An Introduction to Magnetic Reconnection

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Motivation

Solar flares



Earth's magnetosphere



Sawtooth in tokamaks



Motivation: plenty of others!

- Fusion reactors (tokamaks): tearing modes, disruptions, edge-localized modes
- Laser-solid interactions (inertial confinement fusion)
- Magnetic dynamo
- Flares (accretion disks, magnetars, blazars, etc)
- Etc. (actually, there's no need for observational/experimental motivation: it's interesting *per se*.)

<u>Recent review papers</u>: Zweibel & Yamada '09; Yamada *et al.*, '10; also <u>books</u> by Biskamp and Priest & Forbes. Reconnection in <u>exotic HED environments</u>: Uzdensky '11



even more motivation...

"The prevalence of this research topic is a symptom not of repetition or redundancy in plasma science but of the underlying unity of the intellectual endeavor. As a physical process, magnetic reconnection plays a role in magnetic fusion, space and astrophysical plasmas, and in laboratory experiments. That is, investigations in these different contexts have converged on this common scientific question. If this multipronged attack continues, progress in this area will have a dramatic and broad impact on plasma science."

(S. C. Cowley & J. Peoples, Jr., "Plasma Science: advancing knowledge in the national interest", National Academy of Sciences decadal survey on plasma physics, 2010)

RECONNECTION: ESSENTIAL INGREDIENTS

Reconnection: basic idea

Oppositely directed magnetic field lines brought together by plasma flows.

Main features:

- coupling between large and small scales (*multiscale* problem)

- Magnetic energy is converted / dissipated (*energy partition*: what goes where?)

- Reconnection rate ~ $0.01 - 0.1 \text{ L/V}_{A}$ (fast)

- often reconnection events are preceded by long, quiescent periods (*two-timescales*, the *trigger problem*)



Frozen flux constraint

Magnetic flux through a surface S, defined by a closed contour C:

C(t+dt)

C(t)

В

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

How does Ψ change in time?

1. the magnetic field itself can change:

$$\left(\frac{\partial\Psi}{\partial t}\right)_1 = \int_S \frac{\partial\mathbf{B}}{\partial t} \cdot d\mathbf{S} = -c \int_S \nabla \times \mathbf{E} \cdot d\mathbf{S}$$

2. the surface moves with velocity **u**:

$$\left(\frac{\partial \Psi}{\partial t}\right)_2 = \int_C \mathbf{B} \cdot \mathbf{u} \times d\mathbf{l} = \int_C \mathbf{B} \times \mathbf{u} \cdot d\mathbf{l}$$
$$= \int_S \nabla \times (\mathbf{B} \times \mathbf{u}) \cdot d\mathbf{S}$$

Frozen flux constraint (cont'd)

Combine the two contributions to get:

$$\frac{d\Psi}{dt} = -\int_{S} \nabla \times (c\mathbf{E} + \mathbf{u} \times \mathbf{B}) \cdot d\mathbf{S}$$

Recognize that \mathbf{u} is an arbitrary velocity. Let me chose it to be the plasma velocity: $\mathbf{u} = \mathbf{v}$, and recall Ohm's law:

$$\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$$

Neglect collisions (RHS) → *ideal Ohm's law*

$$\frac{d\Psi}{dt} = 0$$

Magnetic flux through the arbitrary contour C is constant: magnetic field lines must move with (are *frozen* to) the plasma

Frozen flux vs. reconnection

Reconnection implies breaking the frozen flux constraint, i.e., going beyond Ohm's law.

$$\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$$



But the plasma is a very good conductor, right?

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Right. The RHS becomes important *not* because collisions are large, but because sharp gradients of the magnetic field give rise to a large current (hence the term *current layer*).

ONE WAY TO GET RECONNECTION GOING: THE TEARING MODE



$$\gamma D_x = ik D_{0y} f(x) v_x + \eta \left(\frac{dx^2}{dx^2} - k\right) D_x$$
$$\gamma \left(\frac{d^2}{dx^2} - k^2\right) v_x = ik B_{0y} f(x) \left[\frac{d^2}{dx^2} - k^2 - \frac{f''(x)}{f(x)}\right] B_x$$

Tearing cont'd

Definitions:

$$\begin{aligned} \tau_H &= 1/k B_{0y}; \quad \tau_\eta = a^2/\eta \\ \mathbf{v} &= \hat{\mathbf{z}} \times \nabla \phi; \quad \mathbf{B} = \hat{\mathbf{z}} \times \nabla \psi \end{aligned}$$

Normalize lengths: $x/a \to x; \quad ka \to k$

Rescale (for convenience): $i\phi/\gamma\tau_H \rightarrow \phi$

$$\psi - f(x)\phi = \frac{1}{\gamma\tau_{\eta}} \left[\frac{d^2}{dx^2} - k^2\right]\psi$$
$$\gamma^2\tau_H^2\phi = -f(x) \left[\frac{d^2}{dx^2} - k^2 - \frac{f''(x)}{f(x)}\right]$$

Tearing cont'd

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Ordering: $1/\tau_\eta \ll \gamma \ll 1/\tau_H$

Expect growth rate to be intermediate between resistive diffusion (very slow) and ideal MHD (very fast)



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It's a reconnecting mode: expect *ideal MHD* to be valid *away* from the reconnection layer (**outer region**), and *resistive* effects to be important *in* the reconnection layer (**inner region** = boundary layer)

Tearing cont'd

Outer region:
$$\phi = \frac{\psi}{f(x)}; \quad f(x) \left[\frac{d^2}{dx^2} - k^2\right] \psi = f''(x)\psi$$

Overlap region: $x \ll 1 \rightarrow f(x) \approx x \Rightarrow \psi'' = 0$

For a reconnecting mode, $\psi(0)$ must be finite. Need even solution. $\psi \approx \psi_0 + |x|\psi_0'$

This solution is discontinuous at x=0. A measure of that discontinuity is the *instability parameter*:

$$\Delta' = \left[\frac{d}{dx}\ln\psi\right]_{0_{-}}^{0^{+}} = \frac{2\psi'_{0}}{\psi_{0}}$$

Tearing mode Dispersion Relation:

$$\Delta' = -\frac{\pi}{8} \gamma^{5/4} \tau_H^{1/2} \tau_\eta^{3/4} \frac{\Gamma\left[\left(\hat{\lambda}^{3/2} - 1\right)/4\right]}{\Gamma\left[\left(\hat{\lambda}^{3/2} + 5\right)/4\right]}$$

where $\hat{\lambda} \equiv \gamma \tau_H^{2/3} \tau_\eta^{1/3}$ Two important limits:

$$\hat{\lambda} \ll 1 \Rightarrow \gamma = 0.55 \tau_{\eta}^{-3/5} \tau_{H}^{-2/5} \Delta'^{4/5} \longrightarrow \text{small } \Delta': \text{``FKR''}$$
$$\hat{\lambda} \to 1^{-} \Rightarrow \gamma = \tau_{H}^{-2/3} \tau_{\eta}^{-1/3} \longrightarrow \text{large } \Delta': \text{``Coppi''}$$

Analytical expressions for Δ ' are obtained from solving the outer region eq. for specific equilibrium profiles, f(x). For the Harris sheet:

$$f(x) = \tanh(x) \Rightarrow \Delta' = 2\left(\frac{1}{k} - k\right)$$

NONLINEAR RECONNECTION: THE SWEET-PARKER MODEL

The simplest description of reconnection: the Sweet-Parker model

Peter Sweet ('58) and Eugene Parker ('57) attempted to describe reconnection within the framework of resistive magnetohydrodynamics (MHD).



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 $S = L_{CS} V_A / \eta$ $\delta_{SP} / L_{CS} \sim S^{-1/2}$ $u_{in} / V_A \sim S^{-1/2}$ $E \sim c B_0 V_A S^{-1/2}$



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Typical solar corona parameters yield $S \sim 10^{14}$; this theory then predicts that flares should **last ~2 months**; in fact, flares last **15min – 1h**. (still, Sweet-Parker (SP) theory was a great improvement on simple resistive diffusion of magnetic fields, which would yield ~3.10⁶ years...)

Does the Sweet-Parker model work?





Does the Sweet-Parker model work?



Hmm... maybe it doesn't...



BEYOND SWEET-PARKER: TEARING INSTABILITY OF THE CURRENT SHEET



1- Assume incompressible flow profile of the form:

$$u_x = -v_A x / L_{CS}; \ u_y = V_A y / L_{CS}$$



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3- Linearize RMHD eqs and look for perturbations

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5- Obtain:

 $\gamma_{\max} \tau_A \sim S^{1/4}$ $k_{\max} L_{CS} \sim S^{3/8}$



Current sheet instability: threshold

Linear theory predicts:
$$\gamma_{\max} \tau_A \sim S^{1/4}$$

• To a good approximation, outflows in the CS are linear (Yamada *et al. '00*, Uzdensky & Kulsrud *'00*):

$$v_y \approx V_A y / L_{CS}$$

• For any perturbation to grow, its growth rate needs to exceed the shearing rate:

$$\gamma \tau_A >> 1 \Rightarrow S^{1/4} >> 1$$

> Critical threshold for instability:
$$S_c \sim 10^4$$

(NB: there's a slightly better way to do this, but this makes for a better joke)

Numerical confirmation of linear theory

Numerical simulations confirm scalings predicted by linear theory (Samtaney *et al.*, PRL '09).



Nonlinear stage: hierarchical plasmoid chains

Long current sheets ($S > S_c \sim 10^4$) are violently unstable to multiple plasmoid formation.



(Shibata & Tanuma '01)

• Current layers between any two plasmoids are themselves unstable to the same instability if

 $S_n = L_n V_A / \eta > S_c$

• Plasmoid hierarchy ends at *the critical layer*:

 $L_c = S_c \eta / V_A \quad \delta_c = L_c / \sqrt{S_c}$ $cE_c = B_0 V_A / \sqrt{S_c}$

• $N \sim L / L_c$ plasmoids separated by nearcritical current sheets.



Hierarchical Plasmoid Chains

Long current sheets ($S > S_c \sim 10^4$) are violently unstable to multiple plasmoid formation.





Barta *et al.*, '11 (also Huang *et al.*, '10)

Plasmoid-dominated reconnection: the ULS model

Theoretical model (ULS) (Uzdensky *et al.*, PRL '10) attempts to describe reconnection in stochastic plasmoid chains.

Key results:

• Nonlinear statistical steady state exists; *effective reconnection rate* is:

 $E_{eff} \sim S_c^{-1/2} \sim 0.01 \rightarrow \text{ independent of } S!$

- Plasmoid flux and size distribution functions are: $f(\psi) \sim \psi^{-2}$; $f(w_x) \sim w_x^{-2}$
- *Monster* plasmoids form occasionally: $w_{max} \sim 0.1 L$ --- can disrupt the chain, observable

High-Lundquist-number reconnection

Direct numerical simulations to investigate magnetic reconnection at $S>S_c$ (Loureiro *et al.*, PoP '12)

S=10⁶, res. 16384²



(see also Huang and Bhattacharjee '12,'13)

Reconnection and dissipation rates

Sweet-Parker model breaks down for $S > 10^4$



Monster plasmoid formation



Monster plasmoid formation



Monster plasmoids: application to blazar flares?

Minute-timescale TeV flares appear to be a generic feature of blazar activity. Recent model proposed by Giannios (arXiv: 1211.0296) claims that envelope can be do the reconnection, while the "bursts" could be due to monster plasmoids.



Reality check



There seems to be abundant evidence for plasmoids in solar flares and Earth's magnetotail (see Loureiro PRE '13 and refs. therein).

Karlicky & Kliem '10

Reality check



Takasao et al. '12

RECONNECTION IN A TURBULENT PLASMA

Reconnection in a turbulent background

Many (if not all) environments where reconnection occurs are turbulent – how does that affect reconnection?

Very roughly: it's SP but now the width d is determined by the typical field line wandering:

$$u_{in}L = V_A \Delta x$$

More precisely:

$$u_{in} = \frac{\lambda_{\perp}}{\lambda_{\parallel}} \frac{L}{\lambda_{\parallel}} V_A$$

Plug in your favourite turbulence model (e.g., GS95: $\lambda_{\parallel} \sim \lambda_{\perp}^{2/3}$) Independent of η .







Lazarian & Vishniac '99

Reconnection in a turbulent background



 $S = 10^3$

Kowal et al. '09

Reconnection in a turbulent background



Kowal et al., '09

Turbulent 2D MHD reconnection is also fast!





KINETIC RECONNECTION



Alternatively, even if $\,\delta_{SP} >
ho_i, c/\omega_{pi}\,$, one is almost certain to get:

$$\delta_c < \rho_i, c/\omega_{pi}$$















- MHD is valid at large scales.
- Below c/ω_{pi} , ions and electrons decouple: *plasma is no longer a single fluid*. Electrons remain frozen-in.
- Electrons and field lines decouple below c/ $\omega_{\rm pe}$

GEM challenge

What is the minimal plasma description that yields fast reconnection rates?



GEM challenge, Birn et al. '01

The signature of Hall reconnection: quadrupolar magnetic field

lon and electron streamlines:



(Breslau & Jardin '03)

[Also observed in MRX (see H. Ji's talk)]

Physical explanation of quadrupole field: Uzdensky & Kulsrud '06



Quadrupole out-of-plane magnetic field:

Kinetic means kinetic...



Two-fluid tearing mode theories seem to fail to predict *linear* tearing mode growth rates. The reason is the failure of simple equations of state (e.g., isothermal closure is *not* valid).



Kinetic means kinetic...



Strongly suggests that minimum model for weakly collisional reconnection may be *kinetic ions* + *drift kinetic electrons* (and even that may not be sufficient)



Connection with other topics at this school



Reconnection in accretion disks (Hawley & Balbus '92)



Firehose / mirror in high- β reconnection (Schoeffler '11)

Some open questions

- 3D
- Reconnection onset (the two-timescale problem)
- Energy partition, dissipation mechanisms
- What is the subgrid model that will reproduce the effect of reconnection on small scales?
- Role of background turbulence?

Bibliography

<u>Selected references for topics covered or (mentioned) in this talk</u> (this list is NOT exhaustive; many important papers NOT here)

- General:
 - Books by D. Biskamp and Priest & Forbes
 - Recent review papers: Zweibel & Yamada '09, Yamada et al., '10
 - Tutorial: Kulsrud '01
- Tearing mode (fluid):
 - MHD: Furth et al. '63, Coppi '75
 - Collisionless: Cowley '86, Porcelli '91 (v. good overview in App. B of Zocco & Schekochihin '11)
 - Rutherford '73; Waelbroeck '93 (nonlinear stage)
 - Militello & Porcelli '04, Escande & Ottaviani '04 "POEM", saturation
 - Steinolfson & Van Hoven '84, Loureiro *et al.* '05 (sims.)
- Tearing mode (kinetic):
 - Coppi '65, Drake & Lee '77, Cowley '86, Porcelli '91, Numata '11 (see App. B of Zocco & Schekochihin '11)
- Forced Reconnection:
 - Hahm & Kulsrud '84, Fitzpatrick '03, Cole & Fitzpatrick '04

Bibliography cont'd

- Sweet-Parker:
 - Parker '57, Sweet '58
 - Biskamp '86, Uzdensky '00
- Petschek '64
- Plasmoids:
 - Shibata & Tanuma '01, Loureiro *et al.* '07,'12,'13, Lapenta '08, Bhattacharjee '09, Daughton '09, Cassak '09, Samtaney '09, Huang '10, '12, '13, Uzdensky '10, Barta '08,'11, etc
- Reconnection in a turbulent plasma:
 - Matthaeus & Lamkin '86, Lazarian & Vishniac '99, Kowal '09, Loureiro '09, Karimabadi '13
- Trigger: Bhattacharjee '04, Katz et al. '10
- Reconnection experiments: see H. Ji's talk; Egedal; Brown, etc.
- Reconnection simulations: see R. Grauer's talk

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