle nebula 10 ¹⁸ m							
oy plasma turbulence							
Strong pressure gradie ⇒ Instabilities							
1		$\hat{\nabla}$					
		Turbulent system	n				
1		Л					

It is observed from quantum to cosmological scales!

It controls mass transport, energy transfers & heating, *magnetic reconnection in plasmas*, ...



Prediction & control of turbulent transport



Near-Earth space plasmas

[Scheckochihin et al., ApJ, 2009]

$$\beta = \frac{\text{Pression thermique}}{\text{Pression magnétique}} \approx 0.4 \frac{NT}{B^2}$$



[Vaivads et al., Plasma Phys. Contr. Fus., 2009]

Parameter	Solar wind	Warm	Warm Accretion flow	
Taranicici	at 1 AU ^(a)	ionized ISM ^(b)	near Sgr $A^{*(c)}$	clusters (cores) ^(d)
$n_e = n_i, {\rm cm}^{-3}$	30	0.5	10 ⁶	6×10^{-2}
T_e, K	$\sim T_i^{(e)}$	8000	1011	3×10^7
T_i , K	5×10^{5}	8000	$\sim 10^{12({ m f})}$? ^(e)
<i>B</i> , G	10^{-4}	10^{-6}	30	7×10^{-6}
β_i	5	14	4	130
$v_{\rm this}$ km s ⁻¹	90	10	10 ⁵	700
v_A , km s ⁻¹	40	3	7×10^{4}	60
U, km s ^{-1(f)}	~ 10	~ 10	$\sim 10^4$	$\sim 10^2$
$L, \mathrm{km}^{(\mathrm{f})}$	$\sim 10^5$	$\sim 10^{15}$	$\sim 10^{8}$	$\sim 10^{17}$
$(m_i/m_e)^{1/2}\lambda_{\rm mfpi}$, km	1010	2×10^8	4×10^{10}	4×10^{16}
$\lambda_{mfpi}, km^{(g)}$	3×10^8	6×10^6	10^{9}	10^{15}
ρ_i , km	90	1000	0.4	104
ρ_e , km	2	30	0.003	200



The solar wind

The solar wind plasma is generally:

- Fully ionized (H⁺, e⁻)
- Non -relativistic ($V_A \ll c$), V~350-800 km/s
- Collisionless



Le champs magnétique structure la couronne Eclipse Corona Aug. 11,1999 Iran(IAP-CNRS)/Lasco(SOHO) 1000Speed (km s^{-1}) ULYSSES/SWOOPS Los Alamos 1000 ULYSSES/MAG NASA/GSFC) Imperial College Mauna Loa MK3 (HAO) Outward IMF LASCO C2 (NRL) Inward IMF 1000

La couronne chaude crée l'héliosphère

Phenomenology of turbulence

NS equation:



 $k^{-5/3}$



L = forcing scale

" Big whorls have little whorls That feed on their velocity, And little whorls have lesser whorls And so on to viscosity"



Lewis Fry Richardson (1920)

Courtesy of A. Celani

• Hydro: Scale invariance down to the dissipation scale $1/k_d$

k_i

Inertial range

• Collisionless Plasmas: - Breaking of the scale invariance at $\rho_{i,e} d_{i,e}$

- Absence of the viscous dissipation scale $1/k_d$

k_d

Solar wind turbulence

Typical power spectrum of magnetic energy at 1 AU

Does the energy cascade or dissipate below the ion scale ρ_i ?





How to analyse space turbulence ?

Turbulence theories generally predict spatial spectra: K41 ($k^{-5/3}$); IK ($k^{-3/2}$), Anisotropic MHD turbulence ($k_{\perp}^{-5/3}$), Whistler turbulence ($k^{-7/3}$), ...

 10^{2} 06:14:40-06:25:00 c - 1.65 10^{0} -3.96 10^{-2} nT²/Hz -2.82 10-4 10⁻⁶ -3.53 10^{-8} 10^{-10} 0.01 0.10 1.00 10.00 100.00 Frequency (Hz)

But measurements provide only temporal spectra (generally with different power laws at differe)

How to infer *spatial spectra* from *temporal* ones measured in the spacecraft frame? $B^2 \sim \omega_{sc}^{-\alpha} \Rightarrow B^2 \sim k_{//}^{-\beta} k_{\perp}^{-\gamma}$?

Example of measured spectra in the SW

The Taylor frozen-in flow assumption

In the solar wind (SW) the Taylor's hypothesis can be valid *at MHD scales*

High SW speeds: V ~600km/s >> V_{ϕ} ~ V_{A} ~50km/s \Rightarrow

$$\omega_{spacecraft} = \omega_{plasma} + \mathbf{k.V} \approx \mathbf{k.V} \neq k_V V$$

 \Rightarrow Inferring the *k*-spectrum is possible with one space craft

But only along one single direction

1. At MHD scales, even if the Taylor assumption is valid, inferring 3D *k*-spectra from an *w*-spectrum is impossible

2. At sub-ion and electron scales scales $V\phi$ can be larger than $V_{sw} \Rightarrow$ The Taylor's hypothesis is invalid



1 & 2 ⇒ Need to use multi-spacecraft measurements and appropriate methods to infer 3D k-spectra

Anisotropy and the critical balance conjecture

The critical balance conjecture [Goldreich & Sridhar, 1995]: Linear (Alfvén) time ~ nonlinear (turnover) time $\Rightarrow \omega \sim k_{//} V_A \sim k_\perp u_\perp$ $\Rightarrow k_{//} \sim k_\perp^{2/3}$

See also [Boldyrev, ApJ, 2005] and [Galtier et al., Phys. Plasmas, 2005]



Single satellite analysis \rightarrow use of the Taylor assumption: $\omega_{sc} \sim k. V_{sw} \sim k_v V_{sw}$

 $V//B → k_v = k_{//}$ Assumes axisymmetry $V ⊥ B → k_v = k_⊥$ around B





 $\Theta_{BV} \rightarrow 0 \Rightarrow B^2 \sim k_{//}^{-2} \Rightarrow Partial \text{ evidence of the critical}$ balance [Horbury et al., PRL, 2008]

Results confirmed by Podesta, ApJ, 2009

See also Chen et al., PRL, 2010



The ESA/Cluster mission

The first multispacecraft mission: 4 identical satellites

Objetives:

3D exploration of the Earth magnetosphere boundaries (magnetopause, bow shock, magnetotail) & SW

➤ Mesurements of 3D quantities:
J=∇xB, ...

Fundamental physics: turbulence, reconnection, particle acceleration, ...



Different orbits and separations $(10^2 \text{ to } 10^4 \text{ km})$ depending on the scientific goal

The 4 satellites before launch



The *k*-filtering technique

Interferometric method: it provides, by using a NL filter bank approach, an optimum estimation of the 4D spectral energy density $P(\omega,k)$ from simultaneous multipoints measurements [Pinçon & Lefeuvre; Sahraoui et al., 03, 04, 06, 10; Narita et al., 03, 06,09]



We use $P(\omega, \mathbf{k})$ to calculate

- 1. 3D ω -k spectra \Rightarrow plasma mode identification e.g. Alfvén, whistler
- 2. 3D *k*-spectra (anisotropies, scaling, ...)

Measurable spatial scales

Given a spacecraft separation *d* only one decade of scales $2d < \lambda < 30d$ can be correctly determined

• $\lambda_{min} \cong 2d$, otherwise *spatial* aliasing occurs.

• $\lambda_{max} \cong 30d$, because larger scales are subject to important uncertainties



 $\omega_{sat} \sim kV \Longrightarrow f_{max} \sim k_{max} V / \lambda_{min} (V \sim 500 \text{ km/s})$

- $d \sim 10^4 \text{ km} \Rightarrow \text{MHD scales}$
- $d \sim 10^2 \text{ km} \Rightarrow \text{Sub-ion scales}$

• $d \sim 1 \text{ km} \Rightarrow \text{Electron scales}$ (*but not accessible with Cluster: d>100*)

1- MHD scale solar wind turbulence

Position of the Quartet on March 19, 2006

000		Position			
time:					
2006-03-3	19 20:30)			
Coordinate	System:				
GSE					
Satellite	Color	x	Y	z	
Cluster-1		15.038	-6.569	-9.299	
CI		15 130	-7 034	-8 672	
Cluster-2		13.135	-1.034	-0.072	
Cluster-2 Cluster-3		13.979	-7.397	-10.41	



Data overview

FGM data (CAA, ESA)



Ion plasma data from CIS (AMDA, CESR)









To compute reduced spectra we integrate over 1. all frequencies f_{sc} : $\widetilde{P}(\mathbf{k}) = \sum_{ky,k_z} P(f_{sc},\mathbf{k})$ 2. all $k_{i,j}$: $\widetilde{\widetilde{P}}(k_x) = \sum_{ky,k_z} \widetilde{P}(k_x,k_y,k_z)$

Anisotropy of MHD turbulence along B_o and V_{sw}

Turbulence is not axisymmetric (around B) [see also Sahraoui, PRL, 2006]



[Narita et al., PRL, 2010]



The anisotropy $(\perp B)$ is along $V_{sw} \rightarrow SW$ expansion effect ?[Saur & Bieber, JGR, 1999]

Kinetic (sub-ion scale) turbulence in the SW

I- Theoretical predictions on small scale turbulence

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \frac{1}{en} \mathbf{J} \times \mathbf{B} - \frac{\nabla P_e}{en} + \dots$$

1. Fluid models (Hall-MHD)

- Whistler turbulence (E-MHD): (Biskamp *et al.*, 99, Galtier, 08)
- Weak Turbulence of Hall-MHD (Galtier, 06; Sahraoui et al., 07)

2. Gyrokinetic theory: $k_{//} << k_{\perp}$ and $-\omega << \omega_{ci}$ (Schekochihin *et al.* 06; Howes *et al.*, 11)



Other numerical predictions on electron scale turbulence

2D PIC simulations gave evidence of a power law dissipation range at $k\rho_e$ >1



Figure 4. Spectrum of magnetic fluctuation $|\delta B|^2 / |B_0|^2$ in the parallel direction $k\rho_{e\parallel}$. The noise level curve is in red. The power-law best fits are superimposed. (A color version of this figure is available in the online journal.)



Figure 5. Spectrum of magnetic fluctuation $|\delta B|^2/|B_0|^2$ in the perpendicular direction $k\rho_{dM}$. The noise level curve is in red. The power-law best fits are superimposed.

[Camporeales & Burgess, ApJ, 2011]

3D PIC simulations of whistler turbulence : $k^{-4.3}$ at $kd_e > 1$



Chang & Gary, GRL 2011

2- Observations of kinetic SW turbulence

What happens to the energy at, and below, the ion scale ρ_i (not f_{ci}): a total dissipation or a new cascade?



Leamon et al, 98; Goldstein et al., 94



First evidence of a cascade from MHD to electron scale in the SW $\underline{B_{II}^{2}}($

- 1. Two breakpoints corresponding to ρ_i and ρ_e are observed.
- 2. A clear evidence of a new inertial range ~ $f^{-2.5}$ below ρ_i
- 3. First evidence of a dissipation range ~ f^{-4} near the electron scale ρ_e







STAFF-SC sensitivity floor

[Alexandrova et al., 2009, 2013]



Similar observations from STAFF-SA data, but the spectra were fit by an exponential model



The largest survey of Cluster/STAFF-SC data

A better correlation with ρ_e than with d_e



[Sahraoui+, 2013; Huang+, 2013]



Whistler or KAW turbulence?

- 1. Large (MHD) scales (L> ρ_i): strong correlation of E_y and B_z in agreement with **E**=-**VxB**
- 2. Small scales (L< ρ_i): steepening of B² and enhancement of E² (however, strong noise in E_v for f>5Hz)
 - ⇒ Good agreement with GK theory of Kinetic Alfvén Wave turbulence

Howes *et al*. PRL, 11



FGM, STAFF-SC and EFW data



Theoretical interpretation : KAW turbulence

Linear Maxwell-Vlasov solutions: $\Theta_{\mathbf{kB}} \sim 90^{\circ}, \beta_{i} \sim 2.5, T_{i}/T_{e} \sim 4$

The Kinetic Alfvén Wave solution extends **down to** kρ_e~1 with ω_r <ω_{ci} [See also Podesta, ApJ, 2010]





E/B : KAW theory vs observations

$$\omega_r = k_{//} V_A k_\perp \rho_i / \sqrt{\beta_i + 2/(1 + T_i / T_e)}$$

 \succ Lorentz transform: $\mathbf{E}_{sat} = \mathbf{E}_{plas} + \mathbf{V} \mathbf{X} \mathbf{B}$

> Taylor hypothesis to transform the spectra from f (Hz) to $k\rho_i$

1. Large scale ($k\rho_i < 1$): $\delta E / \delta B \sim V_A$

2. Small scale $(k\rho_i > 1): \delta E/\delta B \sim k^{1.1} \Rightarrow$ in agreement with GK theory of KAW turbulence $\delta E^2 \sim k_{\perp}^{-1/3} \& \delta B^2 \sim k_{\perp}^{-7/3} \Rightarrow \delta E/\delta B \sim k$

3. The departure from linear scaling $(k\rho_i \gtrsim 10)$ is due to noise in Ey data



Magnetic compressibility



[Sahraoui+, ApJ, 2012]

Additional evidence of KAW at $k\rho_i \gtrsim 1$



[Kiyani+, ApJ, 2012; Podesta+, 2012]

3D k-spectra at sub-proton scales of SW turbulence

Conditions required:

- 1. Quiet SW: NO electron foreshock effects
- Shorter Cluster separations (~100km) to analyze subproton scales
- 3. Regular tetrahedron to infer actual 3D *k*-spectra [Sahraoui et al., JGR, 2010]
- 4. High SNR of the STAFF data to analyse HF (>10Hz) SW turbulence.



20040110, 06h05-06h55

3D *k*-spectra at sub-proton scales

We use the *k*-filtering technique to estimate the 4D spectral energy density $P(\omega,k)$



20040110 (d~200km)



We use $P(\omega, \mathbf{k})$ to calculate

1. 3D ω -k spectra

2. 3D *k*-spectra (anisotropies, scaling, ...)



Turbulence is

• \perp B₀ but non axisymmetric

• Quasi-stationary ($\omega_{plas} \sim 0$ although $\omega_{sat} \sim 20 \omega_{ci}$)

Comparison with the Vlasov theory

Turbulence cascades following the Kinetic Alfvén mode (KAW) as proposed in Sahraoui et al., PRL, 2009

→ Rules out the cyclotron heating

→ Heating by p-Landau and e-Landau resonances

[Sahraoui et al., PRL, 2010]



Limitation due to the Cluster separation (d~200km)

3D *k*-spectra at sub-ion scales

- First *direct* evidence of the breakpoint near the proton gyroscale in k-space (no additional assumption, e.g. Taylor hypothesis, is used)
- 2. Strong steepening of the spectra below $\rho_i \rightarrow A$ *Transition Range* to dispersive/electron cascade





Dissipation through reconnection/current sheets

Large scale laminar current sheet: reconnection can occur and the can be heated or accelerated (e..g. jets)



[Zhong+, Nature Physics, 2010]



Turbulent current sheets

[Lazarian & Vishniac, 1999]





2D Hall-MHD simulation of turbulence: evidence of a large number of reconnecting regions



Dissipation by wave-particle interaction or via reconnection?

Good correlation between enhanced T_p and threshold of linear kinetic instabilities

Good correlation between enhanced high shear B angles and the threshold of linear instabilities !!



Statistical approach to small scale SW turbulence

Which statistical description applies to sub-proton scale SW turbulence:

- 1. Weak or strong turbulence?
- 2. If strong, then is it *self-similar/monofratal or intermittent/multifractal*?

1. Strong vs Weak Turbulence:

Often it has been argued that small scale/high frequency turbulence in the solar wind is a weak turbulence because $|\delta \mathbf{B}|/\mathbf{B} <<1$

This is wrong !

Because *only* the ratio nonlinear/linear times (or terms) for *each physical* system can indicate how weak or strong is the turbulence

Let us consider the example of Incompressible MHD

Incompressible Alfvénic Turbulence

$$\partial_{t} z^{\pm} \mp v_{A} \cdot \nabla z^{\pm} + z^{\mp} \cdot \nabla z^{\pm} = -\nabla p$$

Linear term: $k_{//}v_A z^+$

Nonlinear term: $k_{\perp}u_{\perp}z^+$

Ratio of nonlinear to linear terms:

$$\chi = \frac{k_{\perp} u_{\perp}}{k_{\mu} v_{A}}$$

 $\begin{aligned} \chi &<<1 \Rightarrow \text{Weak turbulence with } k_{||}v_A >> k_{\perp}u_{\perp} \\ \chi &\sim 1 \Rightarrow \text{Strong turbulence with } k_{||}v_A \sim k_{\perp}u_{\perp} (\text{or } \omega \sim \omega_{\text{NL}} \Rightarrow Critical balance conjecture) \end{aligned}$ For anisotropy $k_{\perp} >> k_{||}$ we have STRONG turbulence $(\chi \sim 1)$ even when $\frac{u_{\perp}}{v_A} \sim \frac{\delta B}{B_0} <<1$

⇒ One has to give up using mere criteria, e.g. |δB|/B<<1, to discriminate within the data between weak/strong turbulence theories

Other alternatives?

- 1. Estimation of the linear/nonlinear times of the turbulence from the data
 - → But it is difficult because this generally requires to know *accurately* the nature of the turbulence and its spatial scales (|| and \perp)
- Estimating phase coherence *directly* from the measured Fourier phases of the turbulence from the data using, e.g., Surrogate data [Hada et al., 2003; Sahraoui, PRE, 2008; Sahraoui & Fauvarque, in prep.]

2. Monfractality vs multifractality in the dispersive range:



[Kiyani et al., PRL, 2009]

n_e ~ 4 cm⁻³ ion β ~ 2 V_A ~ 50 km s⁻¹ Ti ~ 103 eV |B|~4 nT





Evidence of monofractality (self-similarity) at sub-proton scales, while MHD-scales are multifractal (intermittent)
[See also Alexandrova et al., ApJ, 2008]

Stuctures functions:



$$S^m(\tau) = S^m(1)\tau^{\zeta(m)}$$



Conclusions

The Cluster data helps understanding crucial problems of astrophysical turbulence:

- Its nature and anisotropies in k-space at MHD and sub-ion scales
- Its cascade and dissipation down to the electron gyroscale $\rho_e \Rightarrow$ electron heating and/or acceleration by turbulence
- Strong evidences of KAW turbulence ($\omega < \omega_{ci}, k_{//} < k_{\perp}$) Heating by e-p-Landau dampings (no cyclotron heating)
- Importance of kinetic physics in SW turbulence
- Turbulence & dissipation are at the heart of the future space missions: NASA/MMS (2014), ESA/SO (2017), NASA/SPP (2019), TOR (????)

 \Rightarrow Need of multiscale measurements with appropriate spacecraft separations

Narita et al. PRL, 2010

104

 10^{3}

E [nT²km]



Sahraoui et al. PRL, 2010



La turbulence et les futures missions spatiales

MAGNETOSPHERIC MULTISCALE A SOLAR-TERRESTRIAL PROBE

4 NASA satellites, launch 2014

Higher resolution instrumentations

Small separations (~10km)

Equatorial orbites

UNLOCKING THE MYSTERIES OF Magnetic reconnection



Solar Orbiter Exploring the Sun-Heliosphere Connection

Launch 2017

Distance : 0.28 AU

In-situ measurements & remote sensing

Launch 2019

Distance : ~0.03 AU

In-situ measurements & remote sensing

Solar Probe Plus

National Aeronautics and Space Administration

Goddard Space Flight Center Greenbelt, Maryland 20771

TOR

Solving energy dissipation problem at kinetic scales in the solar wind

Mission proposed to ESA (2012) Currently under review with SNBS Recently proposed to CNES