Radiative processes in high energy astrophysical plasmas

École de physique des Houches

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Why radiation?

- ✓ Why is radiation so important to understand?
  - ✓ Light is a tracer of the emitting media
    - ✓ Geometry, evolution, energy, magnetic field...
  - ✓ Light can influence the source properties
    - ✓ cooling/heating, radiation pressure...
  - ✓ Light can be modified after its emission

- ✓ Why are high energy plasmas so important to study?
  - ✓ High energy particles are the most efficient emitters
  - ✓ They emit over an extremely wide range of frequencies

Outline

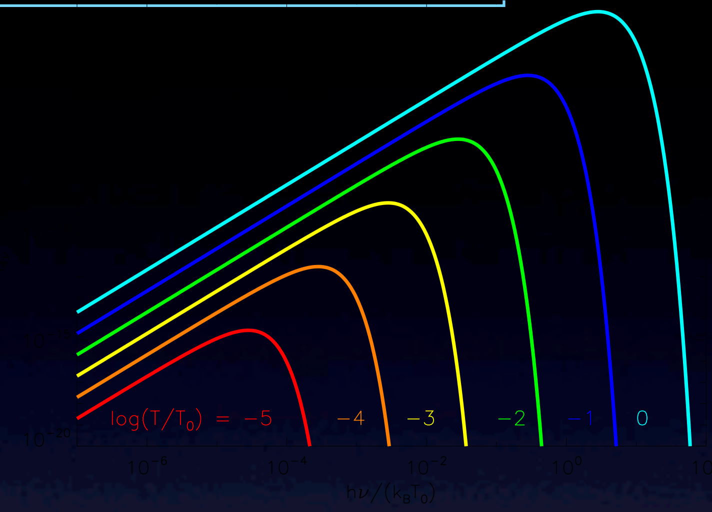
- Goals of this course:
  - Review the main high energy processes for continuum emission
    - Assumptions, approximations, properties
  - Show how they can be used to constrain the physics of astrophysical sources

- Main books/reviews:
    - Rybicki G.B. & Lightman A.P., 1979, Radiative processes in astrophysics, New York, Wiley-Interscience
    - Jauch J.M. & Rohrlich F., 1980, The theory of photons and electrons (2nd edition), Berlin, Springer
    - Aharonian F. A., 2004, Very High energy cosmic gamma radiation, World scientific publishing
    - Heitler W., 1954, Quantum theory of radiation, International Series of Monographs on Physics, Oxford: Clarendon
    - Blumenthal G.R. & Gould R.J., 1970, Reviews of Modern Physics

Contents

Introduction

Non black-body radiation



- ✓ Black-Body radiation is simple ideal limit
  - ✓ independent of internal processes, geometry...
  - ✓ Simple law
- ✓ No black body radiation if:
  - ✓ Optically thin media
    - ✓ <= finite source size and finite interaction cross sections
    - ✓ => absorption/emission/reflection features
  - ✓ Matter not at thermal equilibrium
  - ✓ <= low density, high energy plasmas
- ✓ Then, radiation properties depend on
  - ✓ The particle distributions
  - ✓ The microphysics: what processes?

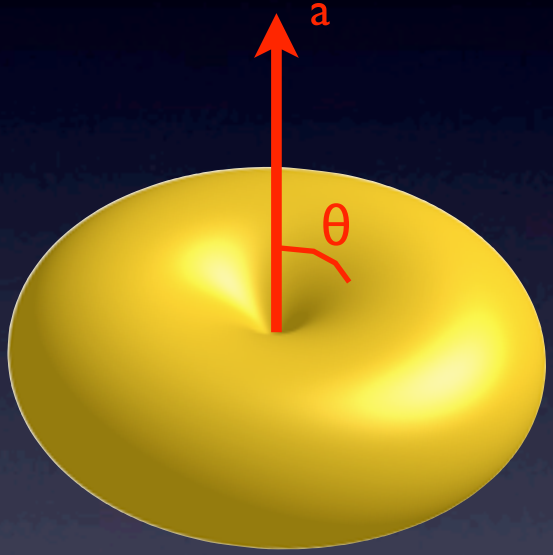
Radiation processes

- ✓ Lines (bound-bound):
  - ✓ Atomic, molecular transitions (radio to X-rays)
  - ✓ Nuclear transitions (γ-rays)
- ✓ Edges (bound-free):
  - ✓ ionization/recombination
- ✓ Nuclear reactions, decay and annihilations:
  - ✓ bosons, pions...
  - ✓ electron-positron, photon-photon
- ✓ Collective processes
  - ✓ Faraday rotation, Cherenkov radiation...
- ✓ <u>Free-free radiation of charged particles in vacuum:</u>
  - ✓ Cyclo-synchrotron radiation
  - ✓ Compton scattering
  - ✓ (Bremsstrahlung radiation)

I. Radiation of relativistic particles

Emission from non-relativistic particles

- ✓ Electro-magnetic field created by charges in motion:
  - ✓ Charged particles in uniform motion do not emit light
  - ✓ Only charged, accelerated particles can emit light
    - ✓ Liénard-Wiechert potentials (1898): (A,Φ) => E-B
    - ✓ Power: $P \propto \vec{E}^2$
    - ✓ Spectrum: $P_\nu \propto \left|\mathrm{FFT}(\vec{E})\right|^2$



- ✓ Emission of low-energy particles:
  - ✓ Total power: $P_e = \dfrac{2q^2a^2}{3c}$
  - ✓ Dipolar field perpendicular to acceleration: $\dfrac{\partial P_e}{\partial \Omega} = \dfrac{q^2a^2}{4\pi c}\sin^2\theta$
  - ✓ Polarization depends on the direction: $\vec{E} \propto \vec{n} \times (\vec{n} \times \vec{a})$
  - ✓ Is also the emission in the particle rest frame...

Change of frame

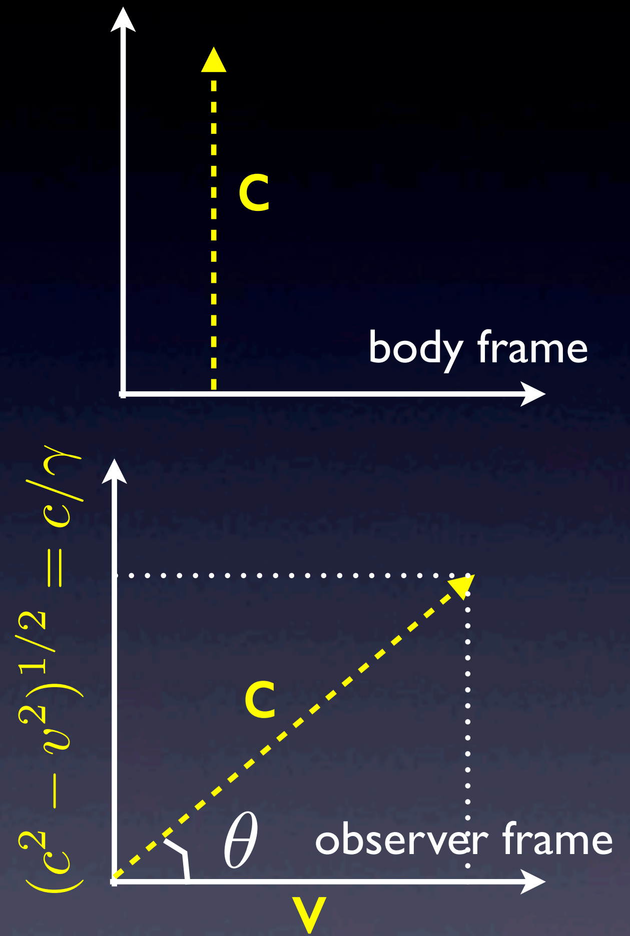
- Relativistic particles:
  - Velocity $\beta = v/c$
  - Lorentz factor $\gamma = 1/(1-\beta^2)^{1/2}$
  - Energy $E = \gamma mc^2 \qquad E_k = (\gamma - 1)mc^2$
  - Momentum $p = (\gamma^2 - 1)^{1/2} = \beta\gamma$

- Let's consider a particle
  - moving at velocity β as seen in the observer frame K in the parallel direction
  - emitting radiation in its rest frame K'

- Total emitted power:
  - Is a Lorentz invariant: $P_e = P'_e = \dfrac{2q^2 a'^2}{3c}$
  - Acceleration is not: $a'_\perp = \gamma^2 a_\perp \quad a'_\parallel = \gamma^3 a_\parallel$
  - Emission of relativistic particles strongly enhanced: $P_e = \dfrac{2q^2}{3c}\gamma^4\left(\gamma^2 a_\parallel^2 + a_\perp^2\right)$
  - High energy sources are amongst the most luminous sources...

Relativistic beaming

- Angles: an example
  - Emitting body moving at velocity v
  - Photon emitted perpendicular to motion in the body frame
  - Photon
    - must fly at c in all frames
    - observed with parallel velocity v
    - observed with perpendicular velocity c/γ
    - observed with an angle $\sin\theta = 1/\gamma$



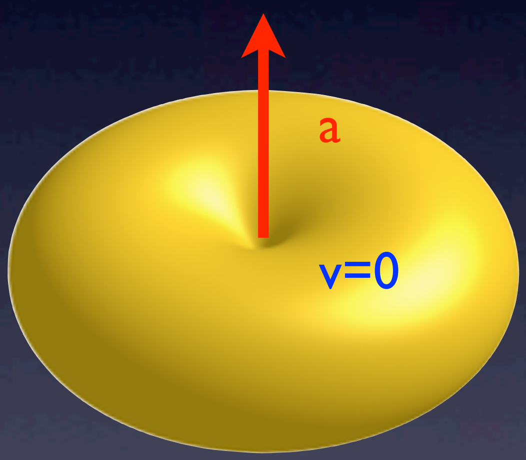
- Angular distribution of emission:

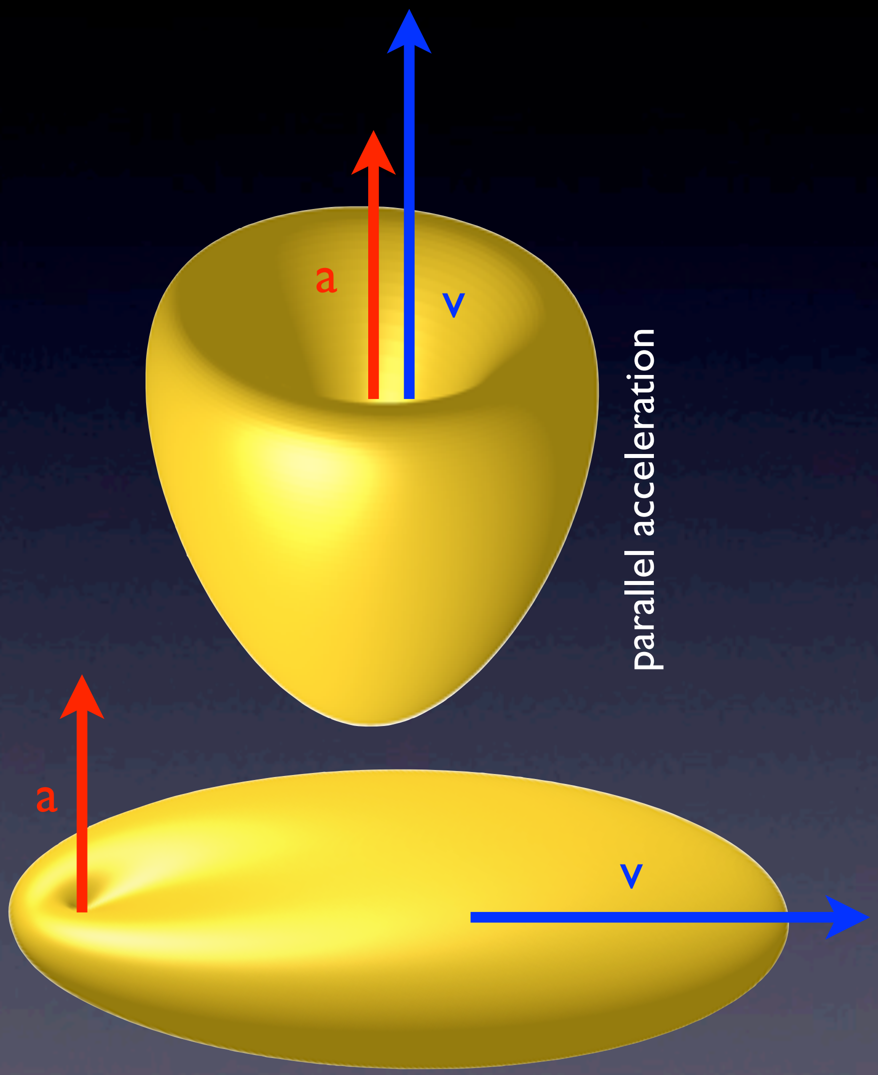
$$\frac{\partial P_r}{\partial\Omega} = \frac{1}{\gamma^4(1-\beta\cos\theta)^4}\frac{\partial P_e'}{\partial\Omega'}$$

  - beaming
  - enhancement

Relativistic beaming

- ✓ The dipolar emission of charge particles:
  - ✓ The angular distribution depends on the (a,v) angle





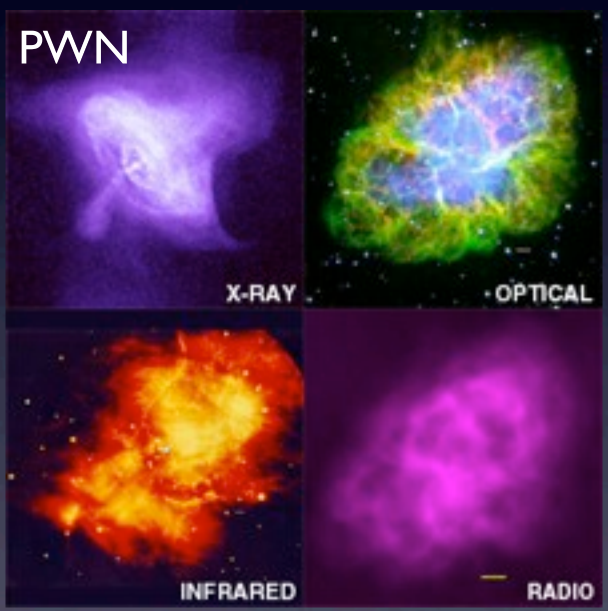
- ✓ Beaming is still present and $\theta \approx 1/\gamma$

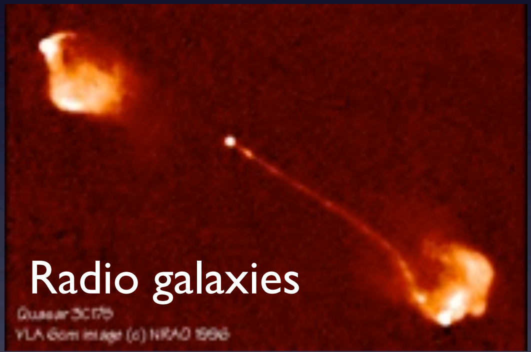
Cyclo-Synchrotron radiation

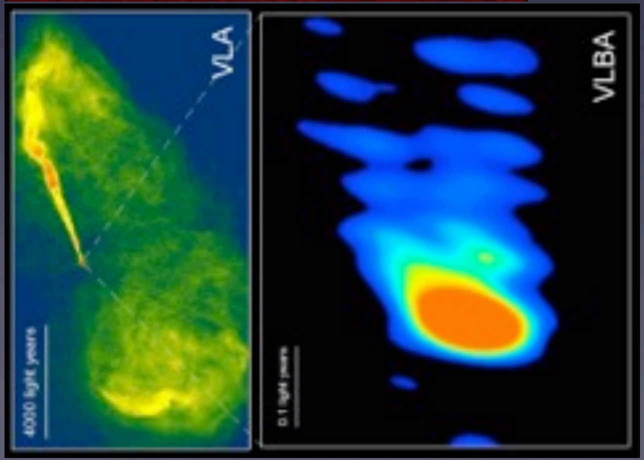
- Emitted power
- Spectrum
- Radiation from many particles
- Polarization
- Self-absorption

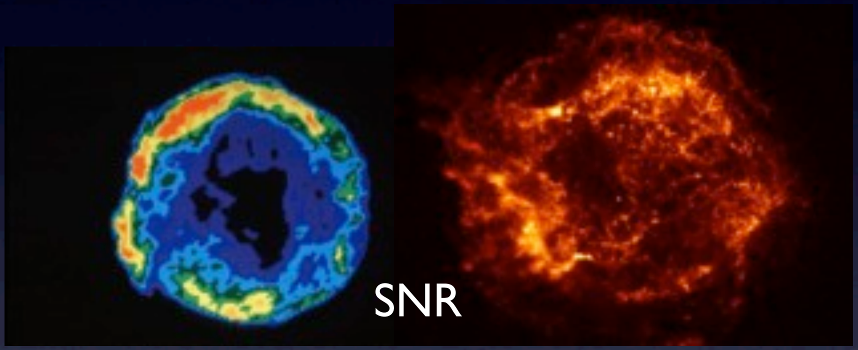
Synchrotron in astrophysics

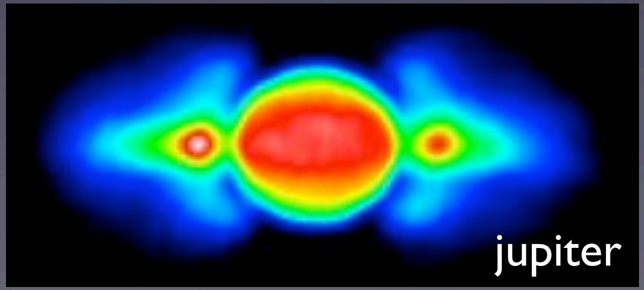
- ✓ Applications to astrophysical sources started in the mid 20th
- ✓ Most observations in radio but also at all wavelengths



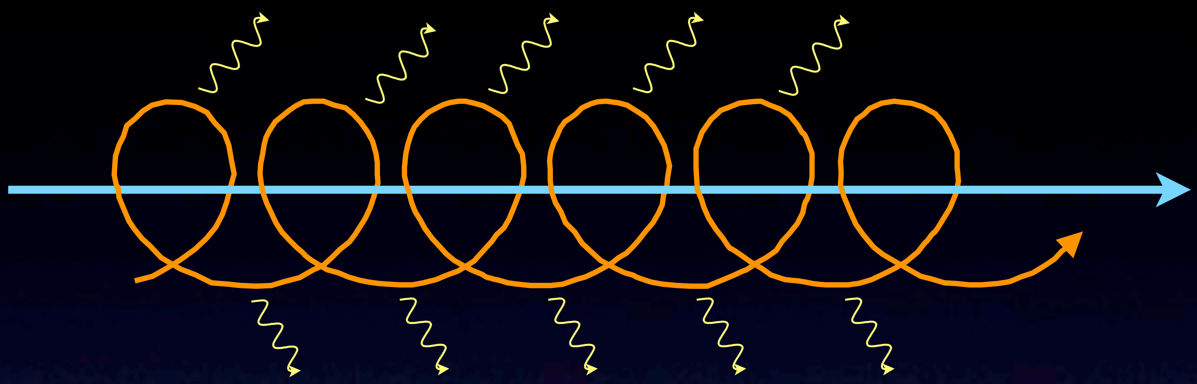


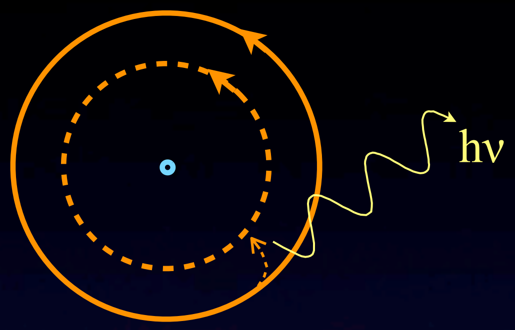






Cyclo-synchrotron radiation

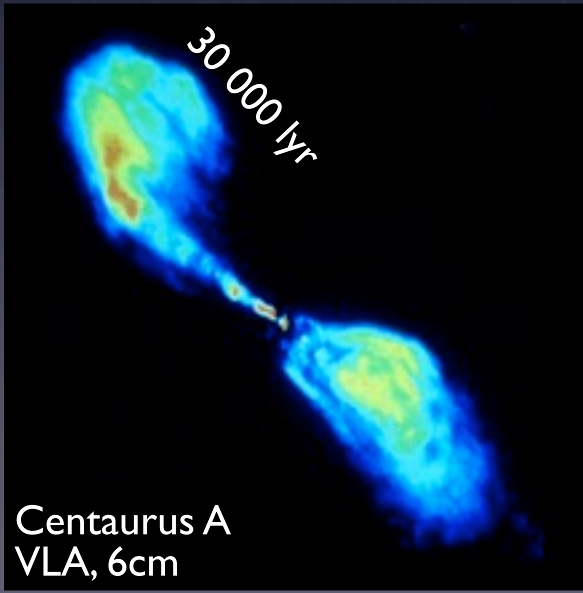




- Radiation from particles gyrating the magnetic field lines
- Relativistic gyrofrequency $\nu_B = \dfrac{qB}{2\pi mc}$ $\quad \nu_{B,r} = \dfrac{1}{\gamma}\dfrac{qB}{2\pi mc}$
- Assumptions:
  - Classical limit: $h\nu_B << mc^2$ $\quad B < B_c = 12 \times 10^{12}$ G
    - Otherwise: quantization of energies, Larmor radii...
    - Observable cyclotron scattering features in accreting neutron stars...
  - B uniform at the Larmor scale (parallel and perp)
    - ! no strong B curvature (pulsars and rapidly rotating neutron stars)
    - ! no small scale turbulence (at the Larmor scale)
    - ! no large losses ($t_{cool} >> 1/\nu_{B,r}$)
- Emission/Absorption

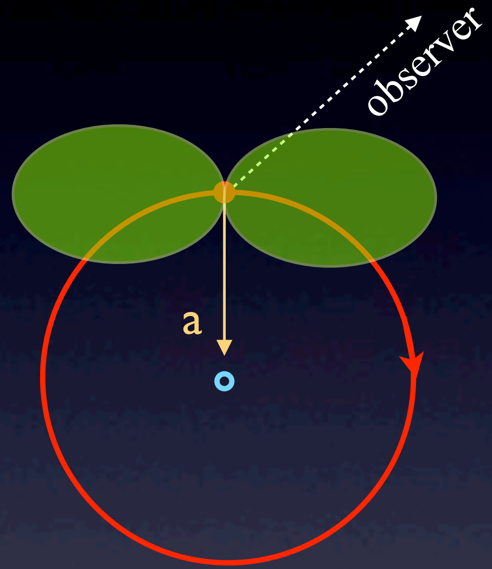
Emitted Power and cooling time

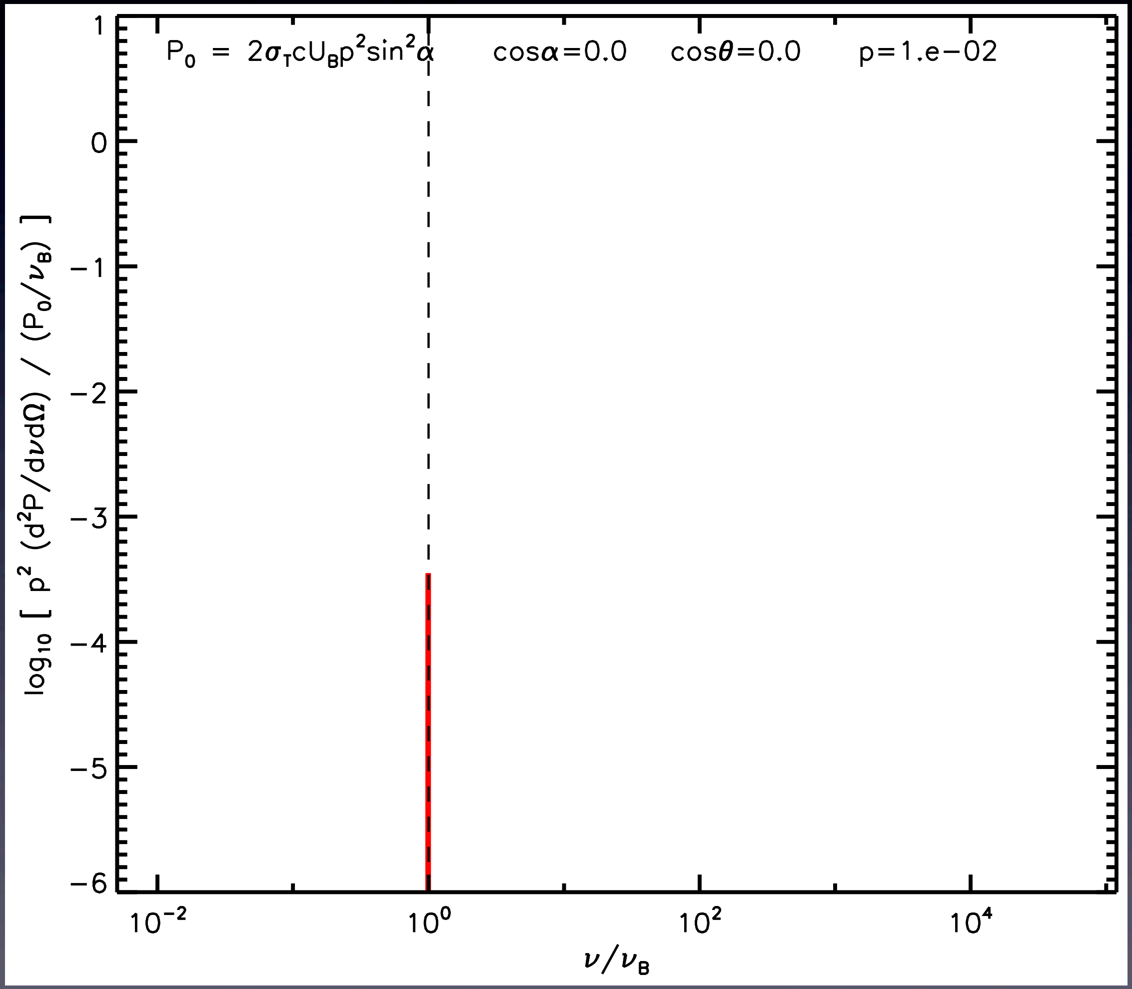
- Circular motion: $a = a_\perp = \frac{\nu_B}{2\pi\gamma} v_\perp$
- Emission of an accelerated particle: $P = \frac{2q^2}{3c^3}\gamma^4 a_\perp^2 = 2c\sigma_T U_B p_\perp^2$
  - Power goes as p^2
  - Power goes as B^2
- Isotropic distribution of pitch angles: $P = \frac{4}{3} c\sigma_T U_B p^2$
- Cooling time: $t_{\mathrm{cool}} = \frac{\gamma m c^2}{P_e} \approx \frac{25\mathrm{yr}}{B^2\gamma}$
  - ISM (B=1μG): $t_{cool} > t_{universe}$ as far as $\gamma < 10^3$
  - AGN jets (B=10μG, $\gamma = 10^4$): $t_{cool} = 10^7$yr (=travel time!)
- Maximal loss limit $t_{\mathrm{cool}} >> 1/\nu_{B,r}$ $\gamma^2 B < \frac{2q}{r_0^2}$



Emission Spectrum

Very low energy particles:



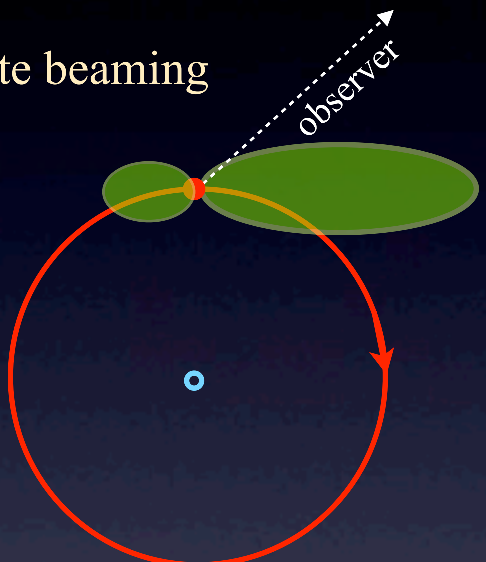


- ✓ Sinus modulation of the electric field at ν_B:
- ✓ Spectrum = one cyclotron line at $\nu_B = \nu_{B,r}$

Emission Spectrum

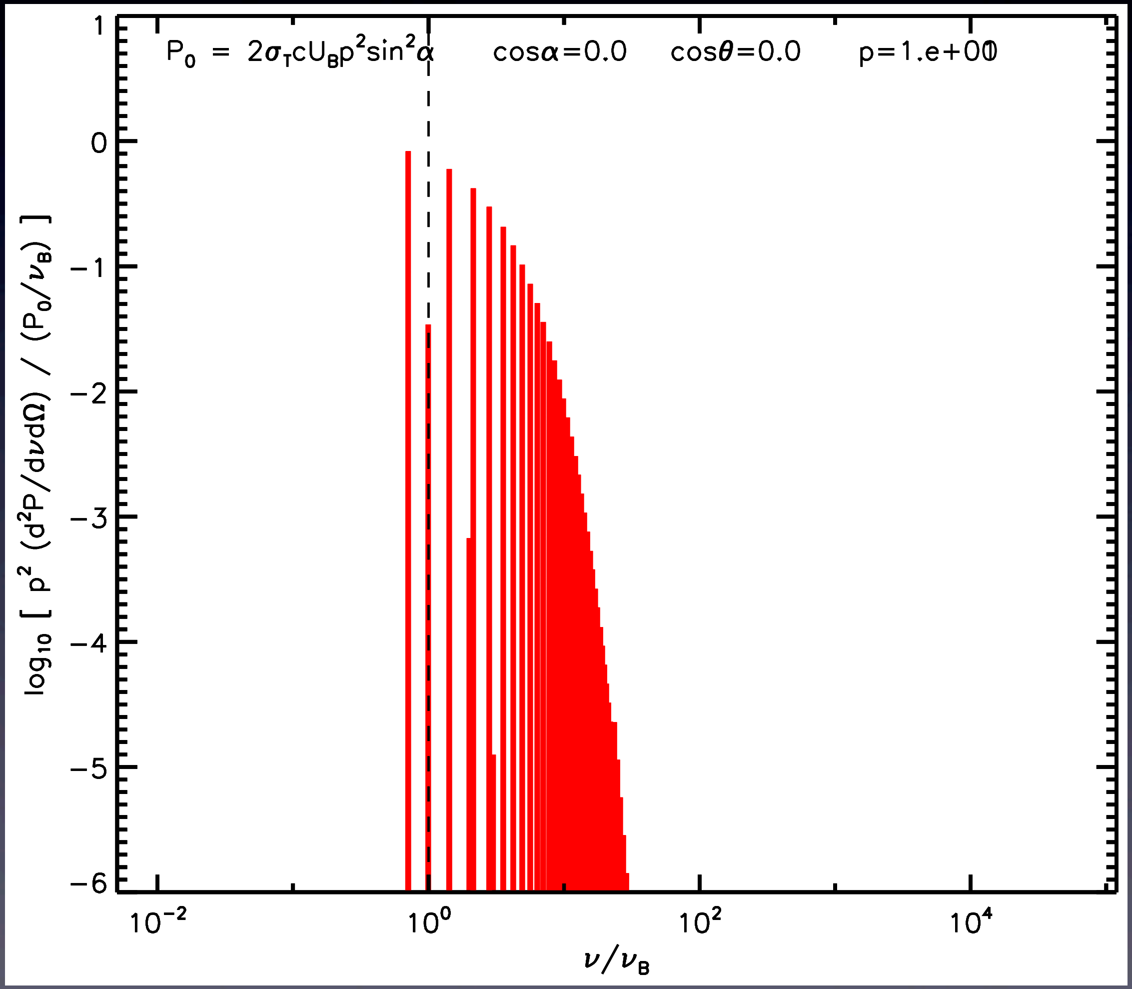
Mid-relativistic particles:

✓ Moderate beaming



✓ Asymmetrical modulation of the electric field at $\nu_{B,r}$:

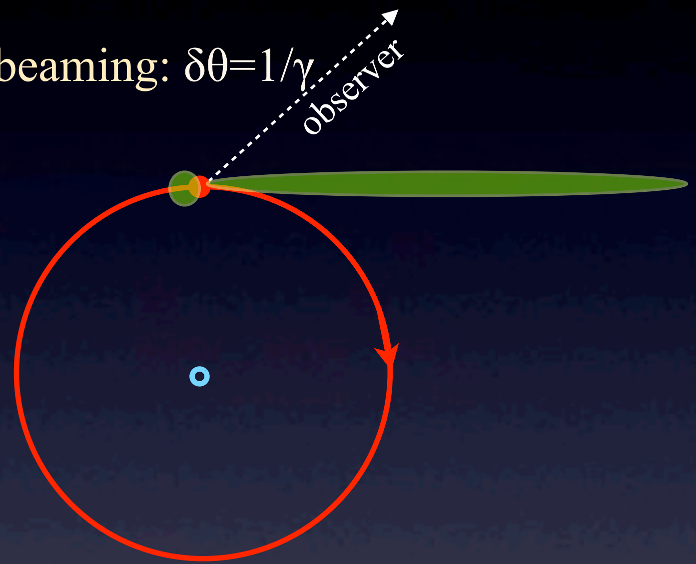
✓ Spectrum = many harmonic lines at $k\nu_{B,r}=k\nu_B/\gamma$



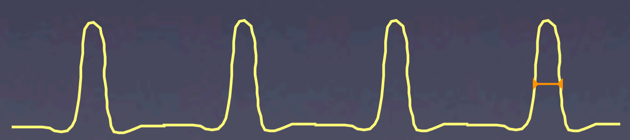
Emission Spectrum

Ultra-relativistic particles:

✓ Strong beaming: δθ=1/γ

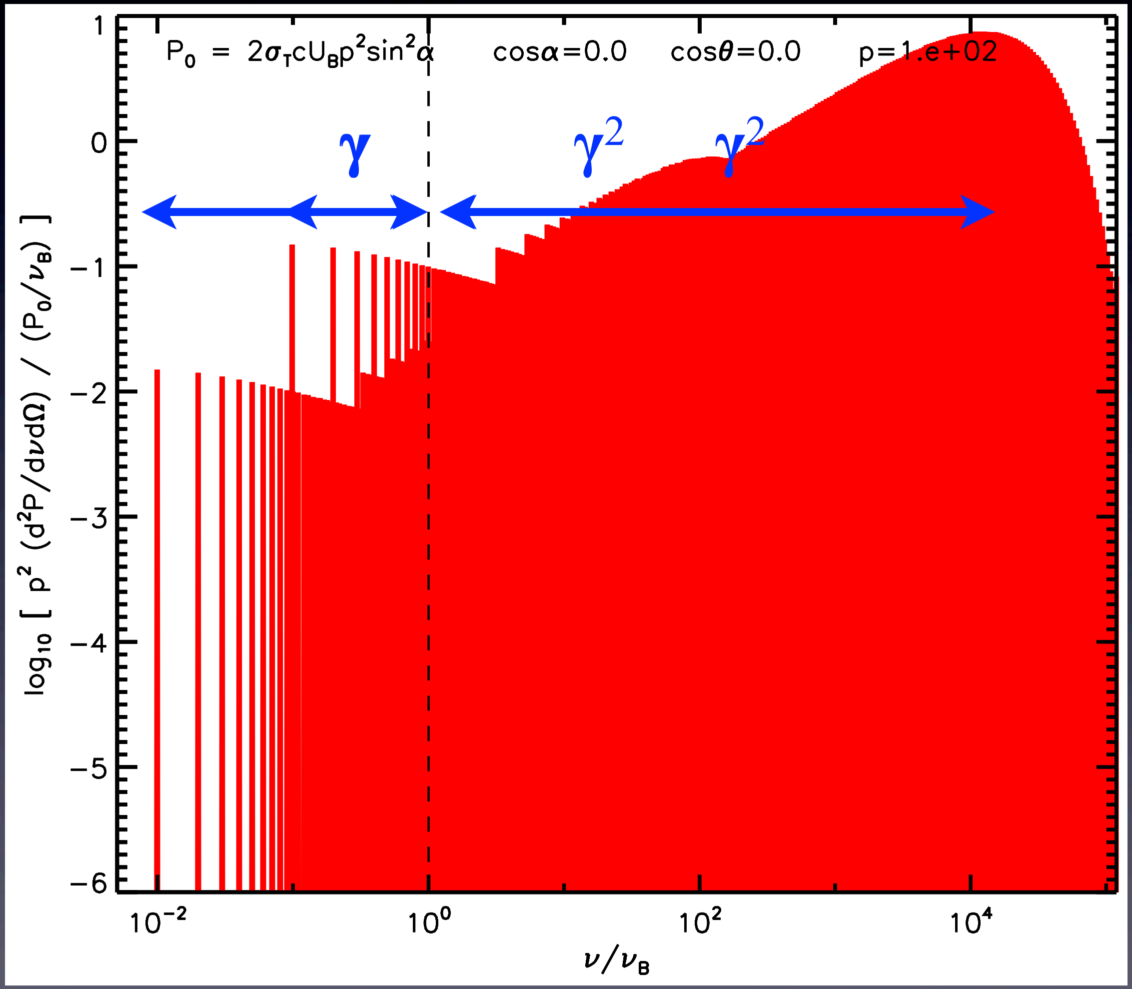


✓ Pulsed modulation of the electric field at ν_B:



$$\delta t \approx 1/(\nu_{B,r}\gamma^3)$$

✓ Spectrum = continuum up to $\nu_c = \frac{3}{2}\gamma^2\nu_B \sin\alpha$

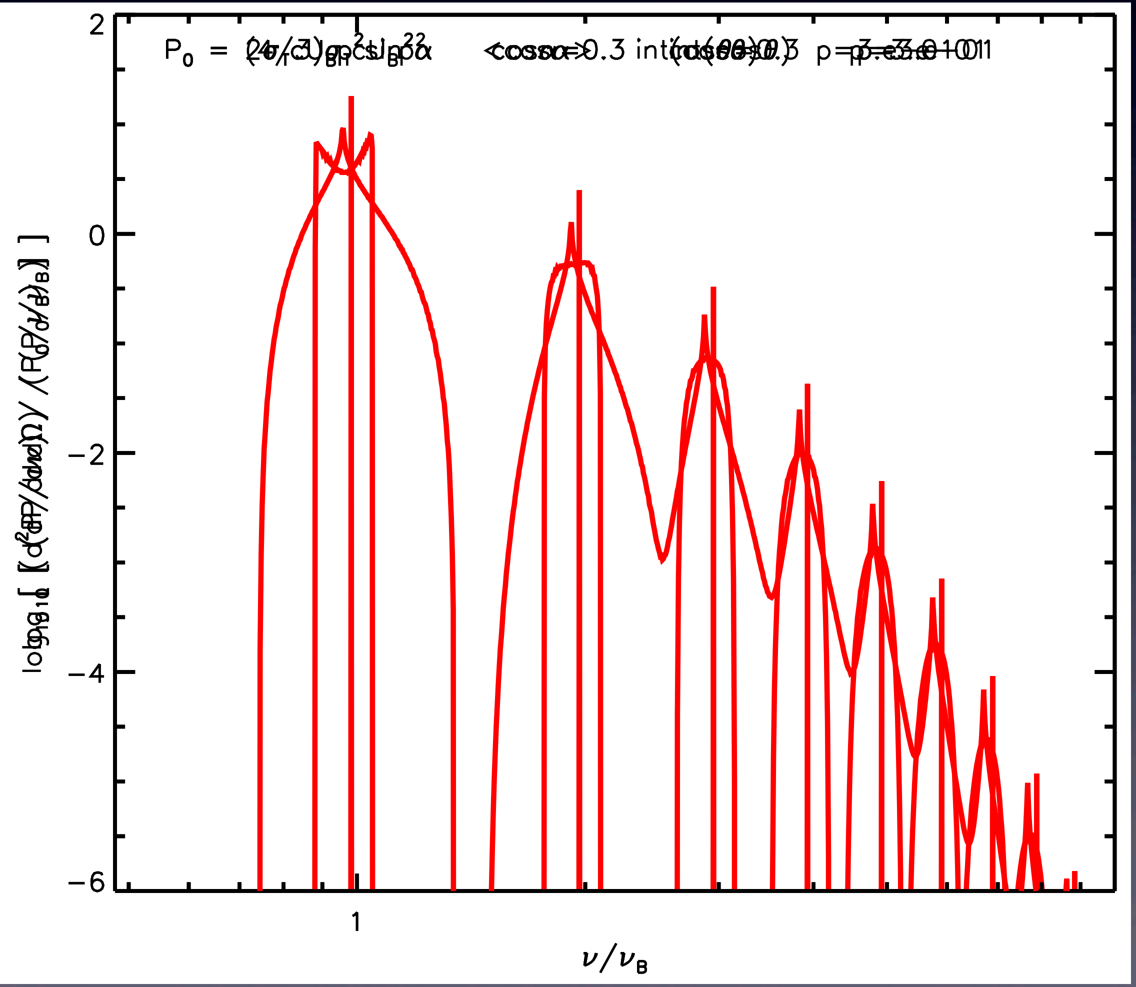


Emission Spectrum

$$\frac{\partial^2 P}{\partial(\nu/\nu_B)\partial\Omega} = 4\sigma_T c U_B \beta_\perp^2 \frac{\nu^2}{\nu_B^2} \sum_{n=1}^{\infty} \left[n^2 \frac{(\cos\theta - \beta_\parallel)^2}{(1-\beta_\parallel \cos\theta)^2} \frac{J_n^2(x)}{x^2} + J_n'^2(x) \right] \delta\left[\frac{\nu}{\nu_B}(1-\beta_\parallel\cos\theta) - \frac{n}{\gamma} \right]$$

$$x = (\nu/\nu_B)\beta_\perp \sin\theta$$

- Exact spectrum depends on:
  - the particle energy
  - the pitch angle α: $\cos\alpha = \beta_\parallel/\beta$
  - the observation angle θ with respect to B



- Line broadening results from integration over:
  - Observation direction
  - Particle pitch angle
  - Particle energy

Spectrum of Relativistic Particles

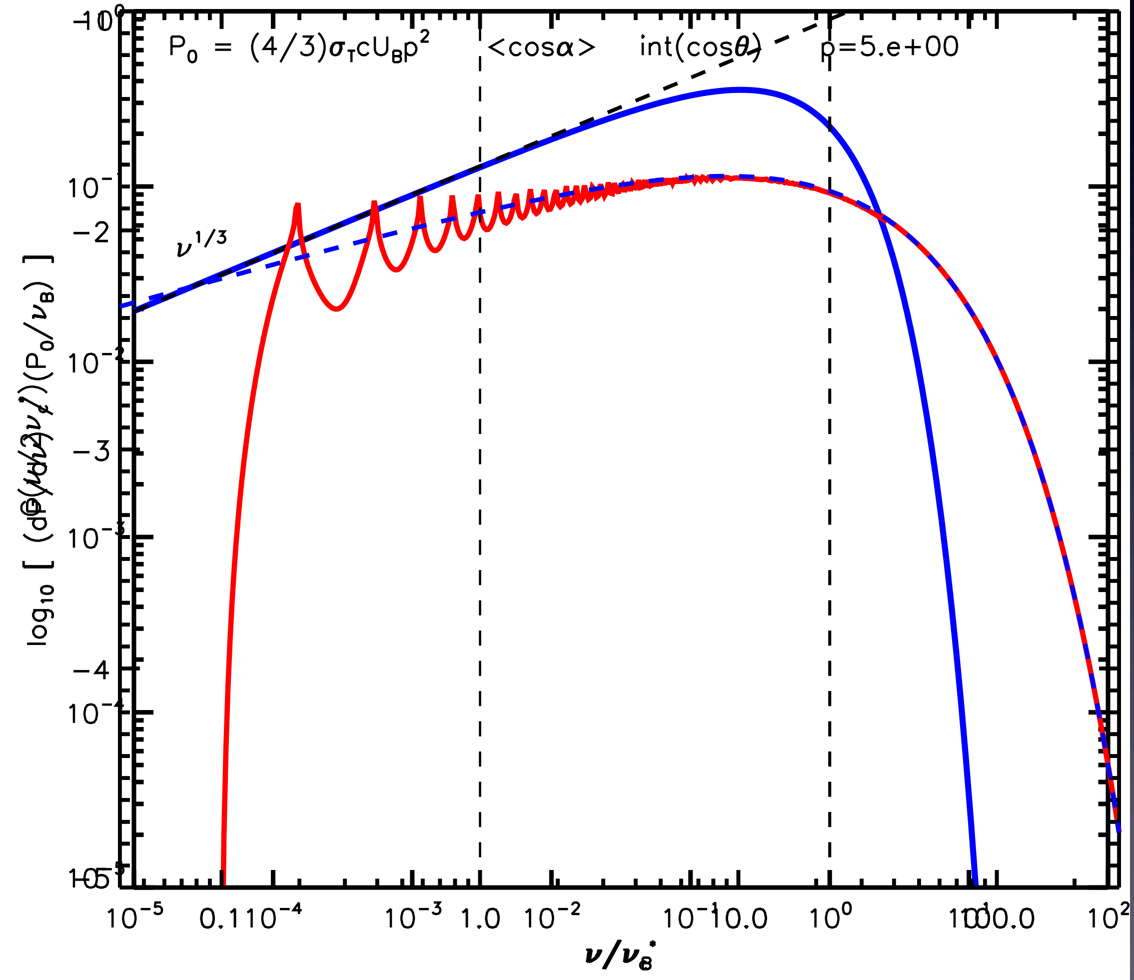
- High energy plasmas have a continuous spectrum: $\dfrac{\partial P}{\partial(\nu/\nu_B)} = 12\sqrt{3}\sigma_T c U_B G\left(\dfrac{\nu}{2\nu_c^*}\right)$

$$G(x) = x^2\left[K_{4/3}(x)K_{1/3}(x) - \frac{3x}{5}\left(K_{4/3}^2(x) - K_{1/3}^2(x)\right)\right]$$

- Peaks at the critical frequency:

$$\nu_c^* = \frac{3}{2}\nu_B\gamma^2 \propto B\gamma^2$$

- Most of the emission is at ν_c^*
  - AGN (B=10μG, γ=10^4): 10cm
  - Crab nebula (B=0.1mG, γ=10^7): 10 keV



- Highest possible energy of photons:

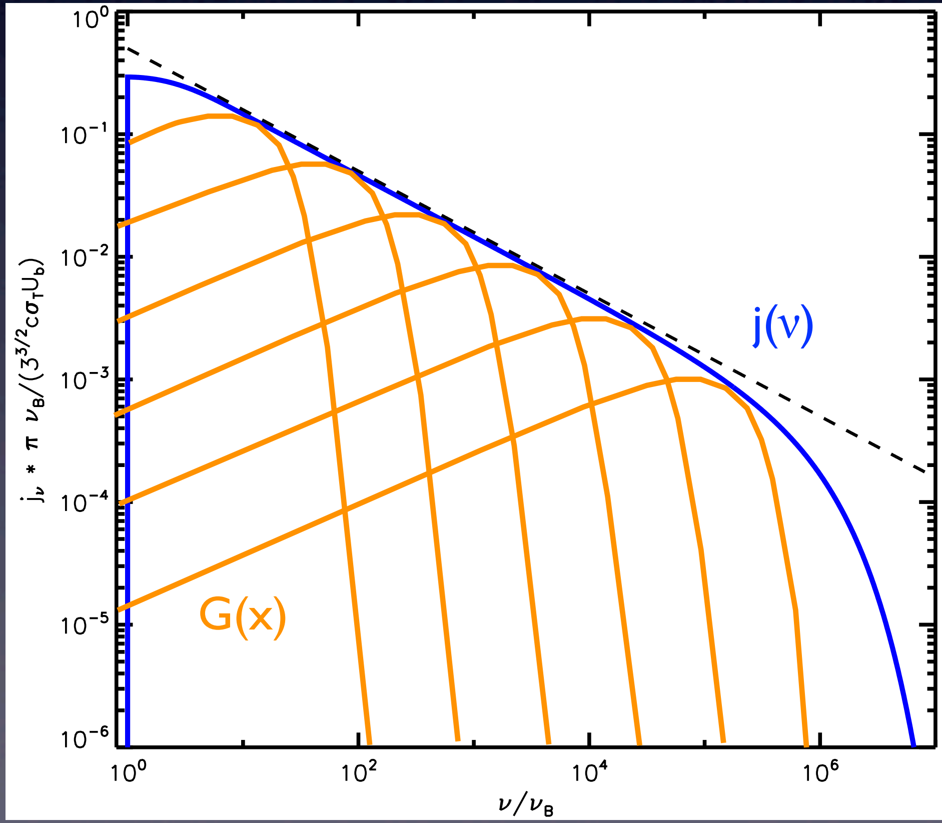
$$\gamma^2 B < \frac{2q}{r_0^2} \qquad h\nu = 3m_e c^2/\alpha_f \approx 70\text{MeV}$$

Spectrum of many particles

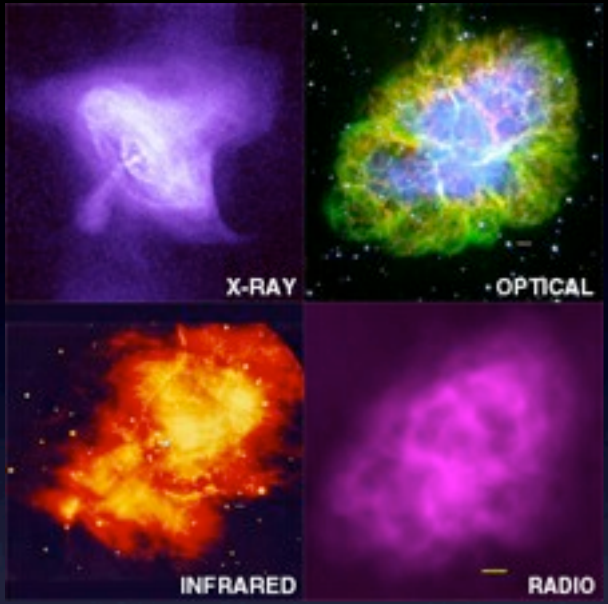
- Emission integrated over the particle distribution: $j_\nu = \int P_\nu(\nu,\gamma) f(\gamma) d\gamma$
- Thermal distribution: $f(\gamma) \propto \gamma^2 e^{-\gamma/\theta}$
  - Same as for a single particle for the mean energy..
- Power-law distribution: $f(\gamma) \propto \gamma^{-s}$
  - Integrated emission:

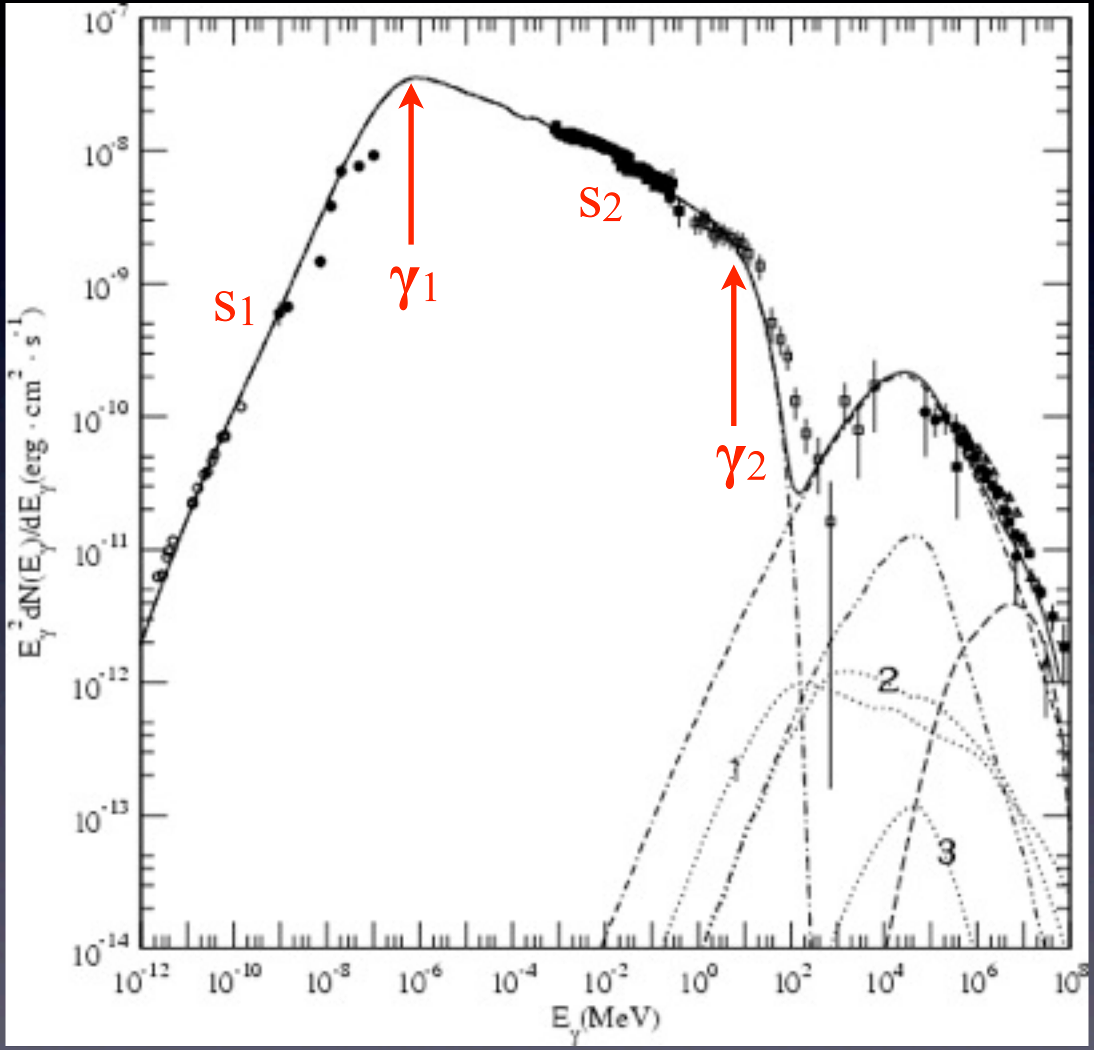
$$\begin{aligned} j_\nu &\propto \int G(3\nu/2\nu_B\gamma^2)\gamma^{-s} d\gamma \\ &\propto \nu^{-(s-1)/2} \int x^{(s-3)/2} G(x) dx \\ &\propto \nu^{-\alpha} \end{aligned}$$

  - Power-law spectrum with
    - slope: $\alpha = \frac{s-1}{2}$
    - minimal energy: $\nu_{\min} \propto B\gamma_{\min}^2$
    - maximal energy: $\nu_{\max} \propto B\gamma_{\max}^2$



The Crab Nebula





- ✓ Pulsar wind nebula
  - ✓ Outflow of high energy electrons
  - ✓ Magnetized medium (0.1 mG)
- ✓ Synchrotron emission from radio to γ-rays
- ✓ Broken power-law distribution
  - ✓ Two slopes: s_1, s_2
  - ✓ Two breaks: γ_1, γ_2

Polarization

- ✓ One single particle produces a coherent EM fluctuation
  - ✓ Intrinsically polarized: elliptically
  - ✓ Depends on p, α and θ
- ✓ Turbulent magnetic field: no net polarization
- ✓ Ordered magnetic field:
  - ✓ Ensemble of particles with random pitch angles => partially linearly polarized
  - ✓ Polarization angle perpendicular to observed B: $P_\perp >> P_\parallel$
  - ✓ High polarization degree: $\Pi(\nu, p) = \frac{P_\perp - P_\parallel}{P_\perp + P_\parallel}$
    - ✓ depends on frequency and particle energy
    - ✓ averaged over all frequencies: Π=75% !
    - ✓ In average over PL distribution of particles: Π=(s+1)/(s+7/3)

Polarization

✓ Such high polarization is characteristic of synchrotron radiation

✓ Measure of the direction gives the direction of B

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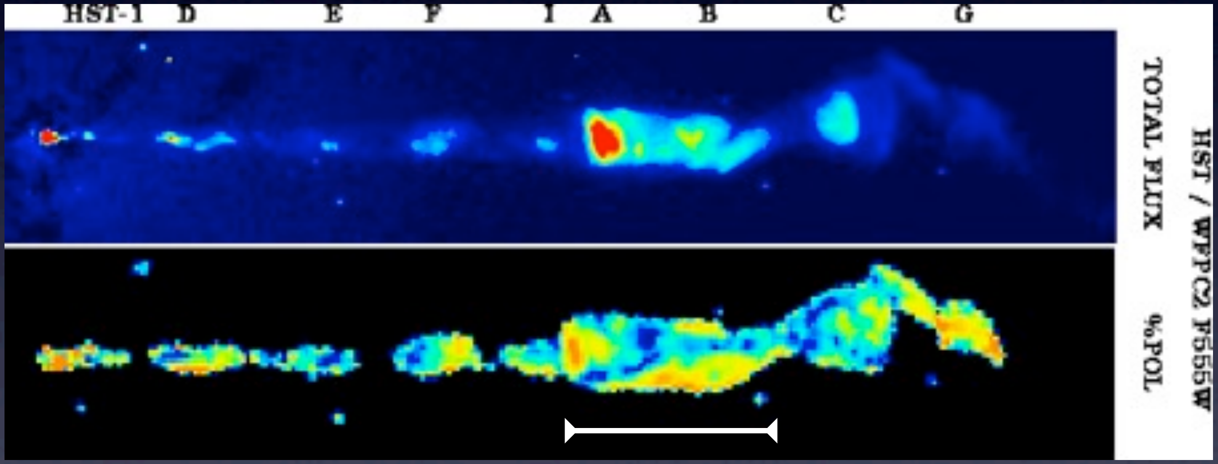
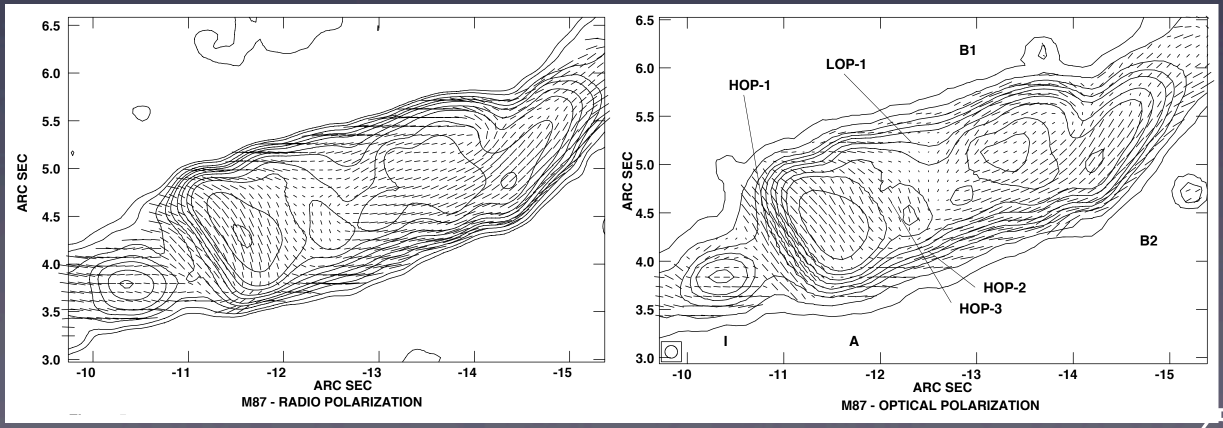
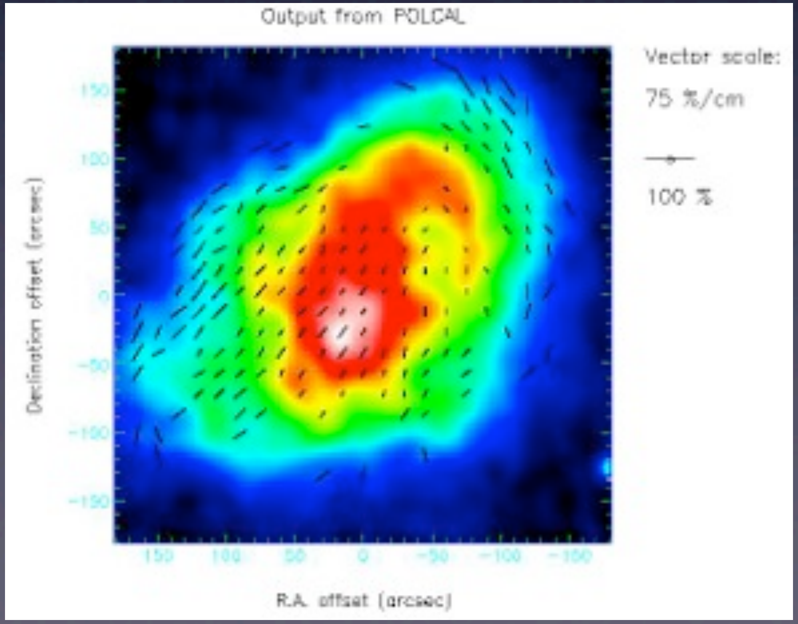
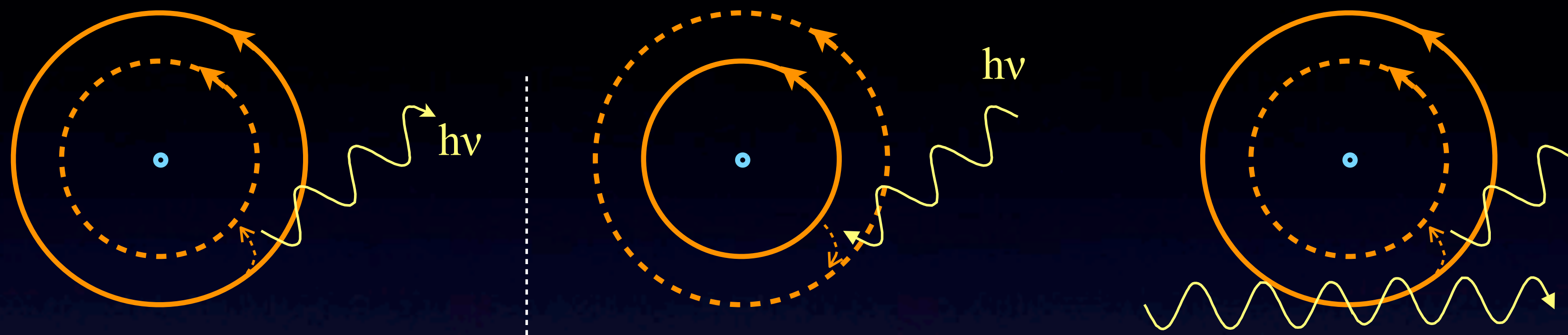




Crab nebula



Synchrotron self-absorption



Spontaneous emission | True absorption | Stimulated emission (negative absorption)

emission coefficient:

$$j_\nu = n \frac{\partial P}{\partial \nu \partial \Omega}$$

absorption coefficient:

$$\alpha_\nu(p,\nu) = \frac{c^2}{2h\nu^3}\frac{1}{p\gamma}\left[\gamma p j_\nu\right]_\gamma^{\gamma + h\nu/mc^2}$$

True absorption

Stimulated emission (negative absorption)

$$\approx \frac{1}{2m\nu^2}\frac{1}{p\gamma}\partial_\gamma\left(\gamma p j_\nu\right)$$

- ✓ Absorption decreases with frequency
  - ✓ high energy photons are weakly absorbed
  - ✓ low energy photons are highly absorbed

Synchrotron self-absorption

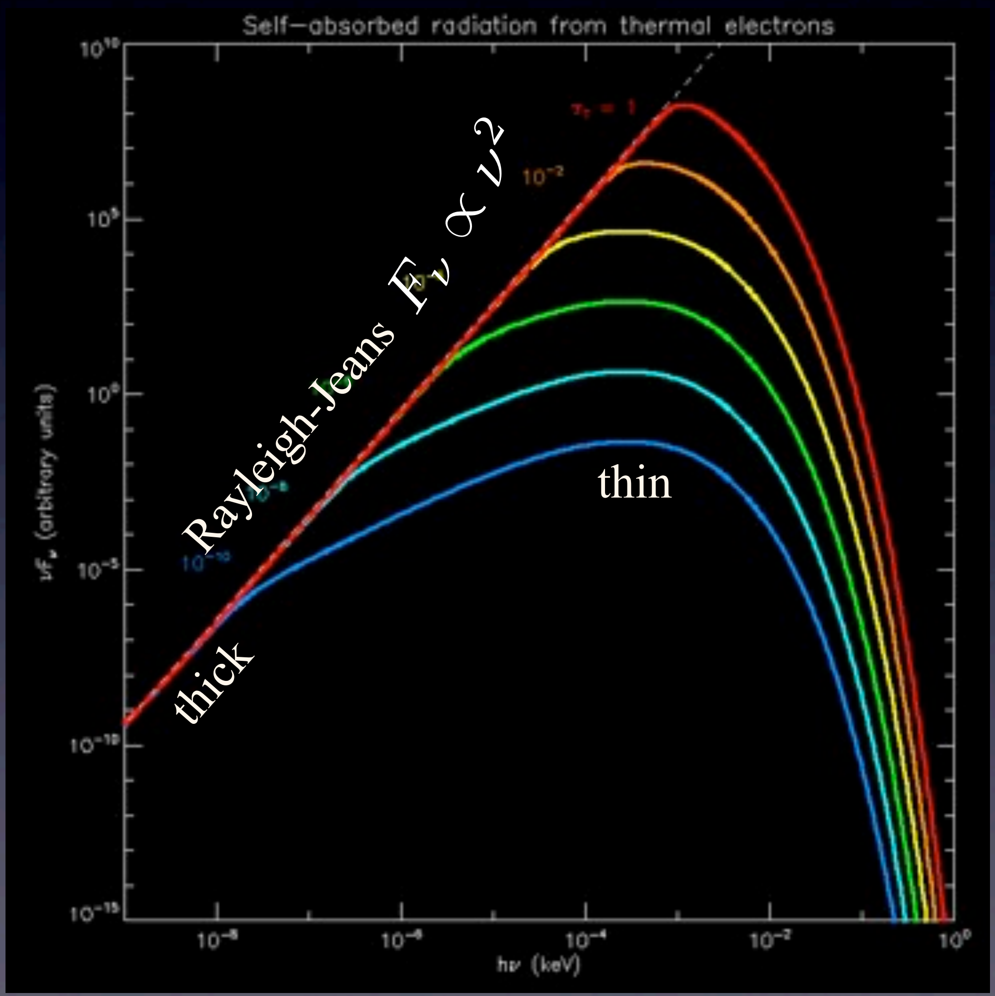
- Radiation transfer:
  - Equation for specific intensity I_ν: $\frac{\partial I_\nu}{dl} = j_\nu - \alpha_\nu I_\nu$
  - Solution for a uniform layer of thickness L: $I_\nu = \frac{j_\nu}{\alpha_\nu}(1 - e^{-\alpha_\nu L})$

- $\tau_\nu = \alpha_\nu L$ is the *optical depth* at energy $h\nu$
  - When $\tau_\nu << 1$: thin spectrum
  - When $\tau_\nu >> 1$: thick spectrum
  - transition for: $\tau_\nu = \alpha_\nu L \approx 1$

- The transition energy increases with optical depth, i.e. with:
  - Physical thickness of the layer
  - Density of the medium

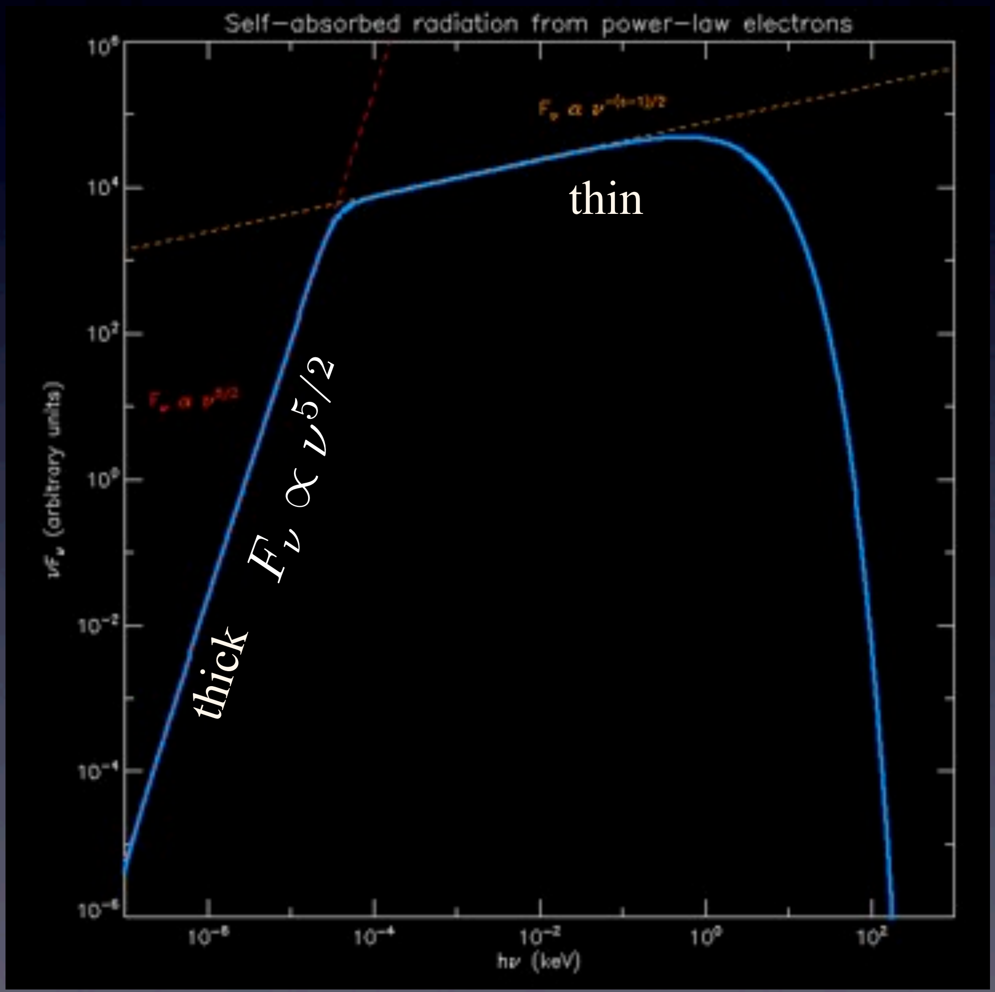
Self-absorbed Spectra

✓ Thermal distribution



Tends to BB radiation in the thick part

✓ Power-law distribution



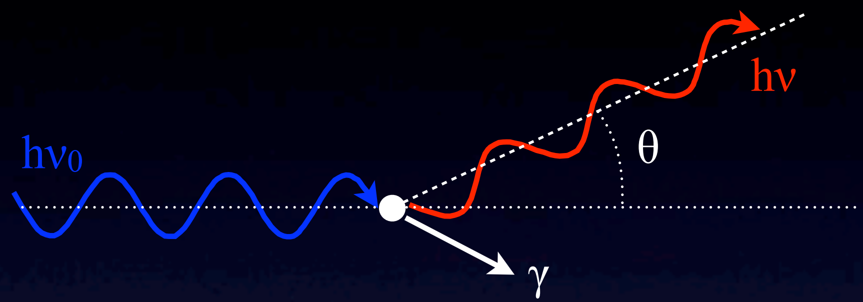
Does not tend to BB radiation in the thick part

Compton Scattering

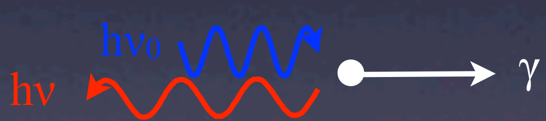
- Thomson/Klein-Nishina regimes
- Spectrum, angular distribution
- Particle cooling
- Multiple scattering

In the particle rest frame

- ✓ Scattering of light by free electrons
- ✓ Result described by 6 quantities
- ✓ 4 Conservation laws:
  - ✓ Energy $h\nu_0 + mc^2 = h\nu + \gamma mc^2$
  - ✓ Momentum $\frac{h\nu_0}{c}\vec{n}_0 = \frac{h\nu}{c}\vec{n} + mc\vec{p}$



- ✓ 1 symmetry
- ✓ One quantity is left undetermined, e.g. the scattering angle θ
- ✓ Direction/energy relation $\frac{h\nu}{h\nu_0} = \frac{1}{1 + \frac{h\nu_0}{m_e c^2}(1 - \cos\theta)}$



backward scattering (θ=π)

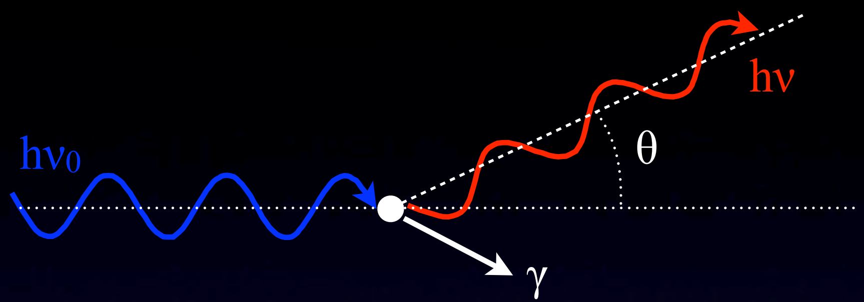
$$\frac{h\nu_0}{1 + 2h\nu_0/mc^2} \leq h\nu \leq h\nu_0$$

forward scattering (θ=0)

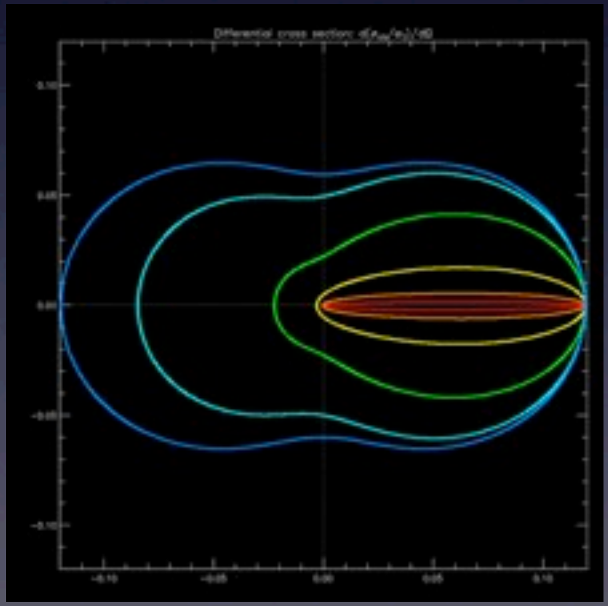
- ✓ Photon loose energy in the particle rest frame
- ✓ Two regimes:
  - ✓ The Thomson regime ($h\nu_0 < mc^2$): coherent scattering: $h\nu = h\nu_0$
  - ✓ The Klein-Nishina regime ($h\nu_0 > mc^2$): incoherent scattering: $h\nu < h\nu_0$

Thomson scattering

- Scattering of linearly polarized waves:
  - Harmonic motion of the particle
  - Emission of light in all directions



- Scattering of unpolarized waves:
  - Average of linearly polarized waves with random directions
  - Scattered power: $\dfrac{\partial P}{\partial \Omega} = \dfrac{\partial \sigma}{\partial \Omega} F$
  - Thomson cross section: $\dfrac{\partial \sigma}{\partial \Omega} = \dfrac{3}{8\pi} \sigma_T \dfrac{1 + \cos^2 \theta}{2}$
  - Total cross section: $\sigma_T = 6.65 \times 10^{-25} \mathrm{cm}^2$
  - Partially polarized: $\Pi = \dfrac{1 - \cos^2 \theta}{1 + \cos^2 \theta}$



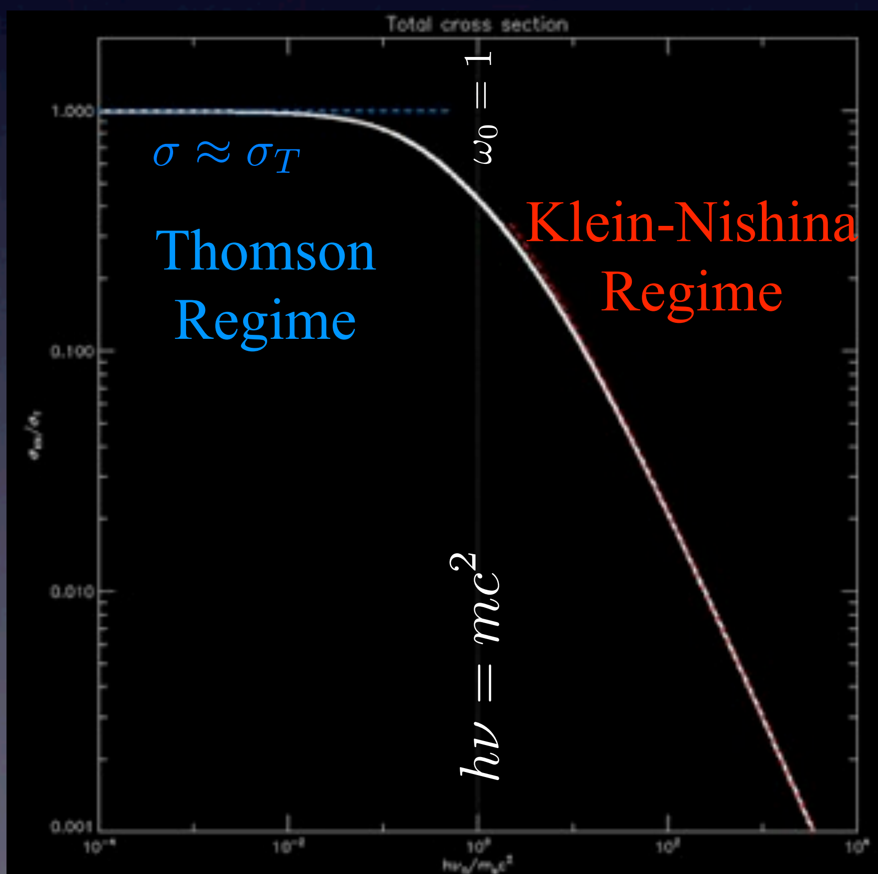
- Spectrum: a line at the incident frequency

Klein Nishina scattering

✓ Requires quantum mechanics but still analytical formulae

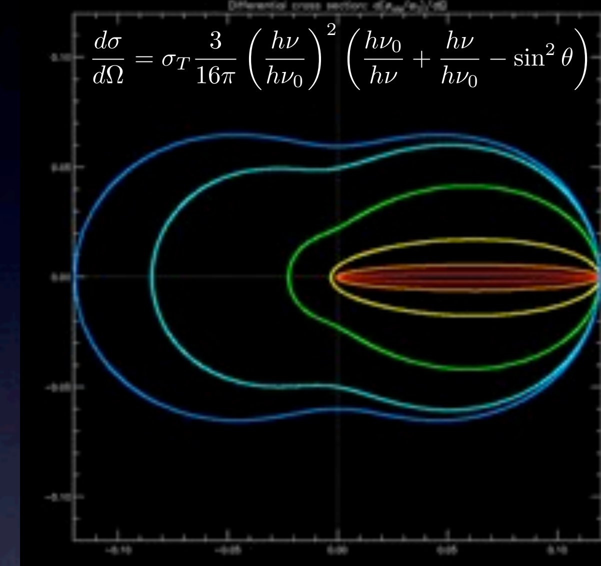
Total cross section:

$$\sigma = \sigma_T \frac{3}{4}\left[\frac{1+\omega_0}{\omega_0^3}\left(\frac{2\omega_0(1+\omega_0)}{1+2\omega_0} - \ln(1+2\omega_0)\right) + \frac{\ln(1+2\omega_0)}{2\omega_0} - \frac{1+3\omega_0}{(1+2\omega_0)^2}\right]$$

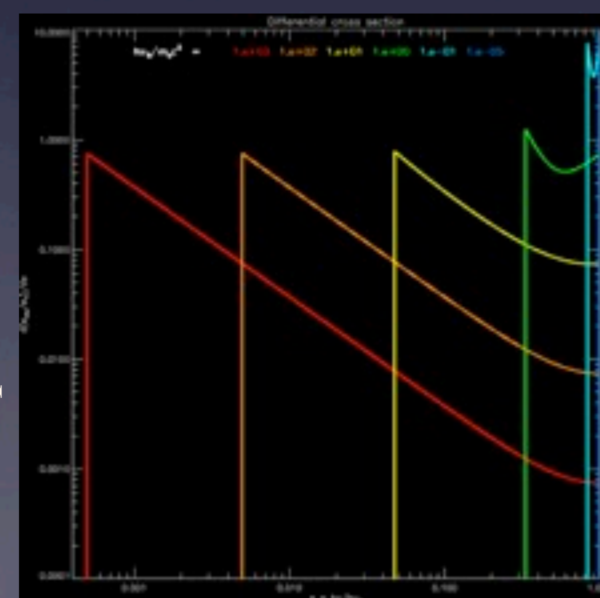


Differential cross sections:

Angular distribution:



Spectrum



$h\nu_0/mc^2$

10^{-5}

10^{-1}

1

10

10^2

10^3

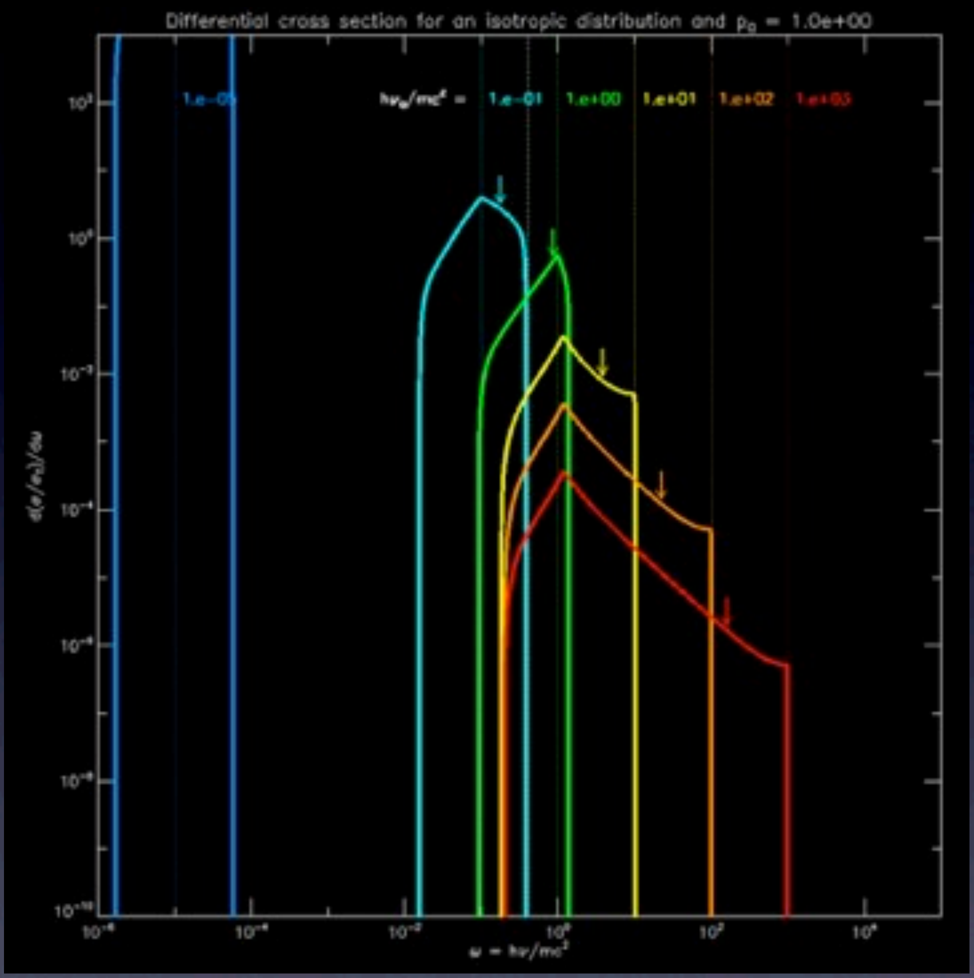
In the source frame

- In the particle frame: one incident parameter ($h\nu_0$)
- In the source frame: new dependance on
  - The particle energy
  - The collision angle
- Photon can now gain/loose energy
- An example:
  - Head-on collision:
    - Cold photon $h\nu_0$ and relativistic electron $\gamma_0 >> 1$
    - Photon energy in the particle frame: $\nu'_0 = 2\gamma_0\nu_0$
  - Thomson backward scattering: $h\nu'_0 << mc^2$
    - Emitted photon energy in the electron frame: $\nu' = \nu'_0$
    - Photon energy in the source frame: $\nu = 2\gamma_0\nu'$
  - In the end: $\nu = 4\gamma_0^2\nu_0$
  - *Compton up-scattering*
- Often: isotropy assumption and average over angles

In the source frame

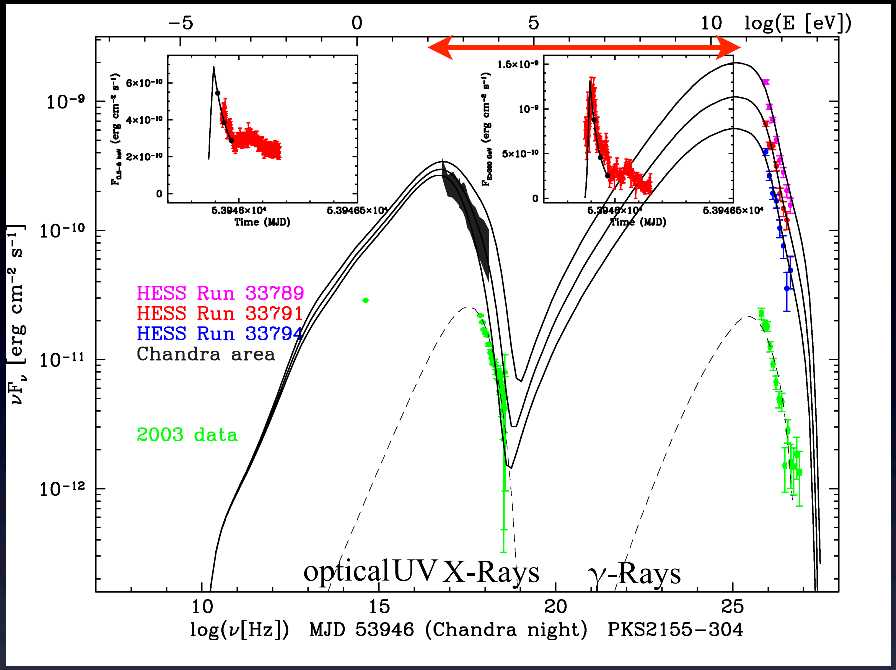
- ✓ For isotropic distributions:
- ✓ The Thomson limit: $\gamma_0 h\nu_0 << mc^2$
- ✓ The scattered spectrum:
- ✓ Average energy of scattered photons
  - ✓ Down scattering: $(\gamma_0 - 1)mc^2 < h\nu_0$
  - ✓ Up-scattering: $(\gamma_0 - 1)mc^2 > h\nu_0$
  - ✓ Amplification factor:

$$A = \frac{< h\nu >}{h\nu_0} \approx \gamma^2$$



- ✓ Scattering by relativistic plasmas produces high energy radiation
- ✓ Particle cooling: $\frac{\partial E_p}{\partial t} = \frac{4}{3} c\sigma_T p^2 U_{ph}$

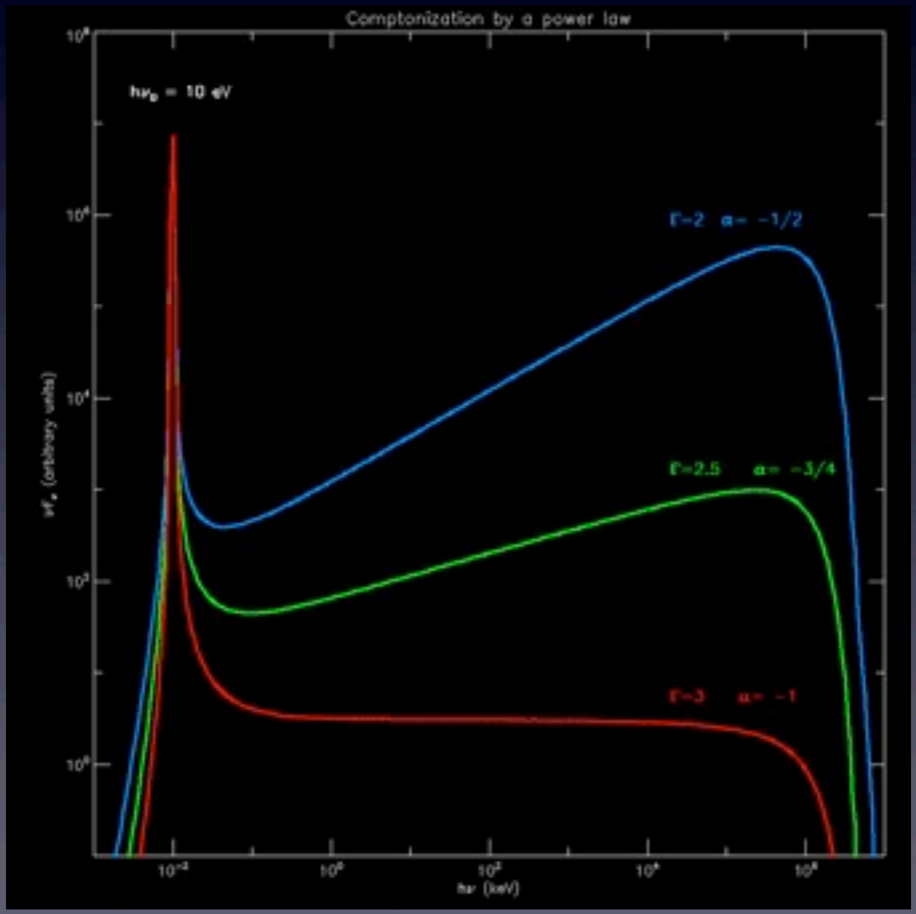
Blazar Spectra



- ✓ E > TeV! Comptonization?
- ✓ Model = Synchrotron Self-Compton (SSC) + Doppler boosting
  - ✓ Seed photons = synchrotron photons
  - ✓ The same particle emit though synchrotron and scatter these photons
- ✓ Here:
  - ✓ $A=10^8 \Rightarrow \gamma=10^4$
  - ✓ $h\nu_0 = 0.1$ keV => KN regime
  - ✓ Synchrotron peaks at $B\gamma^2$, Compton amplifies with $A=\gamma^2 \Rightarrow B$!

Single scattering by many electrons

- ✓ Emission integrated over the particle distribution
- ✓ Thermal distribution: $f(\gamma) \propto \gamma^2 e^{-\gamma/\theta}$
  - ✓ Same as for a single particle for the mean energy..
- ✓ Power-law distribution: $f(\gamma) \propto \gamma^{-s}$
  - ✓ Power-law spectrum with
    - ✓ slope: $\alpha = \frac{s-1}{2}$
    - ✓ minimal energy: $h\nu_0\ \gamma^2_{min}$
    - ✓ maximal energy: $h\nu_0\ \gamma^2_{max}$
- ✓ Scattered photons distributions should also be integrated over the source seed photons

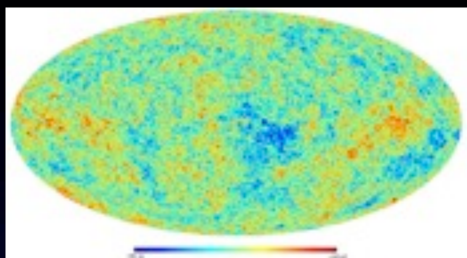


Multiple Scattering

- ✓ Photons can undergo successive scattering events
- ✓ Medium of finite size L: Thomson optical depth: $\tau_T = n_e \sigma_T L$
- ✓ Competition scattering/escape:
  - ✓ τ (or τ^2) = Mean number of scattering before escape
    - ✓ $\tau<1$: single scattering
    - ✓ $\tau>1$: multiple scattering
- ✓ y parameter = <photon energy change> before escape
  - ✓ y= <Energy change per scattering> * <scattering number>
  - ✓ For mono-energetic particles: $y = \tau\gamma^2$
  - ✓ For thermal distributions: $y = 4\tau\theta(1+4\theta)$

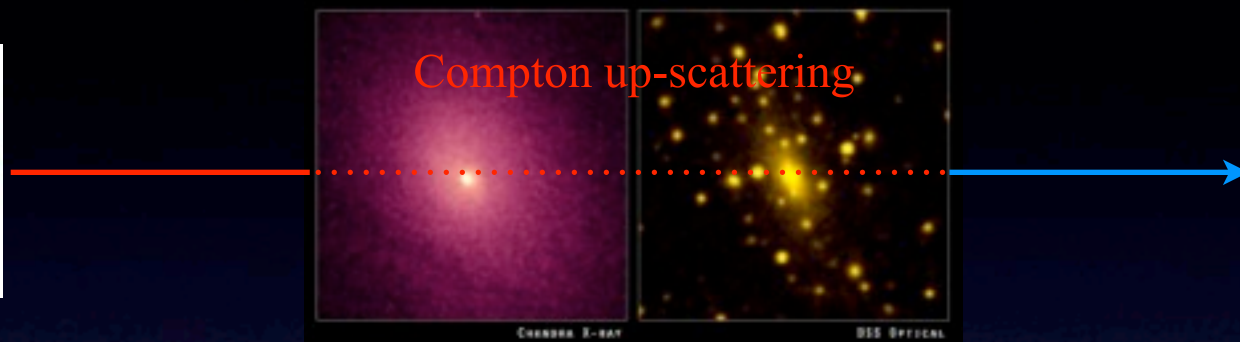
The SZ effect

Cold photons



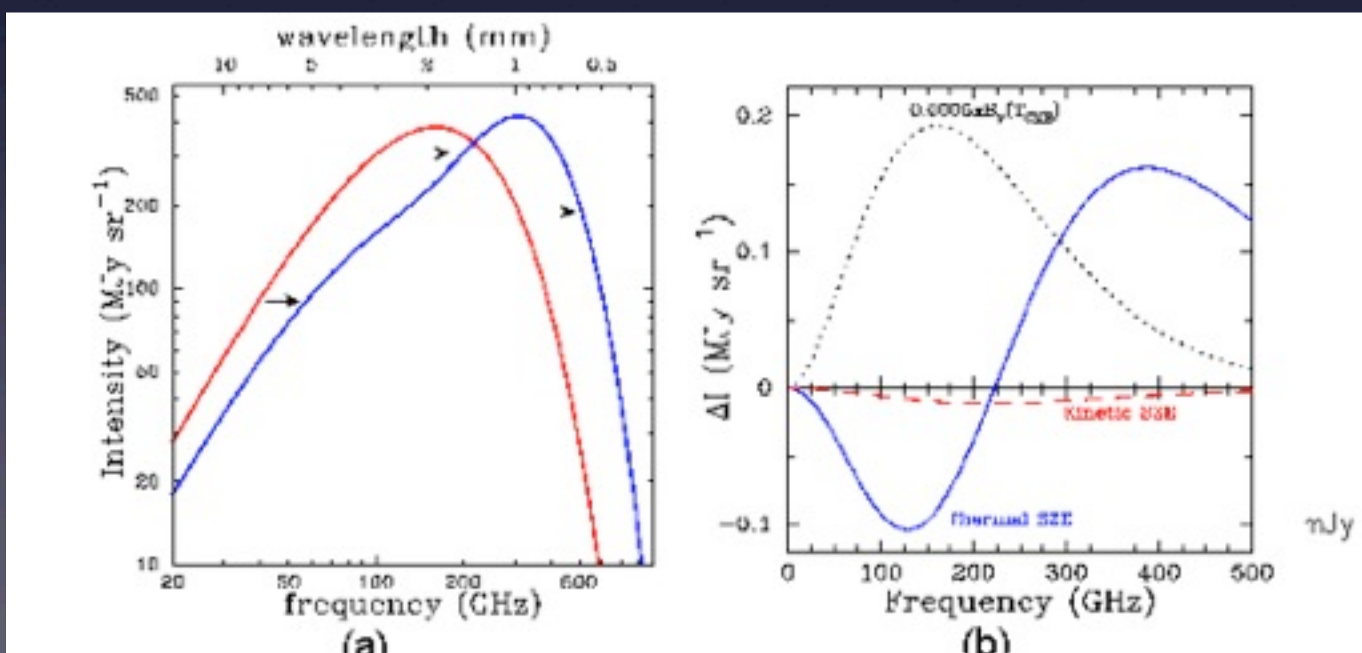
$k_B T_{ph}$=2.7 K

Hot electrons



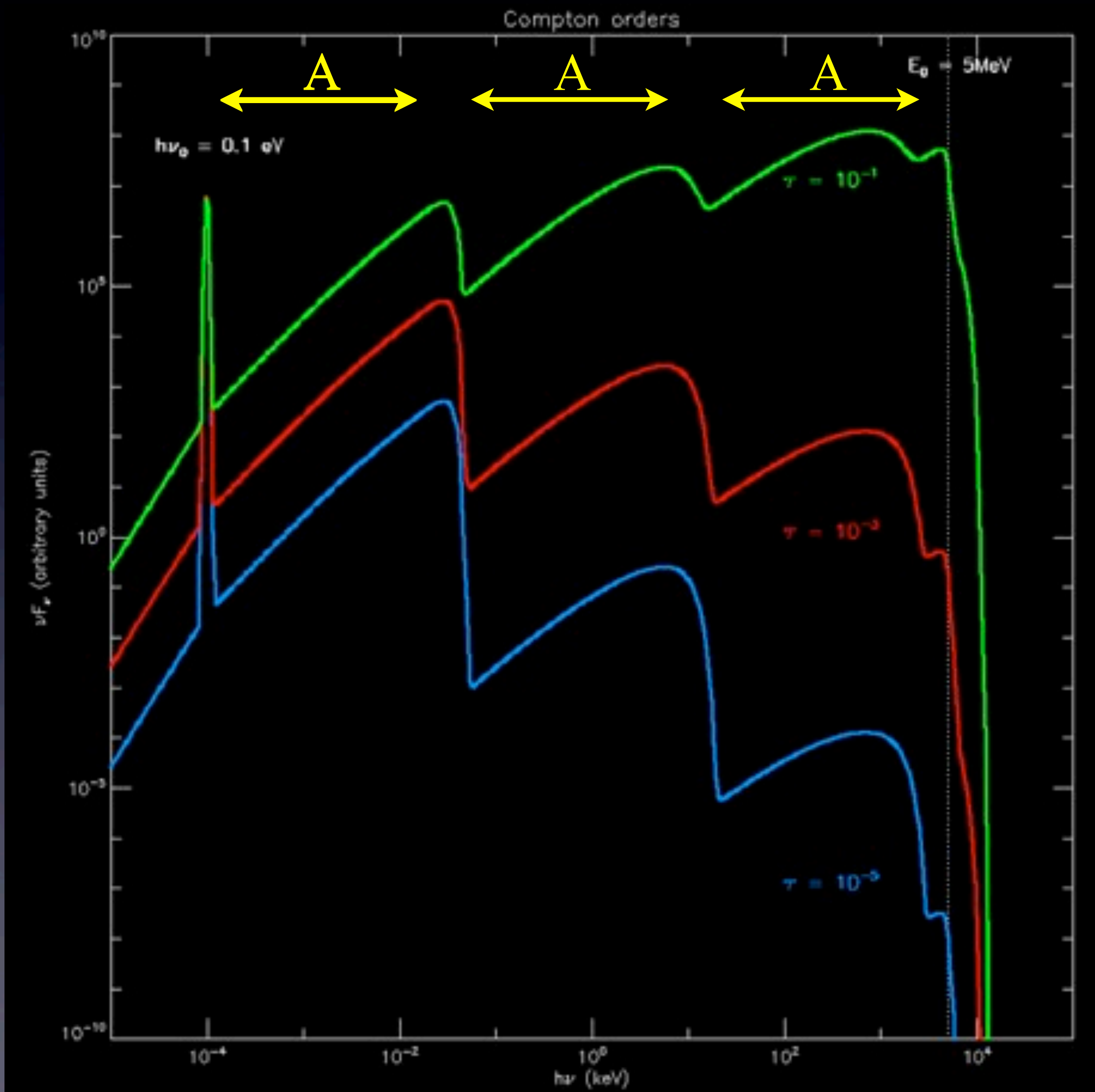
$k_B T_e$=1-10 keV ($\theta_e = k_B T_e / m_e c^2 \approx 10^{-2}$, $p \approx \beta \approx 0.1$)

$\tau = N_e \sigma_T L \approx 10^{-2}$



- ✓ Typical distortion whose amplitude gives: $y \approx \tau\theta \approx 10^{-4}$
- ✓ Bremsstrahlung gives T
- ✓ => density...

Compton orders



$h\nu_0/mc^2=10^{-7}$
$\gamma_0=10$
$A = 100$

bumpy spectrum

cutoff at the particle energy

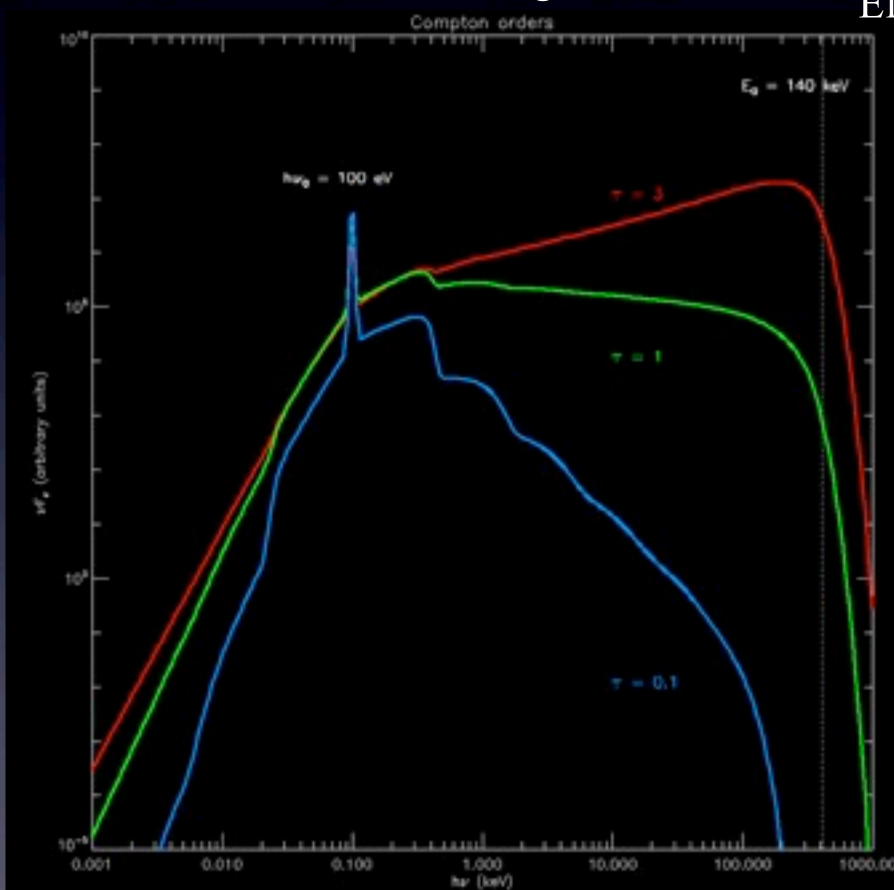
τ => spectrum hardness

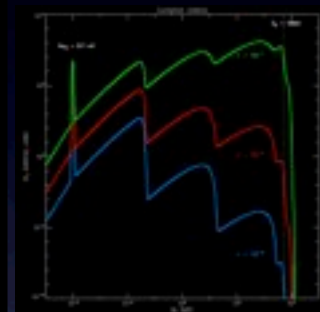
Compton regimes

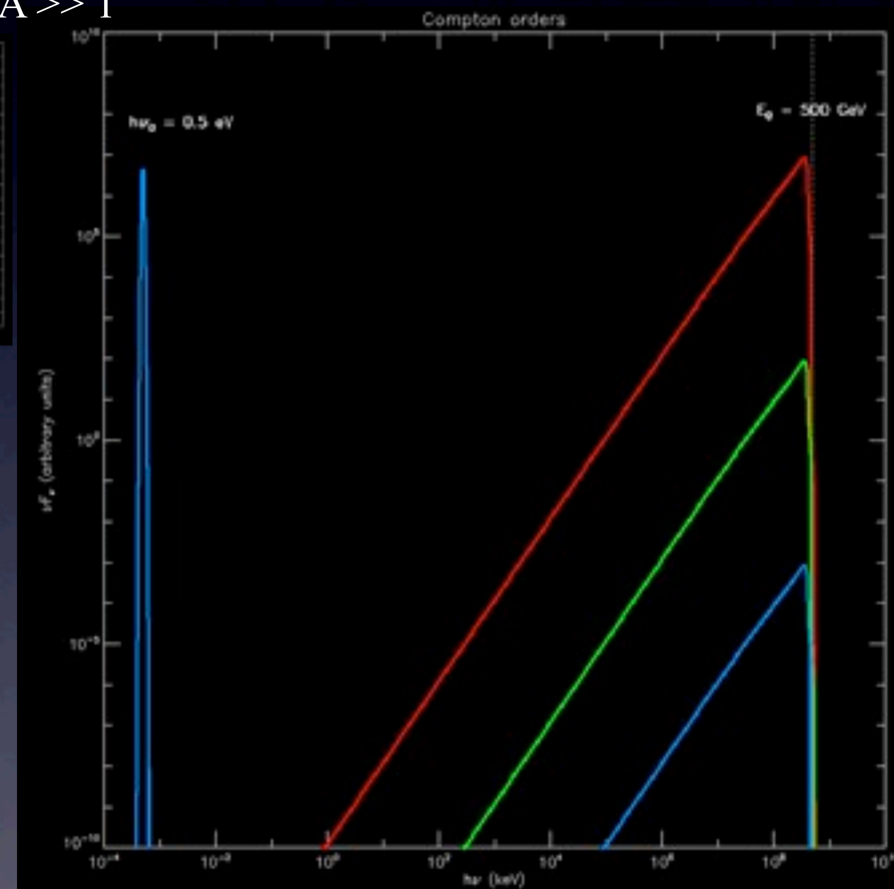
Sub-relativistic particles: $h\nu_0/mc^2 < p < 1$
Thomson regime $h\nu_0\gamma_0 << mc^2$
Inefficient scattering: $A = 1$

Relativistic particles: $\gamma_0 >> 1$
Thomson regime: $h\nu_0\gamma_0 << 1$
Efficient scattering: $A >> 1$

Ultra-relativistic particles: $\gamma_0 >> 1$
KN regime: $h\nu_0\gamma_0 > mc^2$
Efficient scattering: $A >> 1$







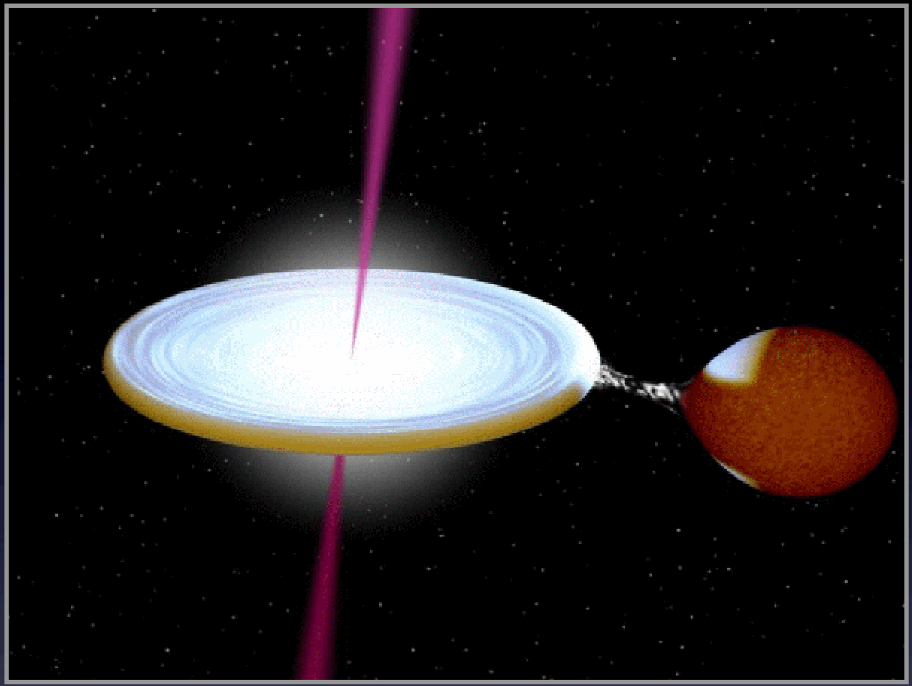
Power-law spectrum
- cutoff at the particle energy
- Slope = $\ln(\tau)/\ln(A)$

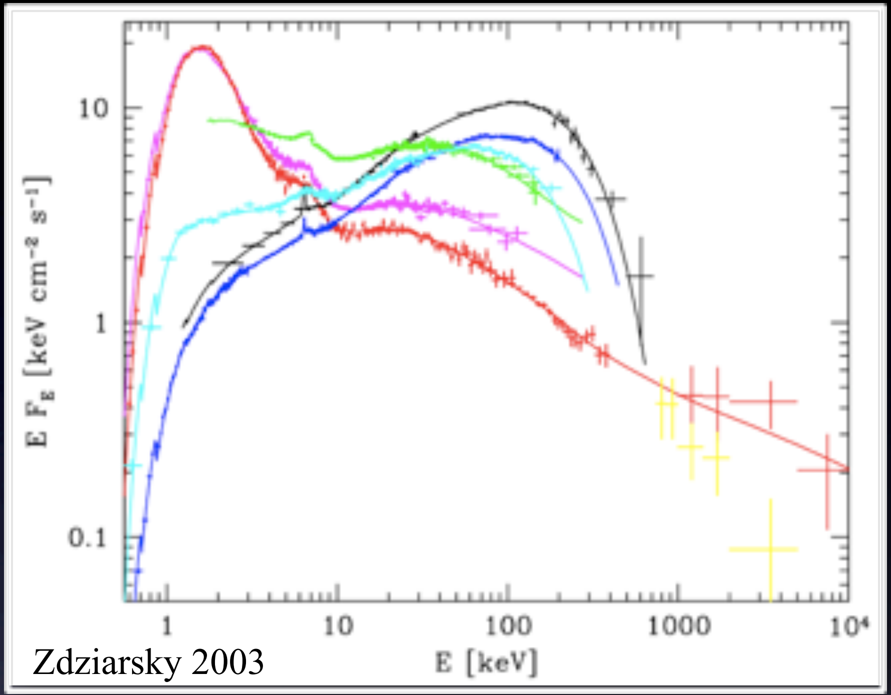
=> X-ray binaries

one-bump spectrum
= single scattering !

=> AGN/Blazars

X-ray binaries



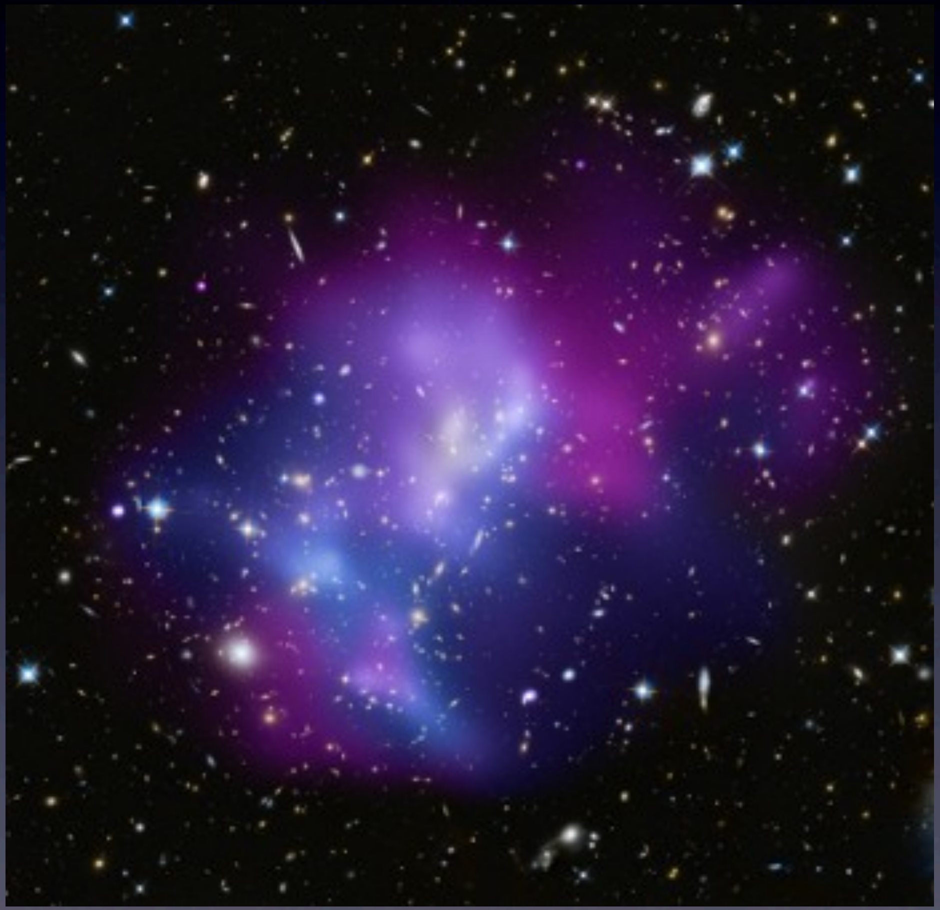


- Soft states:
  - Multi-color black body at 1 keV from the accretion disk
  - Non-thermal comptonization in a hot corona (τ=1)
- Hard states:
  - Soft photons from the accretion disk or synchrotron
  - Inefficient thermal Comptonization in a hot corona (100 keV, τ=0.01)
- What heating acceleration mechanism?

Bremsstrahlung radiation

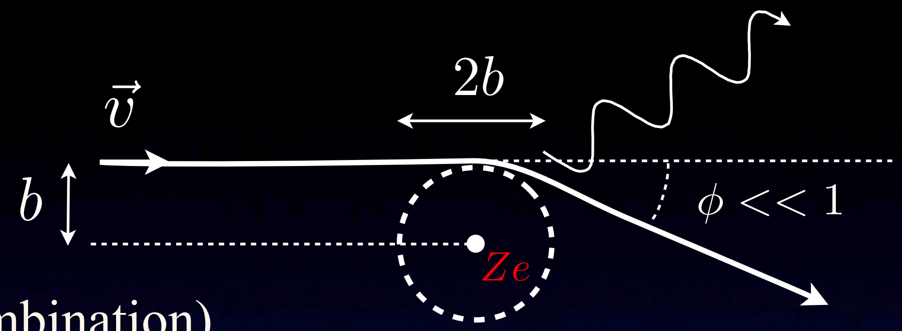
Bremsstrahlung

- Radiation of charged particles accelerated by the Coulomb field of other charges



- Astrophysical sources:
  - Some modes of hot accretion disks
  - Hot gas of intra-cluster medium (1-10 keV)
  - ...

Easy bremsstrahlung



- Assumptions:
  - Classical physics
  - Sub-relativistic particles
  - Far collision (small deviation, no recombination)
  - Small energy change ($\Delta v << v$ i.e. $h\nu << mv2/2$)

- Single event
  - No p+/p+, e-/e-, e+/e+ Bremsstrahlung
  - p+/e, iZ+/e Bremsstrahlung
  - Approximation: static heavy iZ+

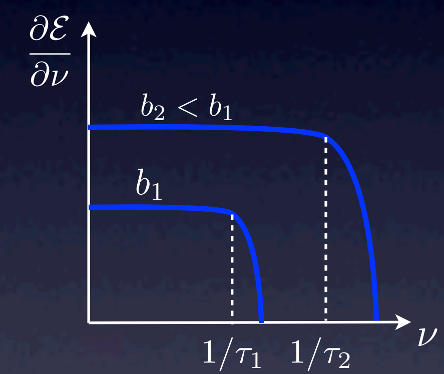
- Approximated motion:
  - Typical collision time: $\tau \approx 2b/v$
  - Typical velocity change: $\Delta v \approx \tau\, a \approx \tau\, F/m \approx \tau\, Z\, e^2/(mb^2) \approx 2Ze^2/(mvb)$
  - τ and Δv characterize the motion = enough to compute the spectrum

Easy bremsstrahlung

- ✓ Spectrum: $\frac{\partial P}{\partial \nu} \propto \left| FFT(\vec{E}) \right|^2 \propto \left| FFT(\vec{a}) \right|^2$

- ✓ Fourier transform: $TF[\dot{\vec{v}}] = \int_{-\infty}^{+\infty} \dot{\vec{v}}(t) e^{-2i\pi\nu t} dt \approx \begin{cases} 0 & \text{if } \nu\tau >> 1 \\ \Delta\vec{v} & \text{if } \nu\tau << 1 \end{cases}$

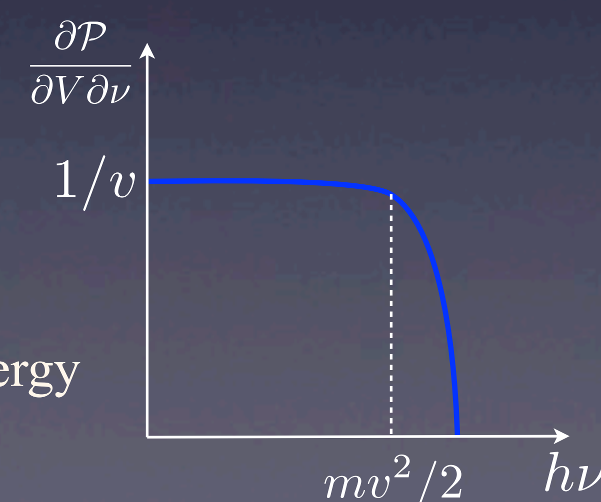
- ✓ One single particle produces a flat spectrum:



$$\frac{\partial \mathcal{E}}{\partial \nu}(v,b) \approx \begin{cases} 0 & \text{if } \nu\tau >> 1 \\ \frac{16e^6 Z^2}{3c^3 m^2 v^2 b^2} & \text{if } \nu\tau << 1 \end{cases}$$

- ✓ Many particles with a range of impact parameters:
  - ✓ with b_{min} from the small angle approximation
  - ✓ with b_{max} from $\nu\tau<1$

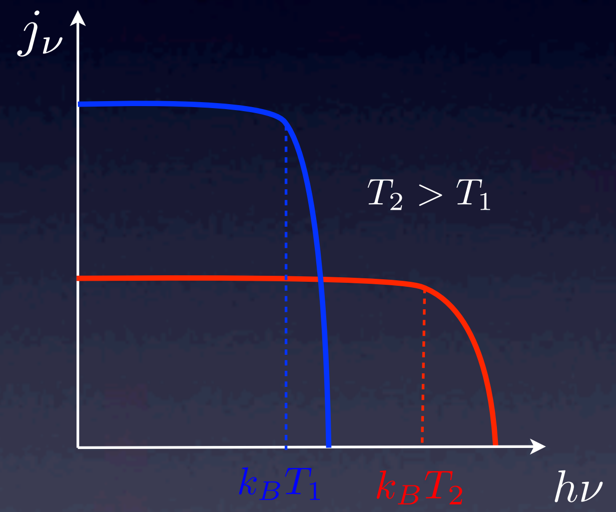
$$\frac{\partial P}{\partial V \partial \nu} = n_i n_e v \int_{b_{min}}^{b_{max}} \frac{\partial \mathcal{E}}{\partial \nu} 2\pi b db = \frac{32\pi e^6}{3m^2 c^3 v} n_i n_e g_{\rm ff}(v,\nu)$$



  - ✓ Produce a flat spectrum that cuts at the electron energy
  - ✓ Total losses: $P_{cool} \propto n_i n_e Z^2 v$

Emission from many electrons

- Emission integrated over the particle distribution
- Power-law distributions produce power-law spectra
- Thermal distributions:
  - Emission coefficient: $j_\nu \propto n_i n_e Z^2 T^{-1/2} e^{-h\nu/k_B T}$
  - Total losses: $P_{cool} \propto n_i n_e Z^2 T^{1/2}$



- Relativistic and quantum correction can be added to give more general spectra
- In media of finite size: bremsstrahlung self-absorption at low energy
  - c.f. synchrotron

Summary

- ✓ Particle cooling:
  - ✓ Synchrotron: $P \propto \sigma_T \, p^2 \, U_B$
  - ✓ Compton in the Thomson regime: $P \propto \sigma_T \, p^2 \, U_{ph}$
  - ✓ Bremsstrahlung: $P \propto \sigma_T \, \alpha_f \, p \, U_i$ (with $U_i = n_i \, m_e c^2$)
- ✓ Photons:
  - ✓ Synchrotron:
    - ✓ Thin spectrum of 1 particle peaks at $\nu_c \propto \gamma^2 B$
    - ✓ Thin spectrum of a power-law distribution is a power-law
    - ✓ Absorption => Thick spectrum at low frequency
  - ✓ Compton
    - ✓ Amplification factor in the Thomson regime: $A = \gamma^2$
    - ✓ Mildly relativistic particles: power-law spectrum
    - ✓ Comptonization by a relativistic power-law distribution is a PL spectrum
  - ✓ Bremsstrahlung
    - ✓ Flat spectrum
    - ✓ up to the particle energy