

A bright yellow sun is centered in a purple sky, with a soft glow around it. The sky transitions from a lighter purple near the sun to a darker purple at the edges.

From Plasma Microphysics to
Global Dynamics
in Clusters of Galaxies

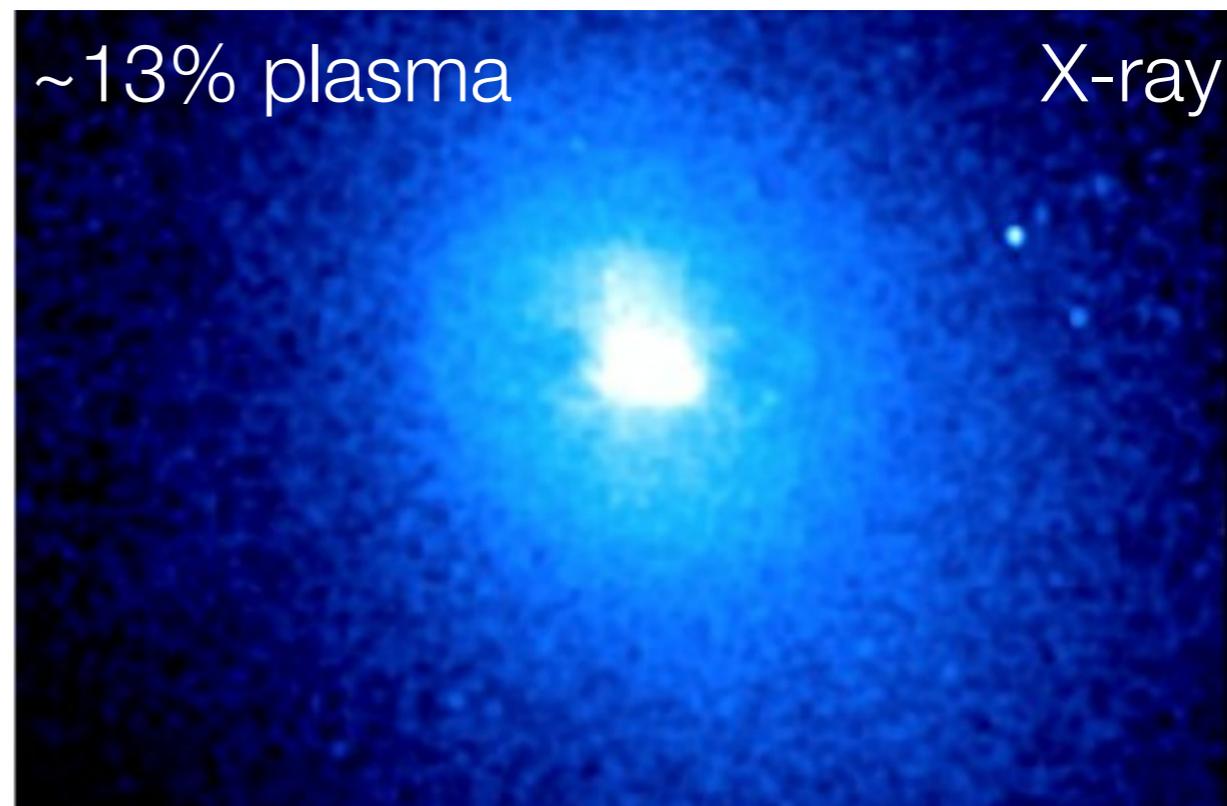
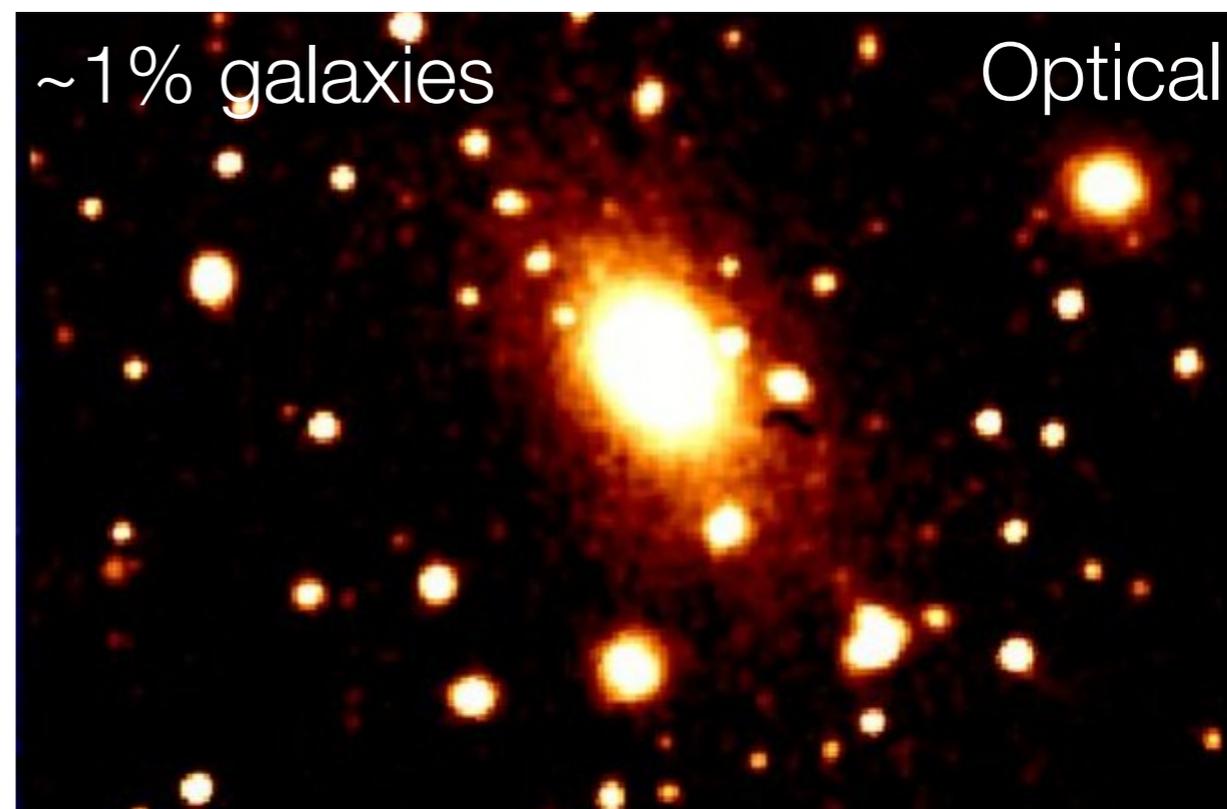
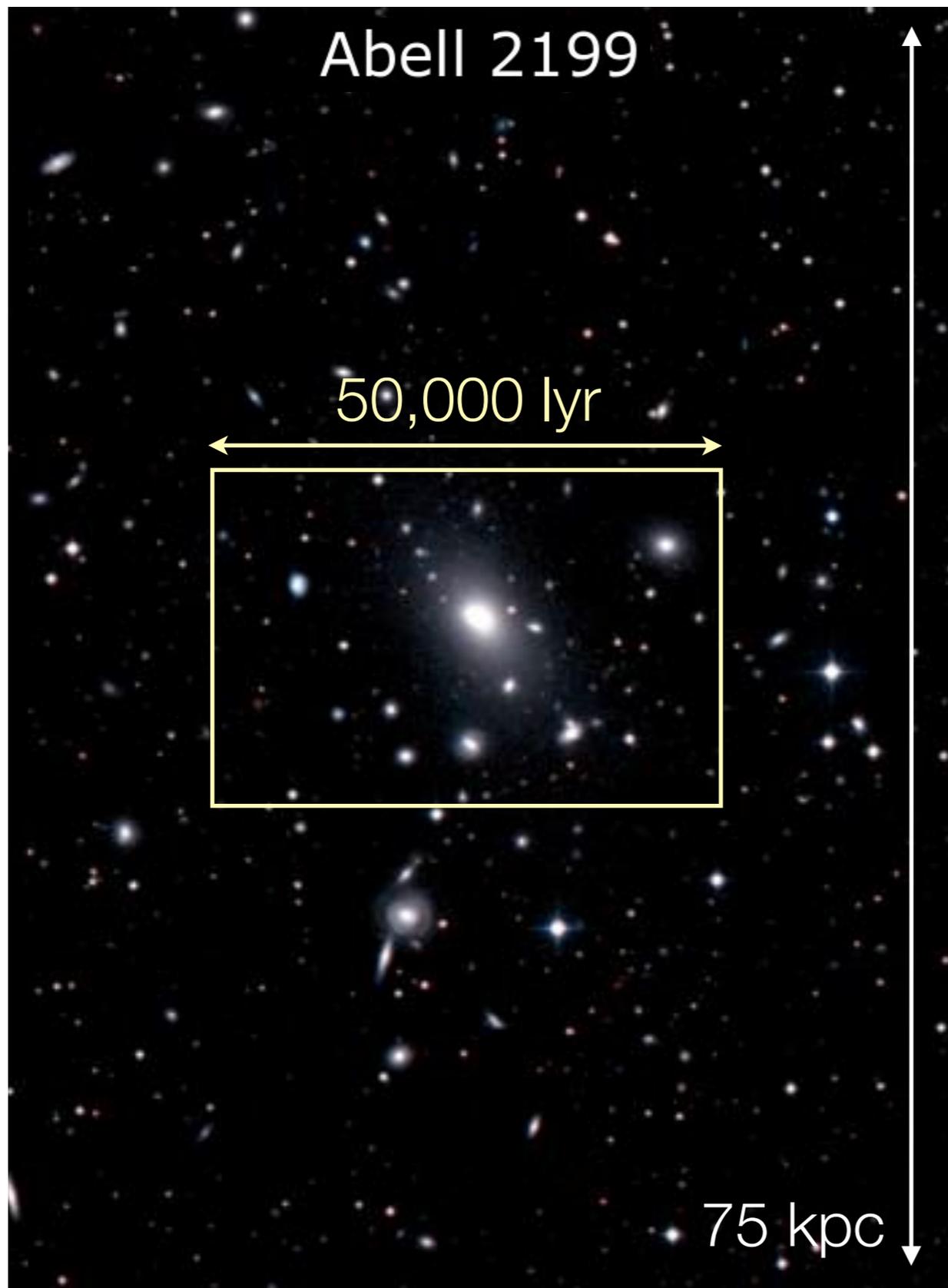
Matthew Kunz
28 Feb 2013

Picking up where we left off...

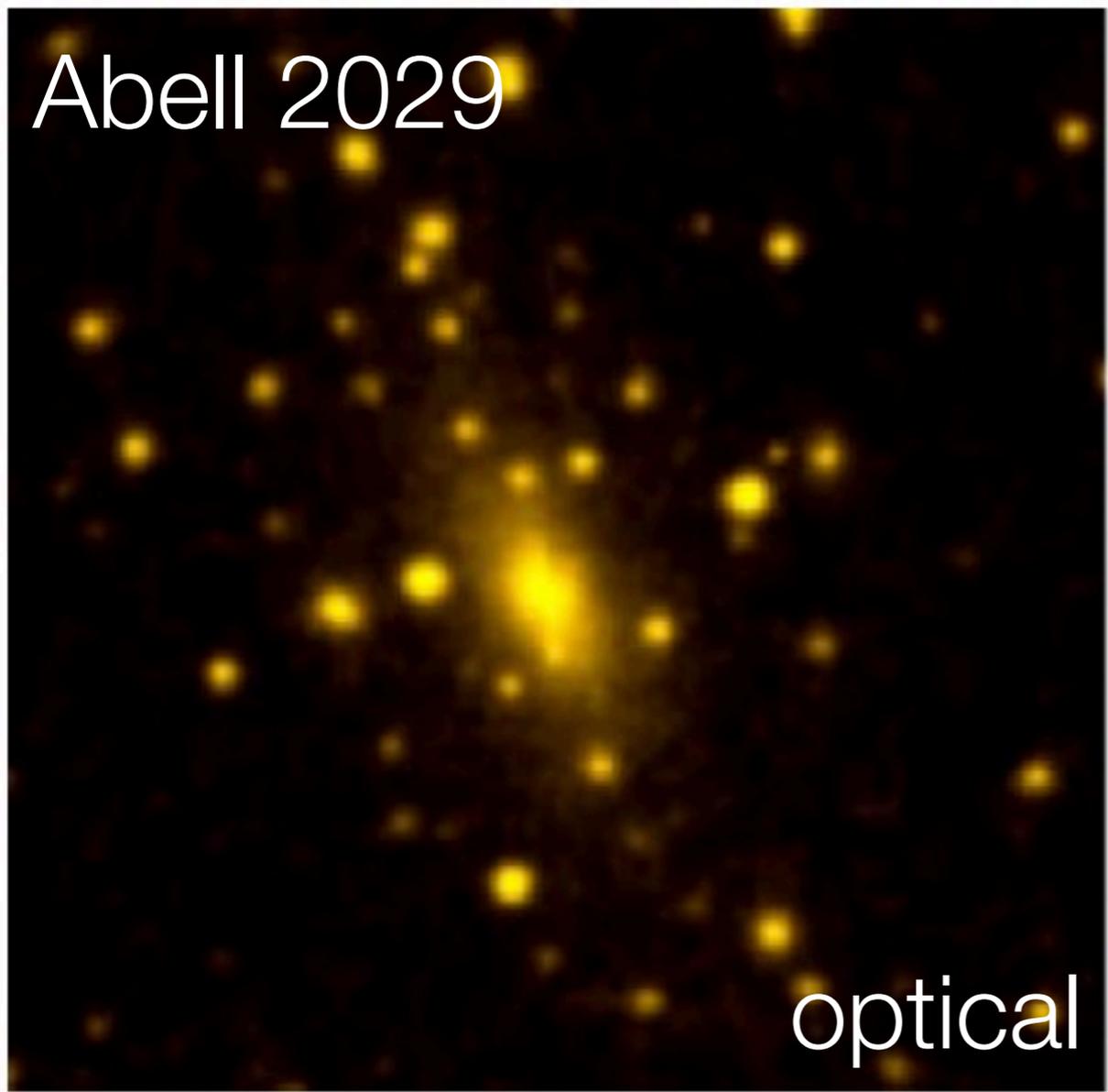
- We derived a system of equations that describe meso- and macroscale plasma physics in weakly collisional systems (including rotation, shear, gravity, thermal stratification).
- We discussed the physical interpretation of the kinetic-MHD closure.
- We applied these equations to elucidate the linear stability of weakly collisional accretion discs.

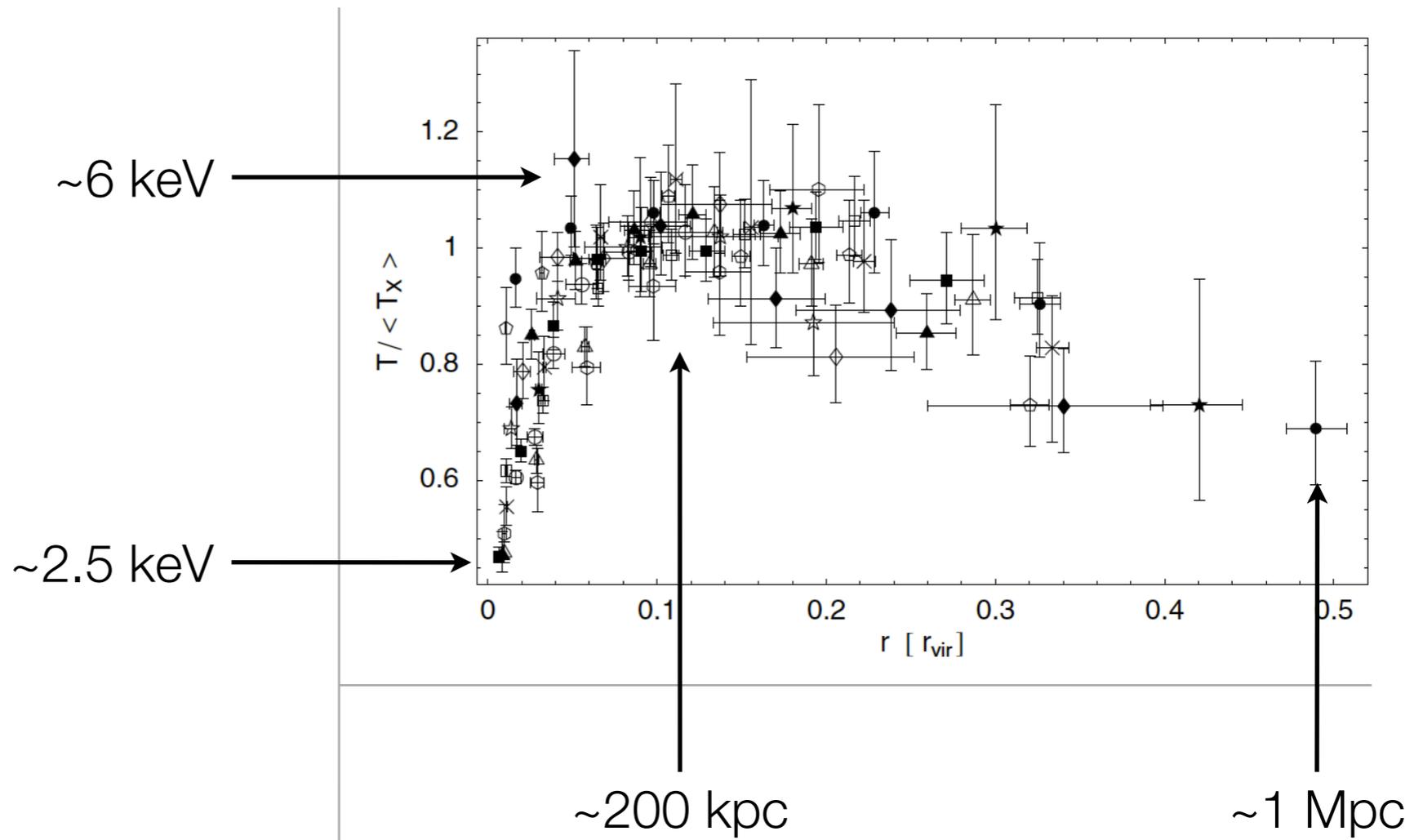
Galaxy Clusters

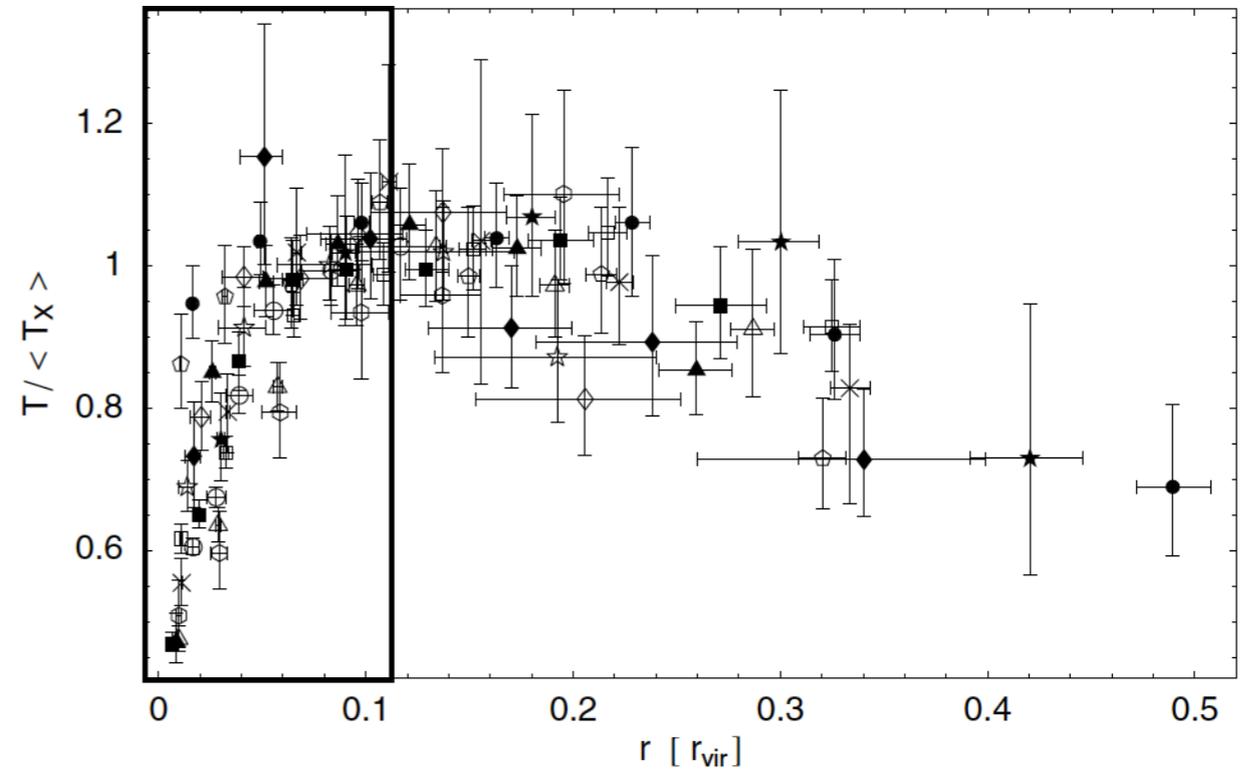
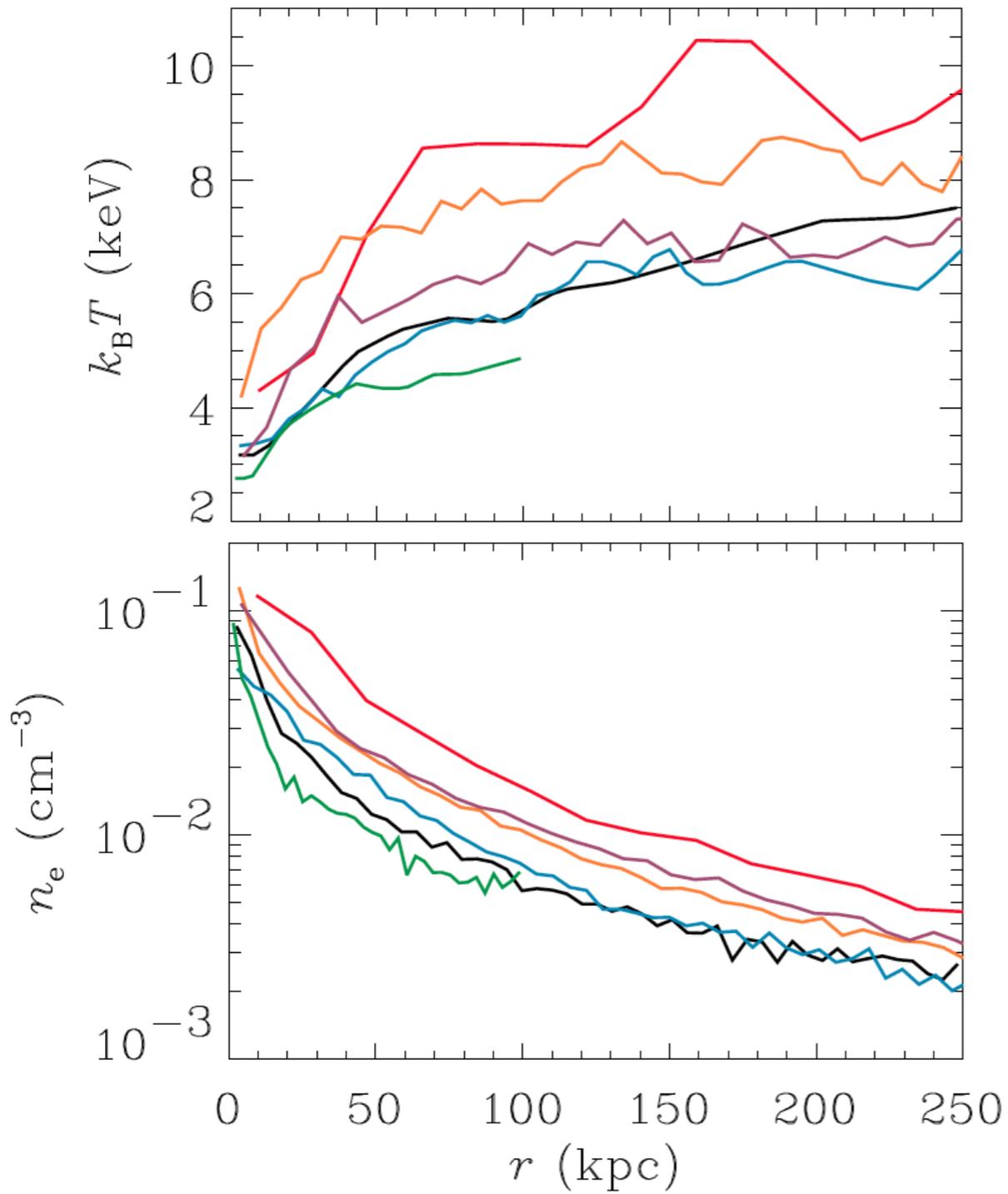
- We will learn the plasma parameters characterising these systems.
- We will investigate conduction, convection, and viscous dissipation in these systems.
- We will investigate the outstanding problems related to cluster plasma physics.
- We will look to the future and see what can be solved with current tools.



$M \sim 10^{14-15} M_{\odot}$ 100s of galaxies in ~ 1 Mpc





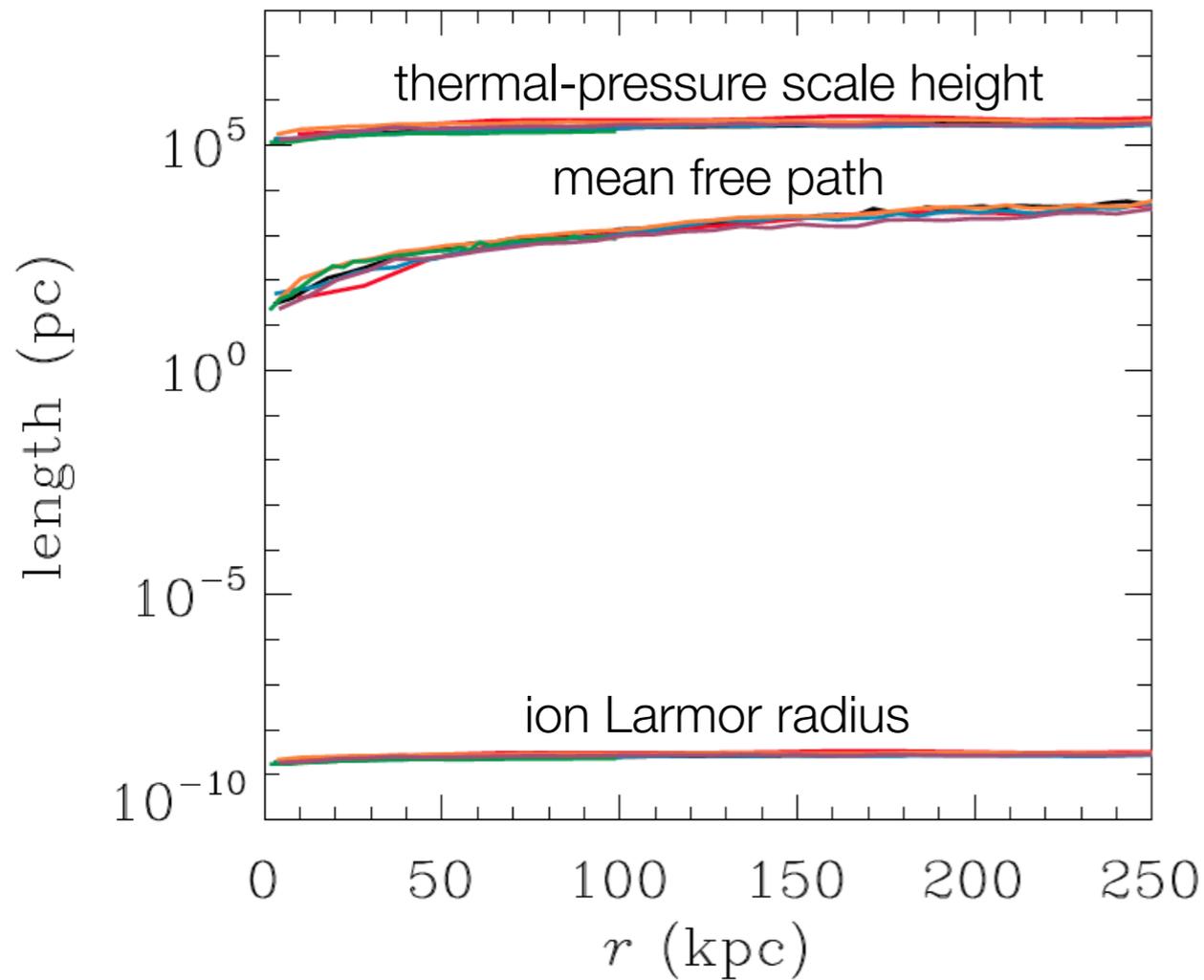


A85
A1835
A1795
A2029
A2199
A478

Cavagnolo et al 2009

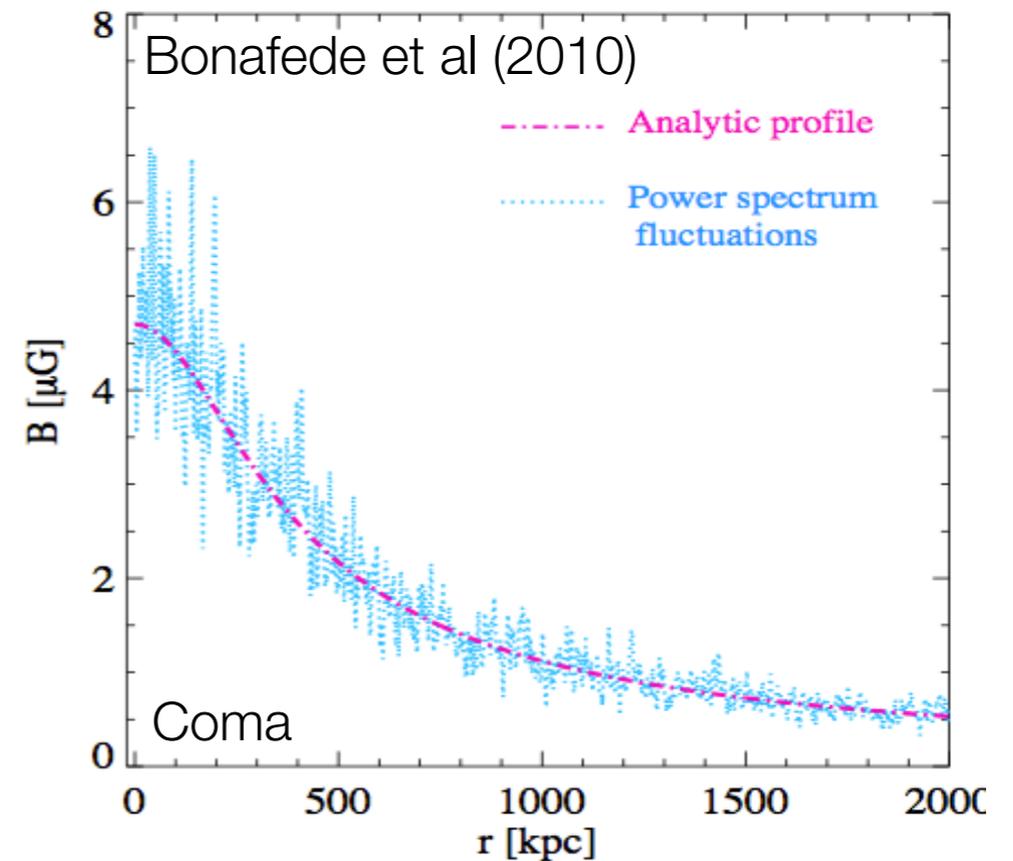
<http://www.pa.msu.edu/astro/MC2/accept/>

ICM Dynamics: A 3-scale problem

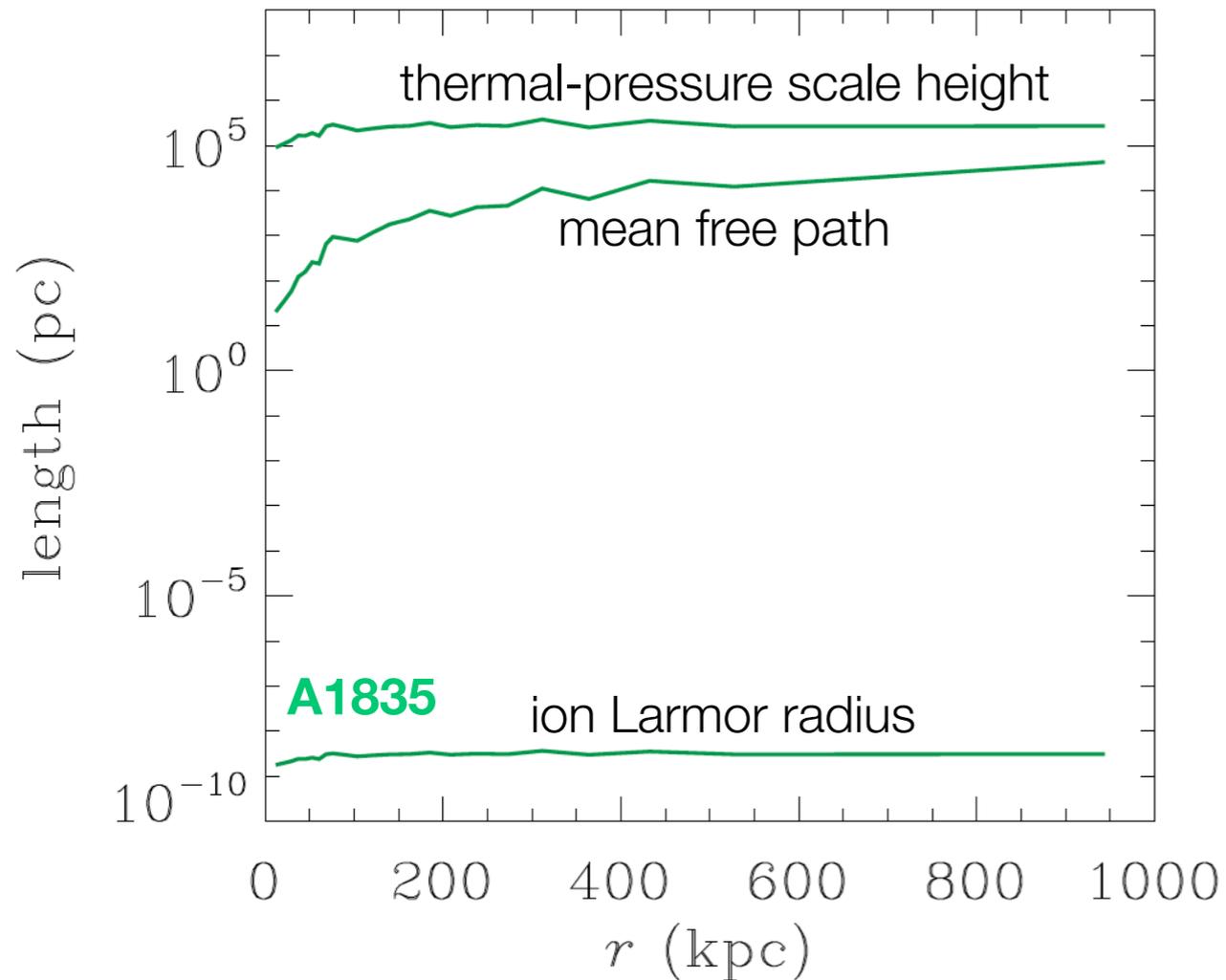


1 npc ~ 20,000 miles
~ 1 trip around the Earth

Cluster name	$n_{e,c}$ (10^{-2} cm^{-3})	T_c (keV)	$B_{c,obs}$ (μG)
Cool-core clusters			
A1835	10	2.85	–
Hydra A	7.2	3.11	12 ^a
A478	15.2	1.72	–
A2199	10	$\simeq 2$	15 ^b
M87	10.8	1.62	35 ^b
A1795	5.4	2.26	9.7 ^b
Centaurus	9.5	1.24	8
A262	3.7	1.54	–

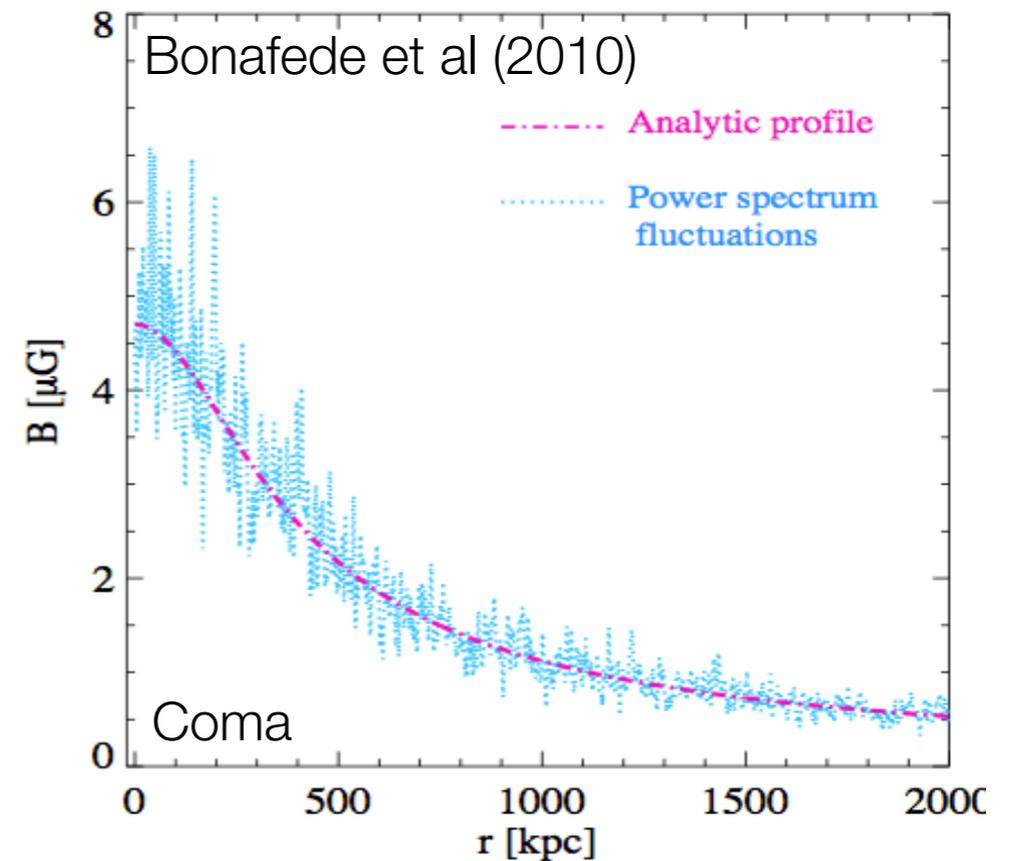


ICM Dynamics: A 3-scale problem



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Braginskii-MHD equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

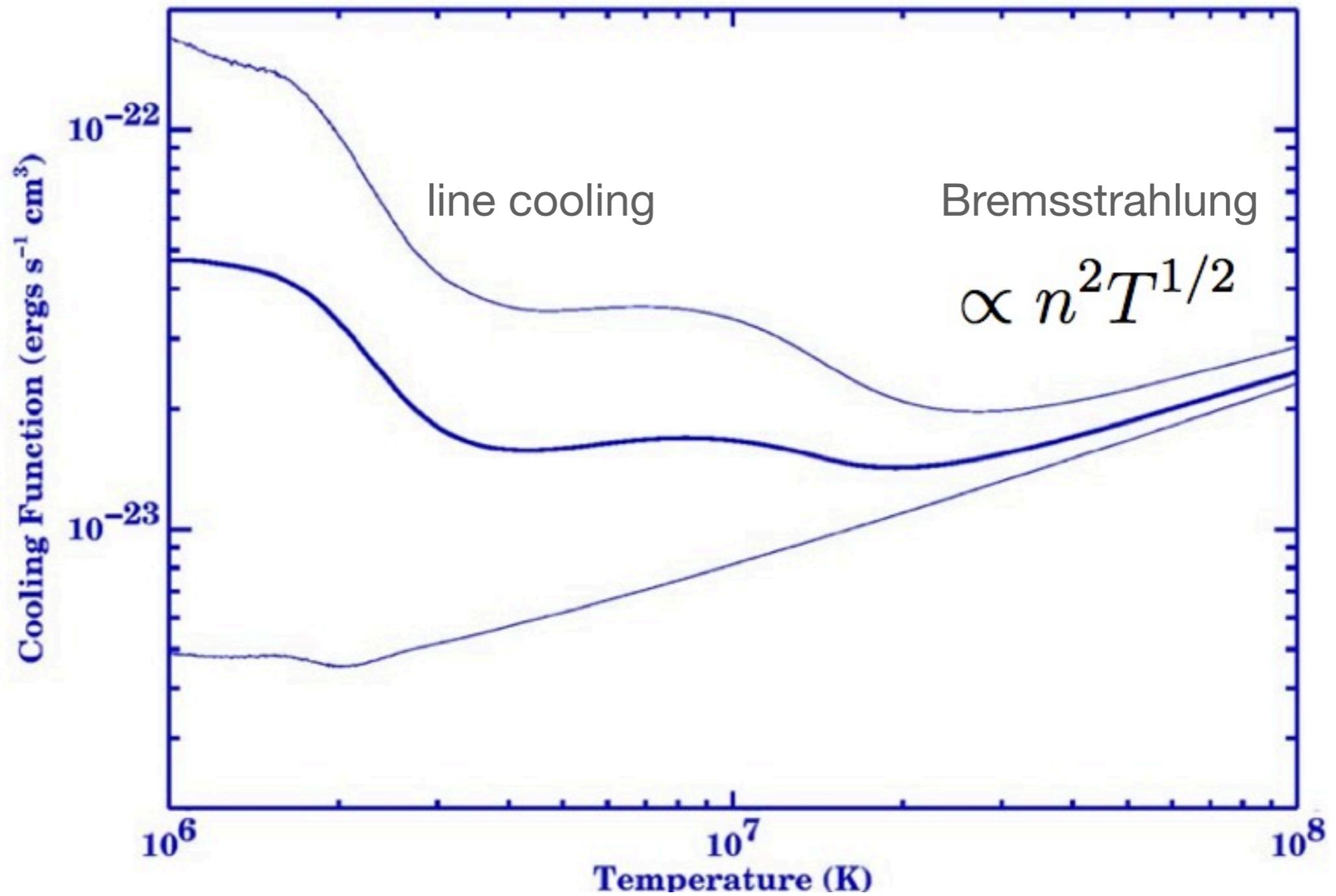
$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla p - \rho \nabla \Phi + \frac{1}{c} \mathbf{j} \times \mathbf{B} + \nabla \cdot \left[\left(\hat{\mathbf{b}} \hat{\mathbf{b}} - \frac{1}{3} \mathbf{I} \right) (p_{\perp} - p_{\parallel}) \right]$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0$$

$$\frac{3}{2} p \frac{d}{dt} \ln \frac{p}{\rho^{5/3}} = \rho \mathcal{L} + \nabla \cdot \left(\chi_{\parallel} \hat{\mathbf{b}} \hat{\mathbf{b}} \cdot \nabla T \right) + (p_{\perp} - p_{\parallel}) \frac{d}{dt} \ln \frac{B}{\rho^{2/3}}$$

$$p_{\perp} - p_{\parallel} = \frac{3}{2} \frac{\rho v_{\text{th}}^2}{\nu_i} \frac{d}{dt} \ln \frac{B}{\rho^{2/3}}$$

Cooling



Physical timescales

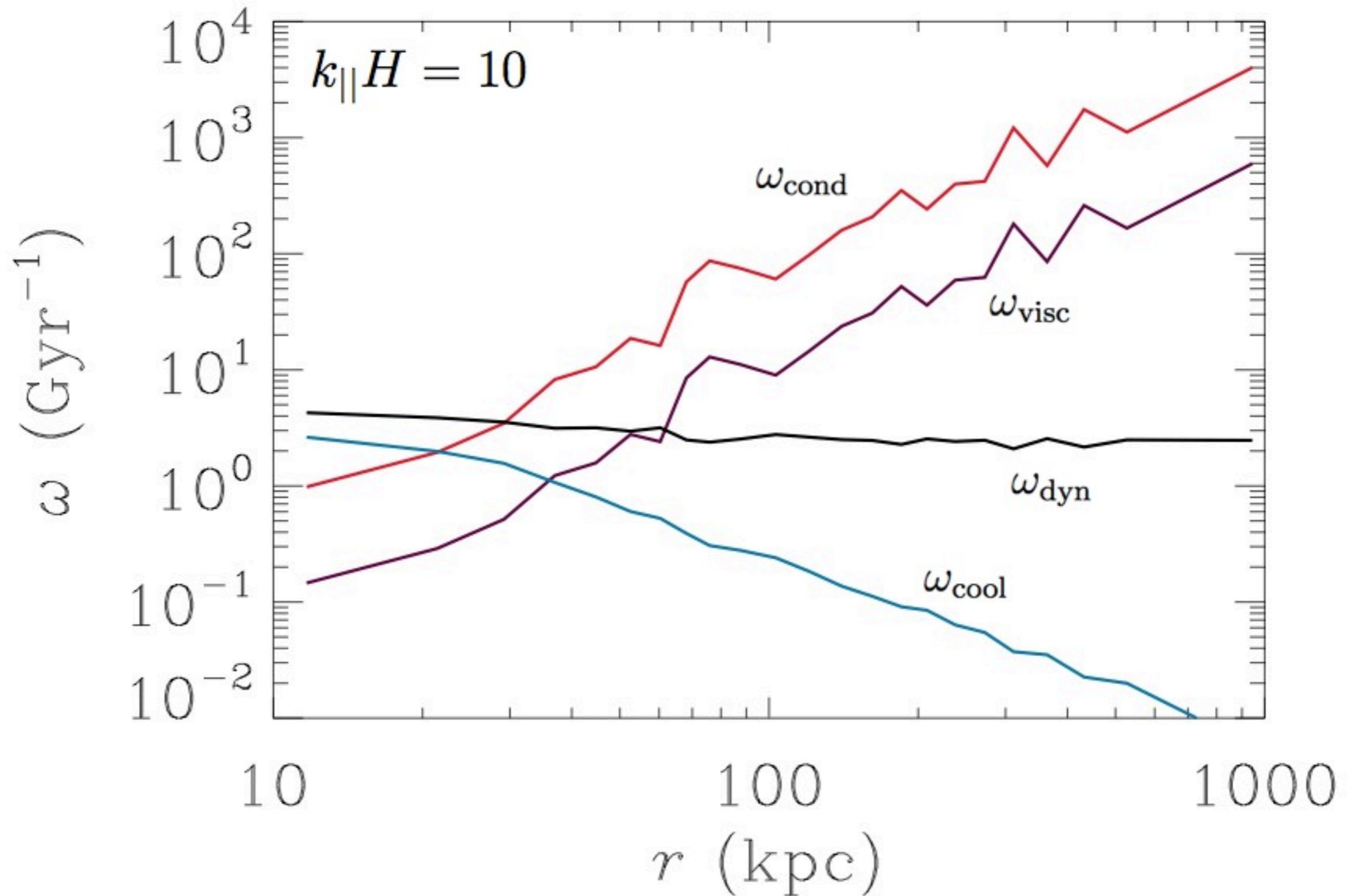
$$\omega_{\text{cond}} \equiv 0.79 \times \frac{2}{5} \frac{k_{\parallel}^2 v_{\text{th,e}}^2}{\nu_e} \approx 11 \left(\frac{k_{\parallel} H}{10} \right)^2 \left(\frac{n_e}{0.01 \text{ cm}^{-3}} \right)^{-1} \left(\frac{k_B T}{3 \text{ keV}} \right)^{5/2} \text{ Gyr}^{-1}$$

$$\omega_{\text{dyn}} \equiv \left(\frac{g}{H} \right)^{1/2} \approx 4 \left(\frac{g}{10^{-8} \text{ cm s}^{-2}} \right) \left(\frac{k_B T}{3 \text{ keV}} \right)^{1/2} \text{ Gyr}^{-1}$$

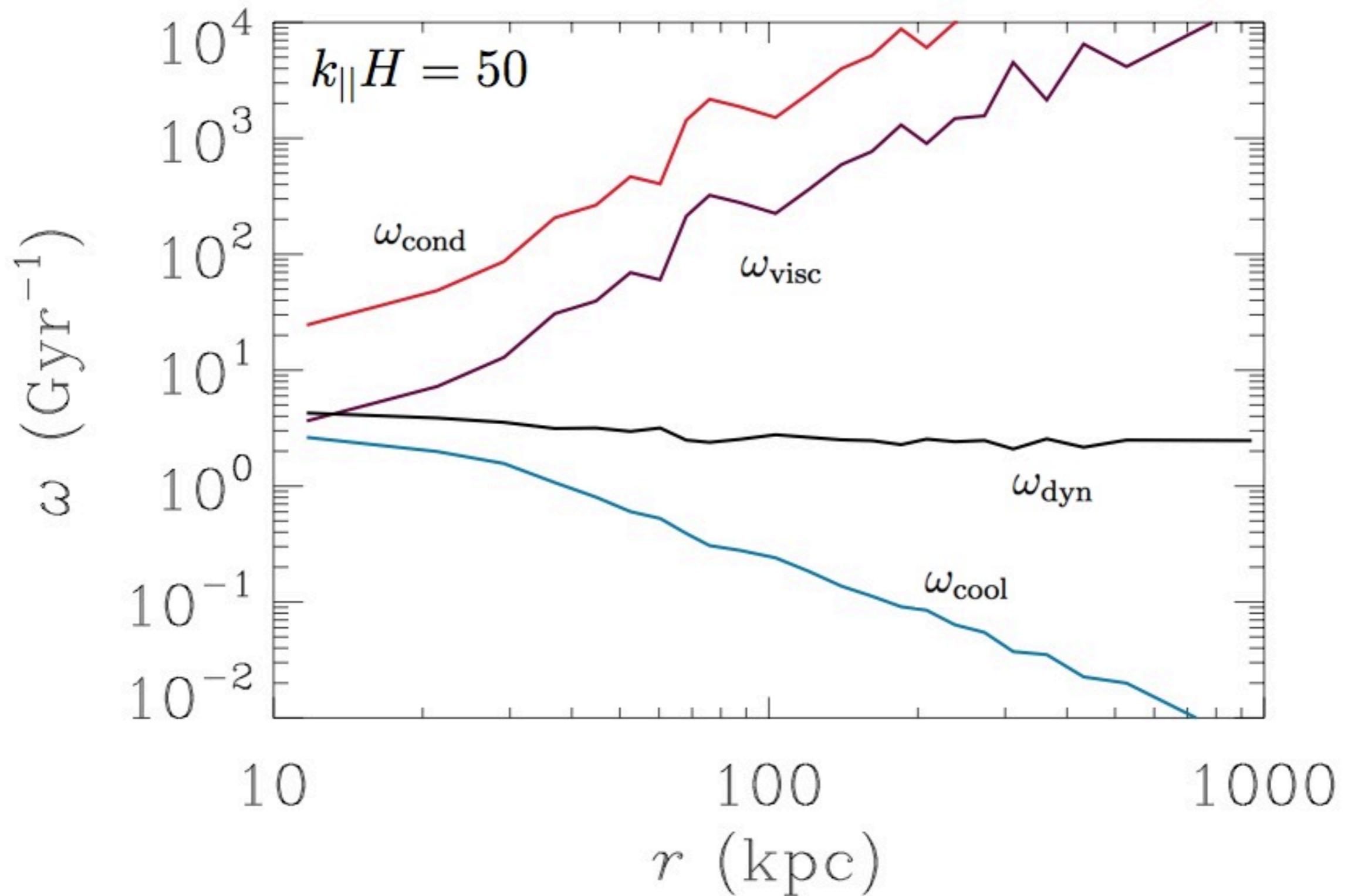
$$\omega_{\text{visc}} \equiv 0.96 \times \frac{3}{2} \frac{k_{\parallel}^2 v_{\text{th}}^2}{\nu_i} \approx 2 \left(\frac{k_{\parallel} H}{10} \right)^2 \left(\frac{n_i}{0.01 \text{ cm}^{-3}} \right)^{-1} \left(\frac{k_B T}{3 \text{ keV}} \right)^{5/2} \text{ Gyr}^{-1}$$

$$\omega_{\text{cool}} \equiv -\frac{3}{5} \frac{\rho \mathcal{L}}{p} \approx -0.3 \left(\frac{n_e}{0.01 \text{ cm}^{-3}} \right) \left(\frac{k_B T}{3 \text{ keV}} \right)^{-1/2} \text{ Gyr}^{-1}$$

Physical timescales



Physical timescales



Cooling flows

$$\rho\mathcal{L} \approx 10^{-25} \left(\frac{n_e}{0.1 \text{ cm}^{-3}} \right)^2 \left(\frac{T}{2 \text{ keV}} \right)^{1/2} \text{ erg s}^{-1} \text{ cm}^{-3}$$

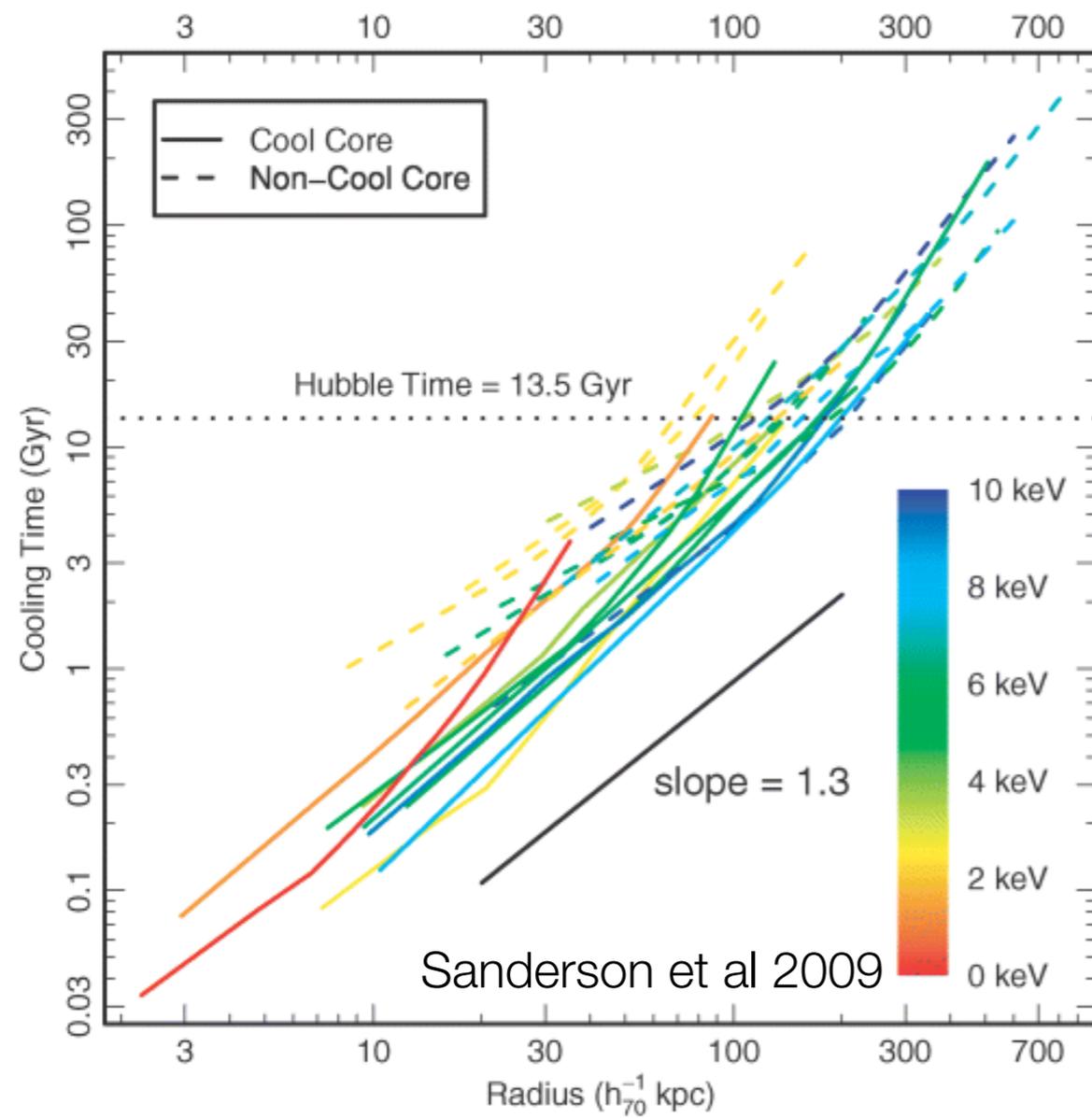
$$t_{\text{cool}} \equiv \frac{2}{3} \frac{\rho\mathcal{L}}{n_e T} \sim 100 \left(\frac{n_e}{0.1 \text{ cm}^{-3}} \right) \left(\frac{T}{2 \text{ keV}} \right)^{-1/2} \text{ Myr}$$

$$\ll t_{\text{age}} \sim H_0^{-1}$$

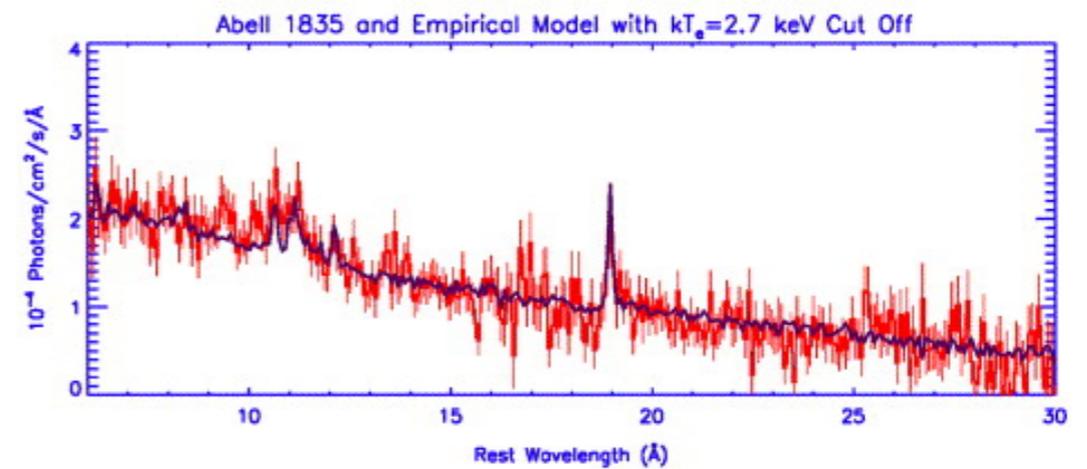
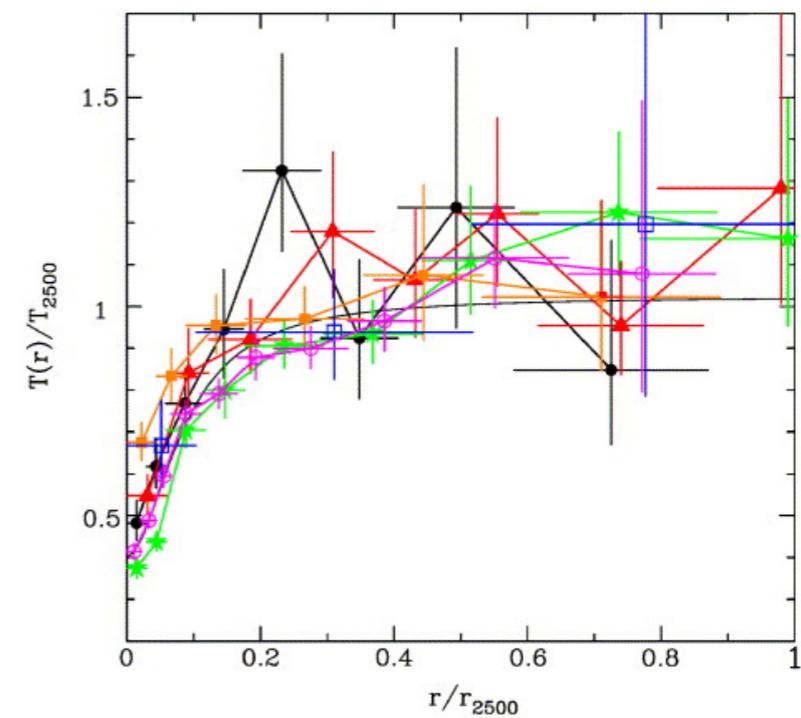
$$L_{\text{cool}} = \frac{5}{2} \frac{\dot{M}}{\mu m} k_B T \stackrel{\sim 3 \text{ keV}}{\rightarrow} \dot{M} \sim 100 M_{\odot} \text{ yr}^{-1}$$
$$\sim 10^{44} \text{ erg s}^{-1}$$

Cooling flows

expect lots of cold gas...



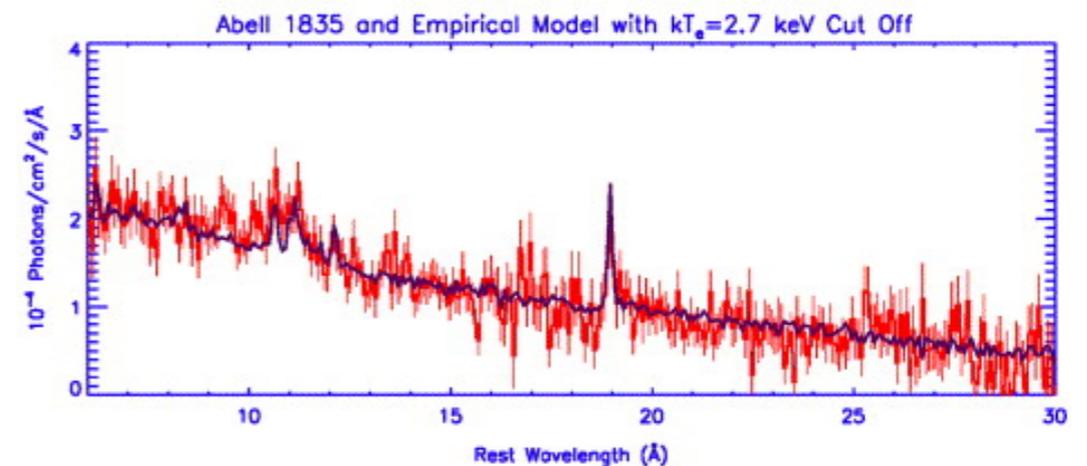
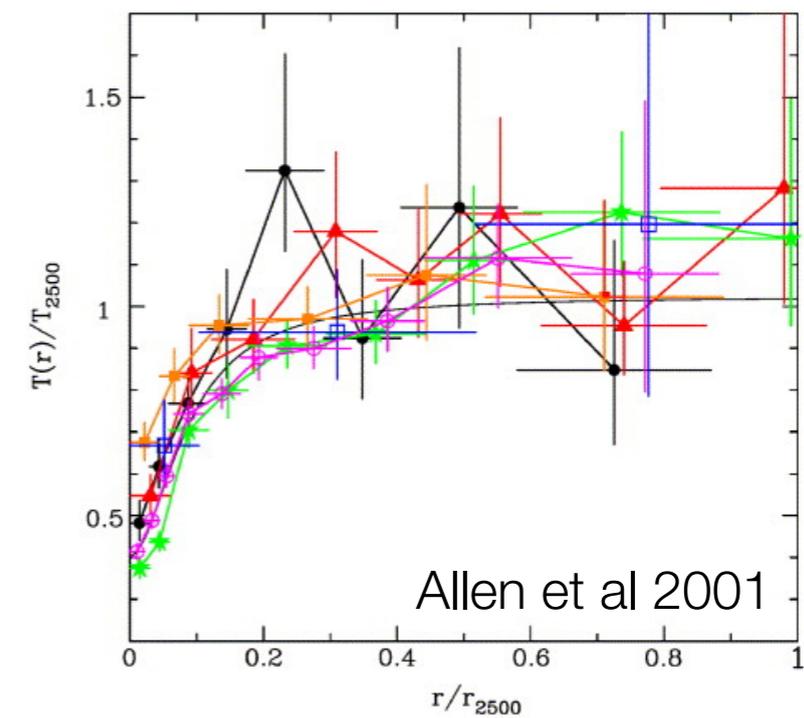
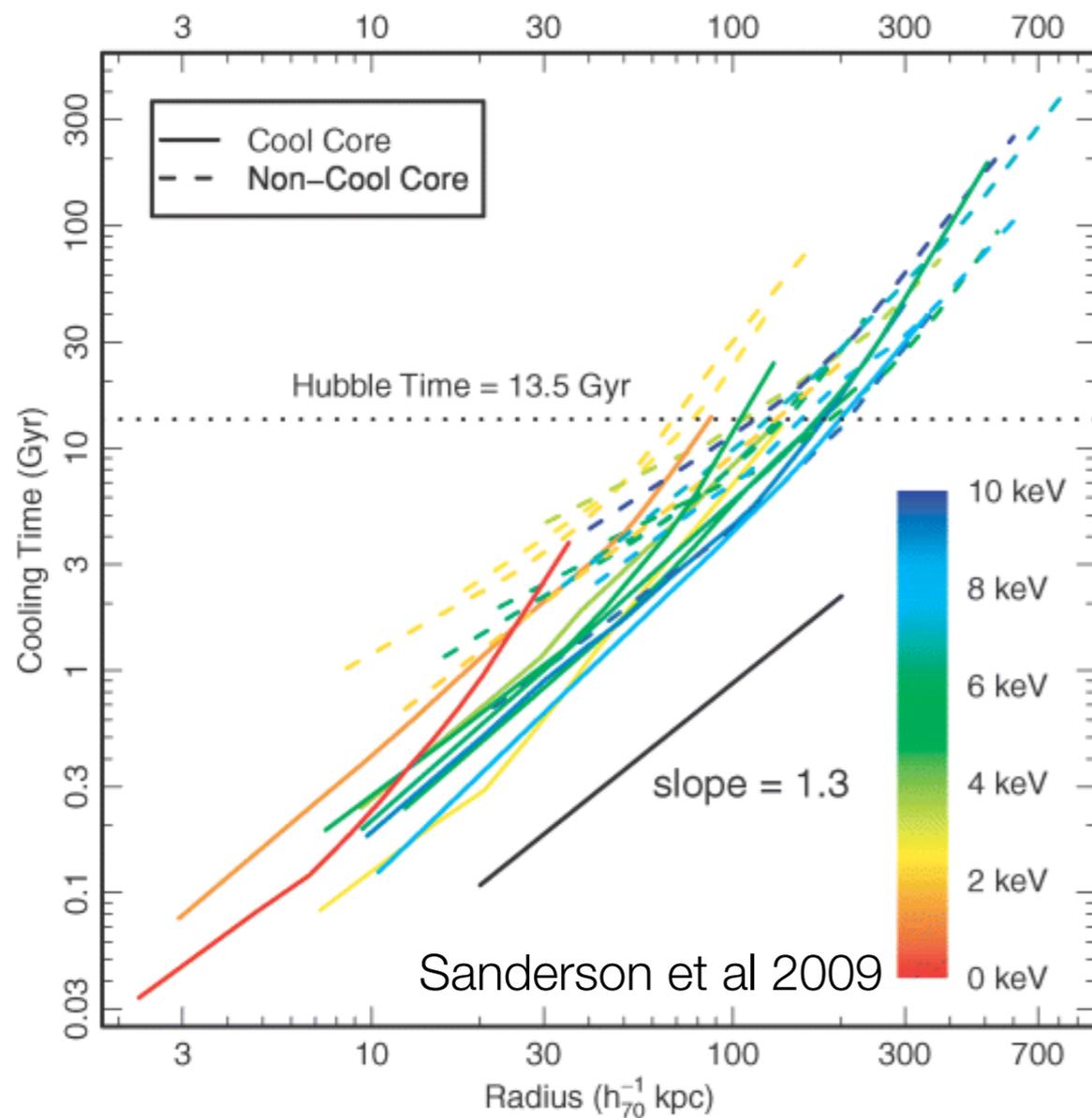
...but not observed



~~Cooling flows~~ “Cooling-flow problem”

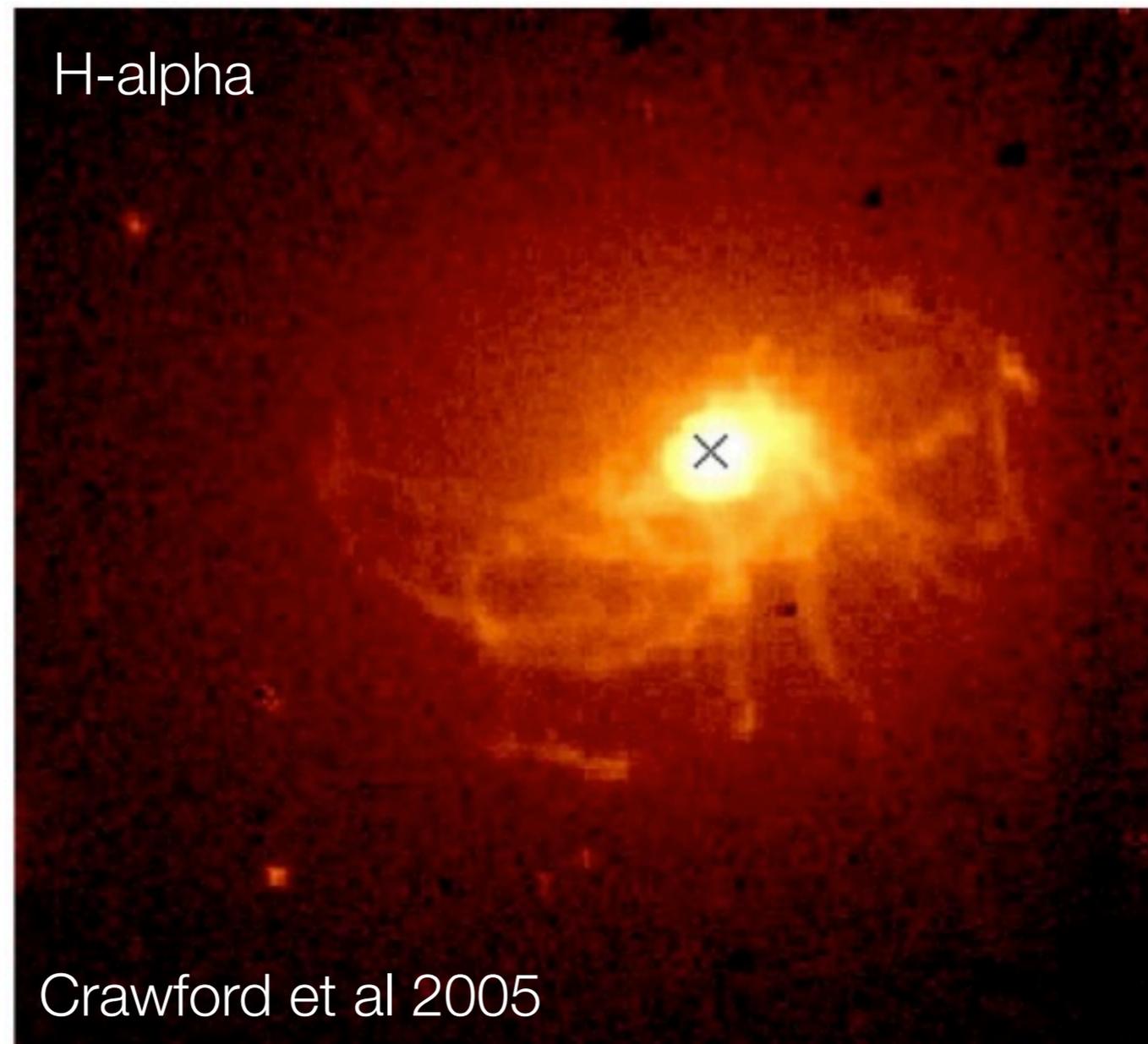
expect lots of cold gas...

...but not observed



There is *some* cold gas...

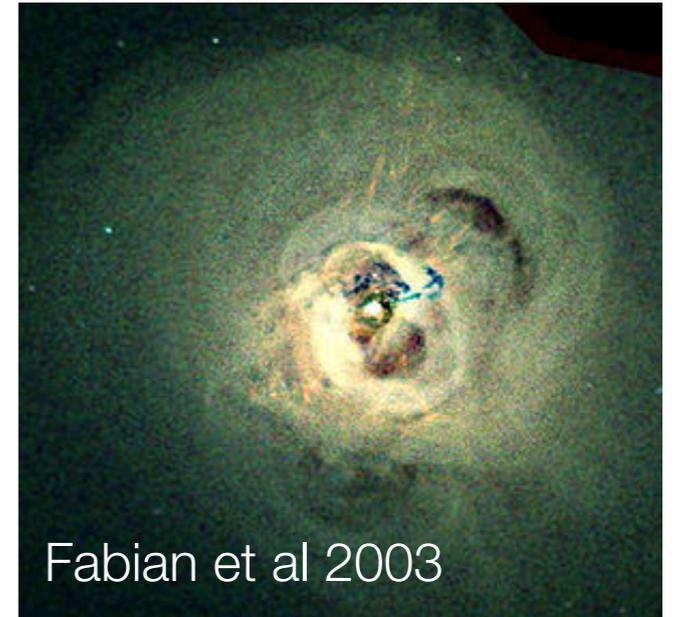
...mostly in filaments



God's thermostat?

- Feedback from A(ctive) G(alactic) N(uclei)...

...but how does it know how much heat is needed?



$$\dot{M} \sim 4\pi n v_r r^2 \sim 4\pi \rho c_s \left(\frac{GM}{c_s^2} \right)^2$$
$$\propto \frac{1}{c_s^5} \text{ at constant pressure}$$

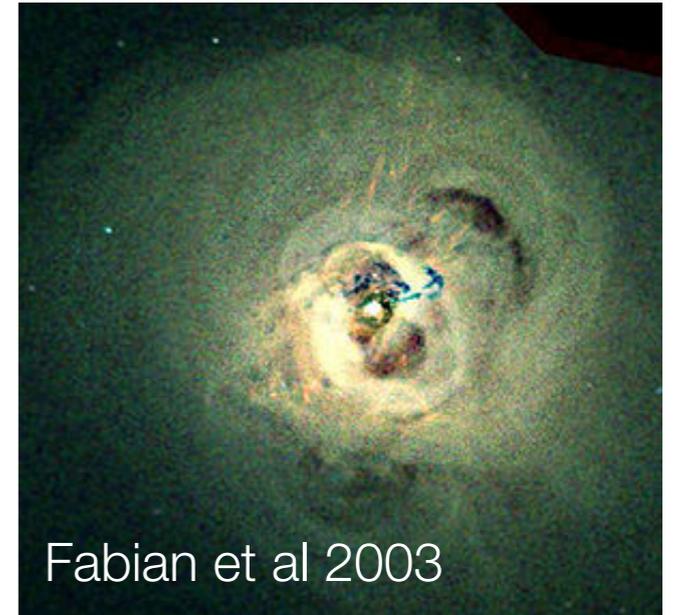
God's thermostat?

- Feedback from A(ctive) G(alactic) N(uclei)...

...but how does it know how much heat is needed?

...and how is it thermalized and distributed?

turbulence? sound waves? weak shocks? bubbles? cosmic rays?



$$\text{Re} = \frac{u_{||} \ell_{||}}{\nu_{||}} \sim 10 - 100$$

God's thermostat?

- Feedback from A(ctive) G(alactic) N(uclei)...

...but how does it know how much heat is needed?

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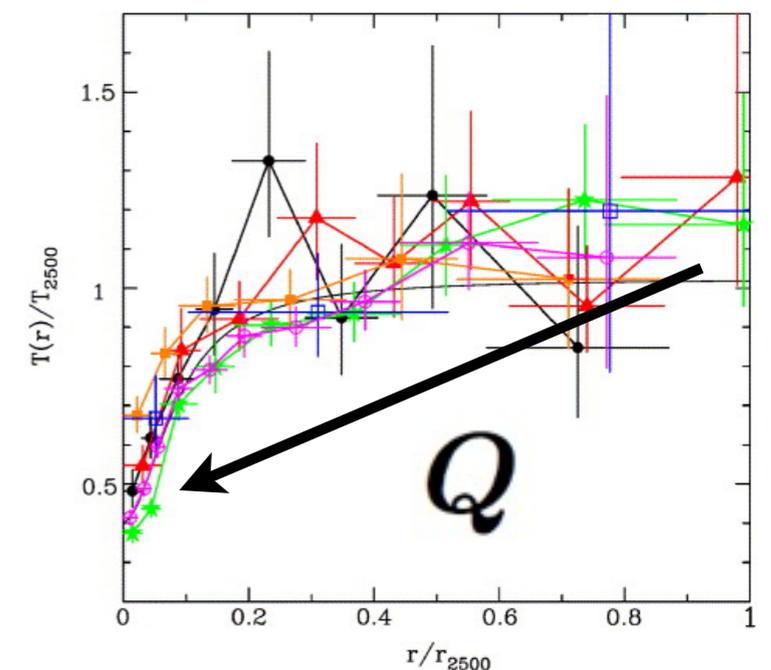
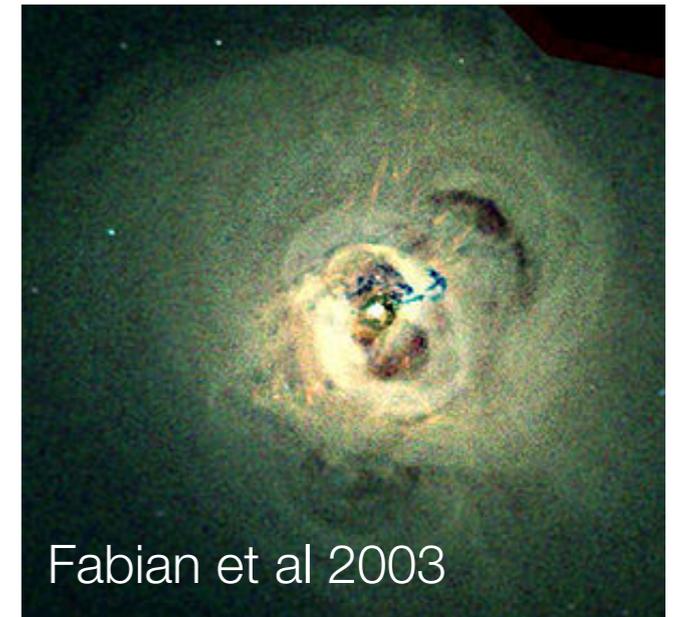
turbulence? sound waves? weak shocks? bubbles? cosmic rays?

- Conduction inwards from bulk of cluster...

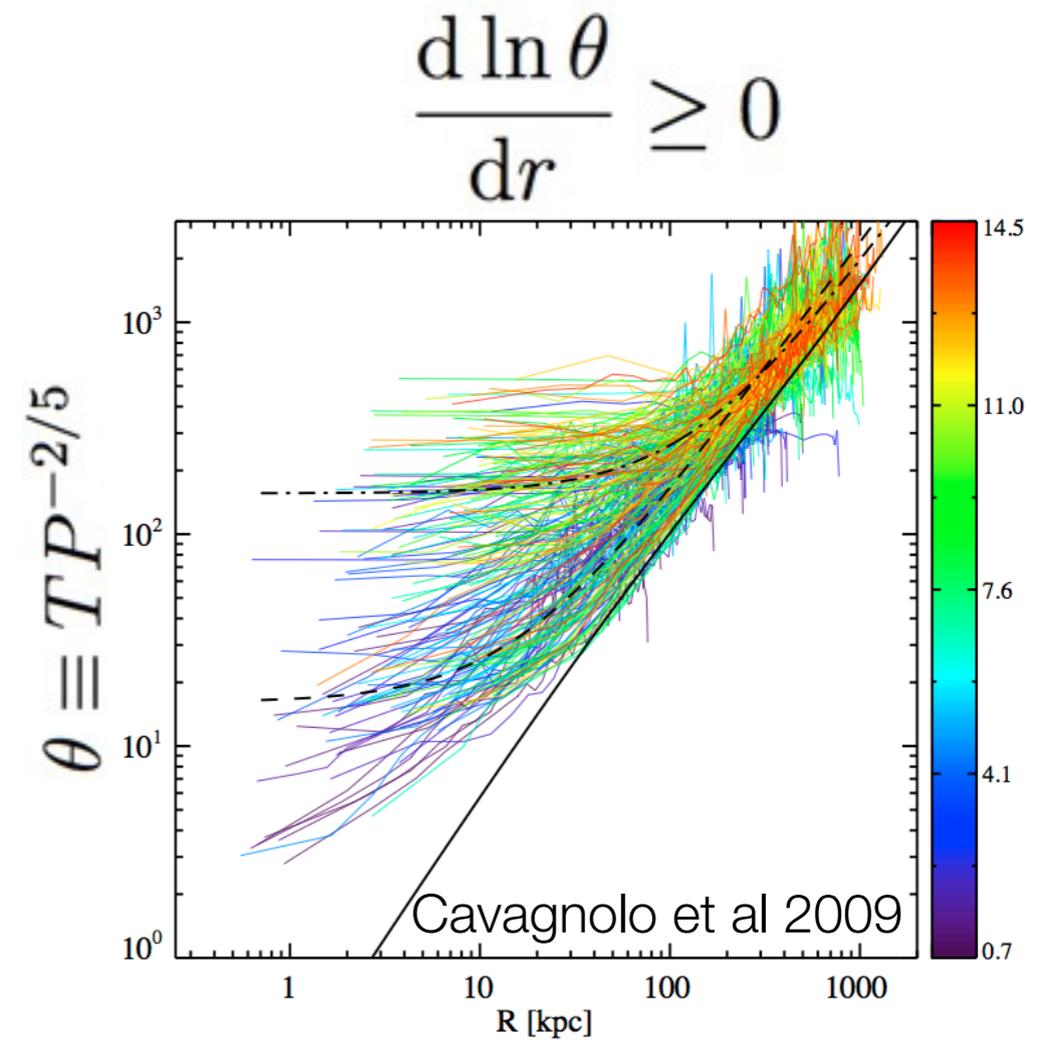
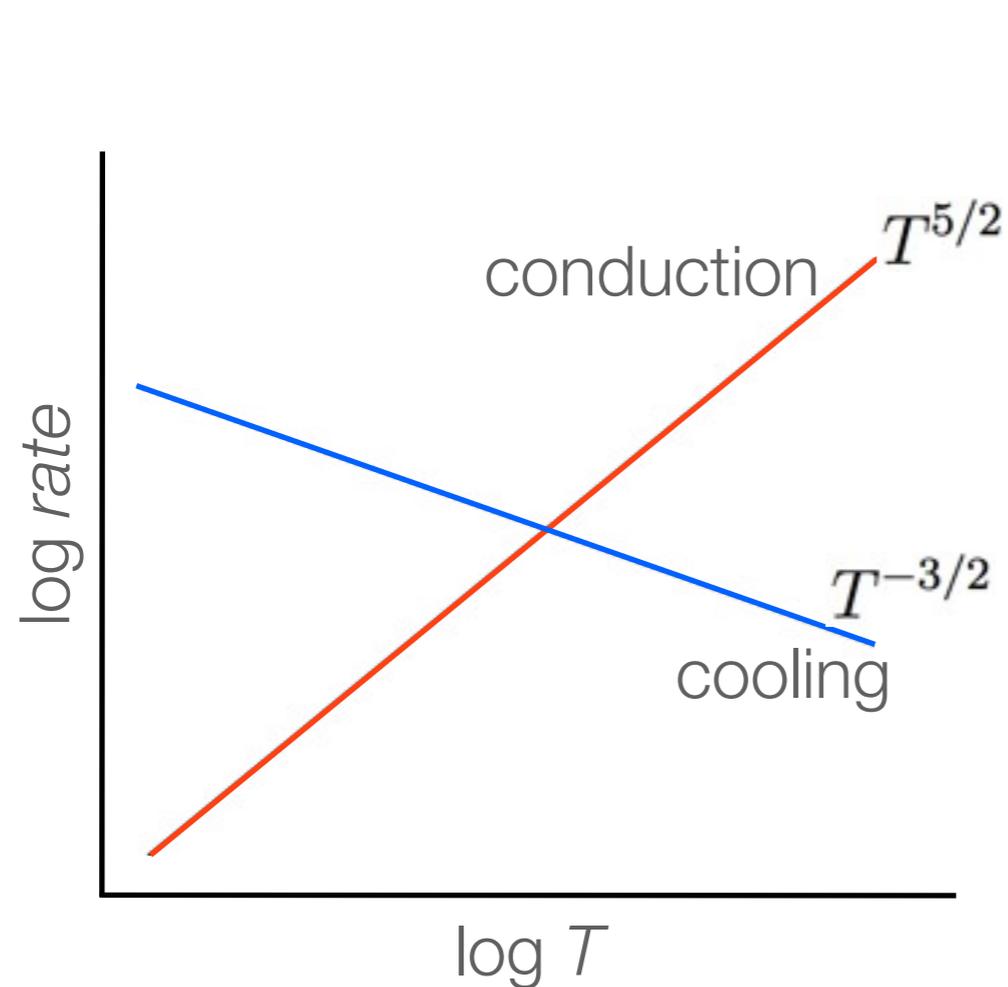
...but can it (stably) offset cooling? (e.g. Fabian et al 2002)

...and what is its efficiency?

(e.g. Chandran & Cowley 1998; Narayan & Medvedev 2001)



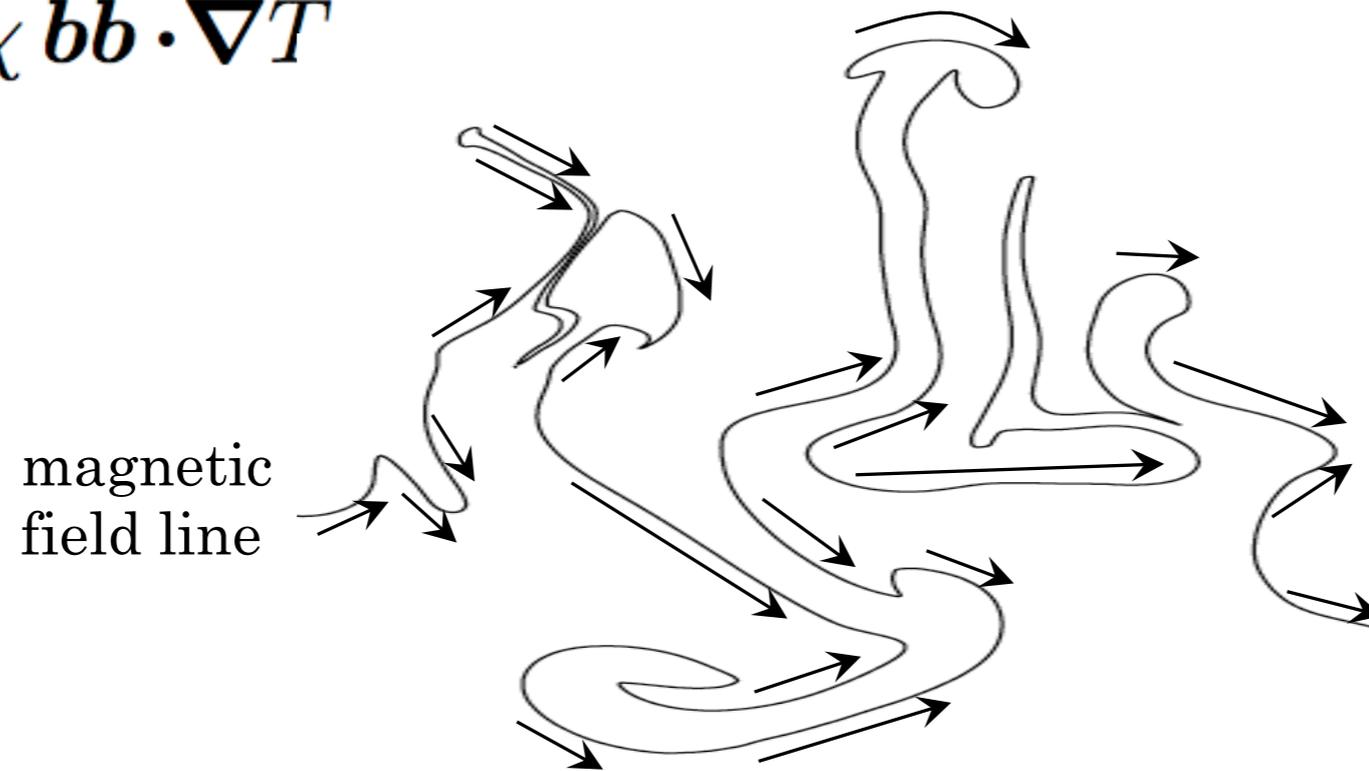
Conduction and convection – stability



but it's a bit more subtle than this...

Heat conduction

$$Q = -\chi \hat{b}\hat{b} \cdot \nabla T$$

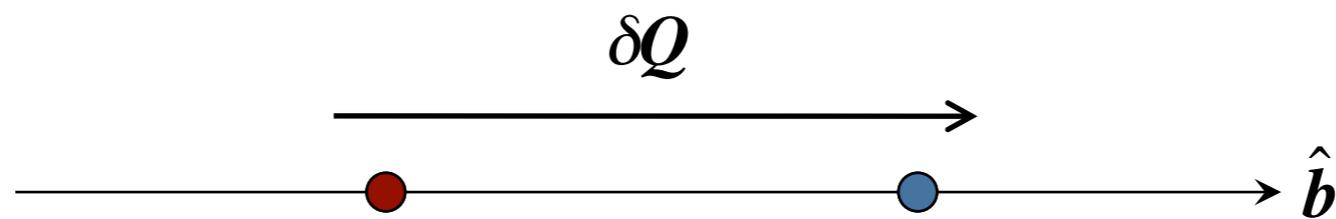


hot  cold

$$\delta Q = -\chi \nabla \delta T \quad \text{vs.} \quad \delta Q = -\chi \hat{b}\hat{b} \cdot \nabla \delta T - \chi \hat{b} \delta \hat{b} \cdot \nabla T - \chi \delta \hat{b} \hat{b} \cdot \nabla T$$

Heat conduction

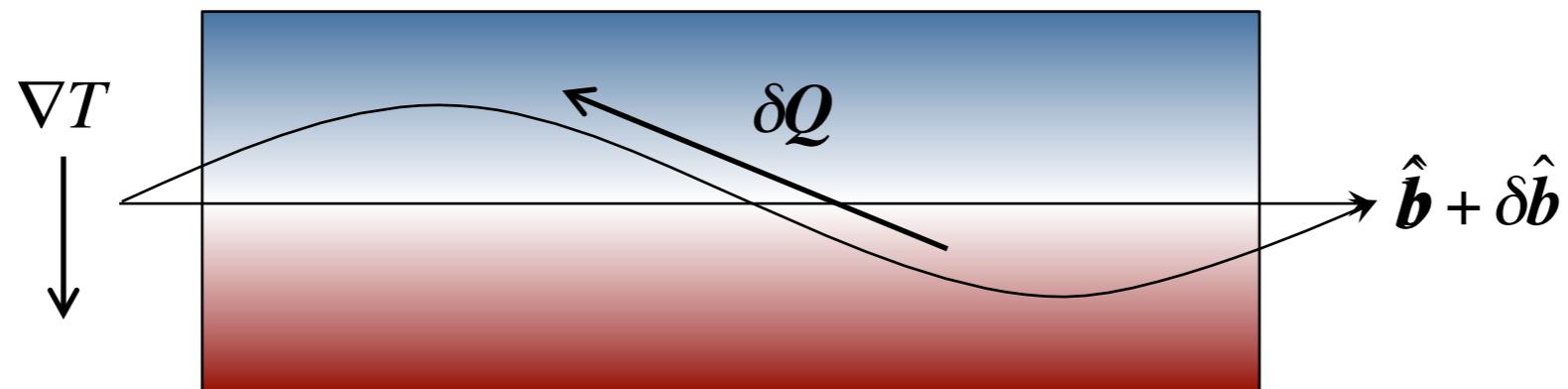
$$\delta Q = \boxed{-\chi \hat{\mathbf{b}} \hat{\mathbf{b}} \cdot \nabla \delta T} - \chi \hat{\mathbf{b}} \delta \hat{\mathbf{b}} \cdot \nabla T - \chi \delta \hat{\mathbf{b}} \hat{\mathbf{b}} \cdot \nabla T$$



create a local temperature gradient along B,
create heat flux along B

Heat conduction

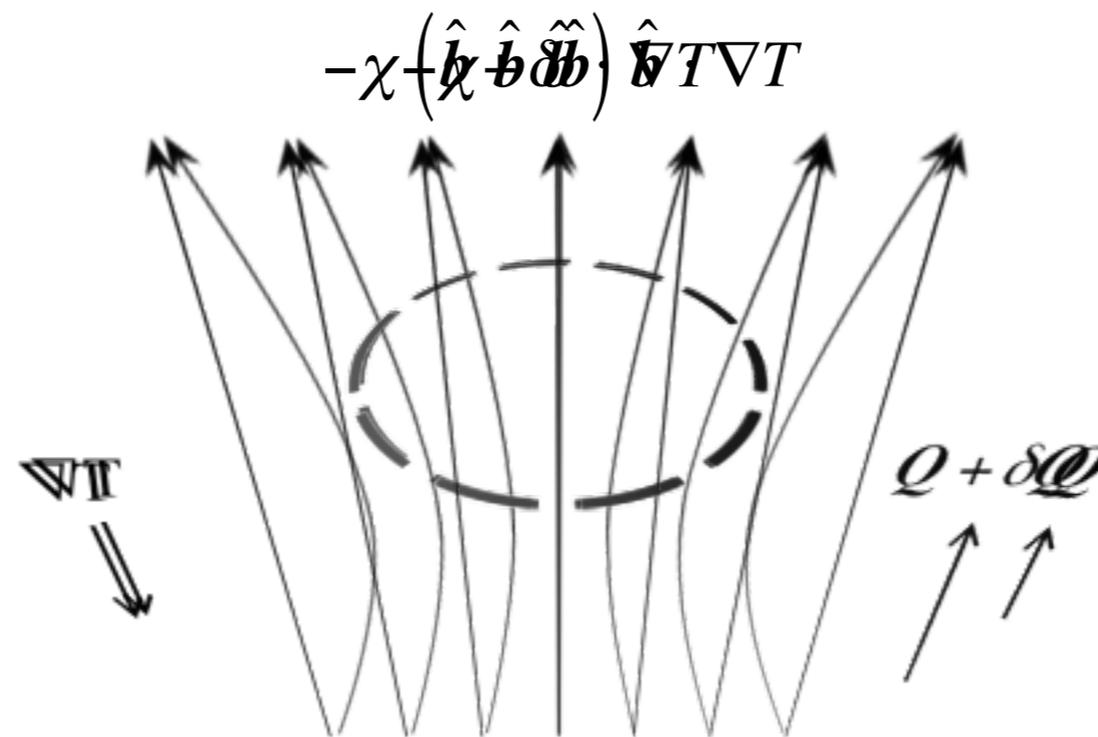
$$\delta Q = -\chi \hat{\mathbf{b}} \hat{\mathbf{b}} \cdot \nabla \delta T \quad \boxed{-\chi \hat{\mathbf{b}} \delta \hat{\mathbf{b}} \cdot \nabla T} \quad -\chi \delta \hat{\mathbf{b}} \hat{\mathbf{b}} \cdot \nabla T$$



align \mathbf{B} with the background temperature gradient,
create heat flux along \mathbf{B}

Heat conduction

$$\delta Q = -\chi \hat{\mathbf{b}} \hat{\mathbf{b}} \cdot \nabla \delta T - \chi \hat{\mathbf{b}} \delta \hat{\mathbf{b}} \cdot \nabla T \quad \boxed{-\chi \delta \hat{\mathbf{b}} \hat{\mathbf{b}} \cdot \nabla T}$$

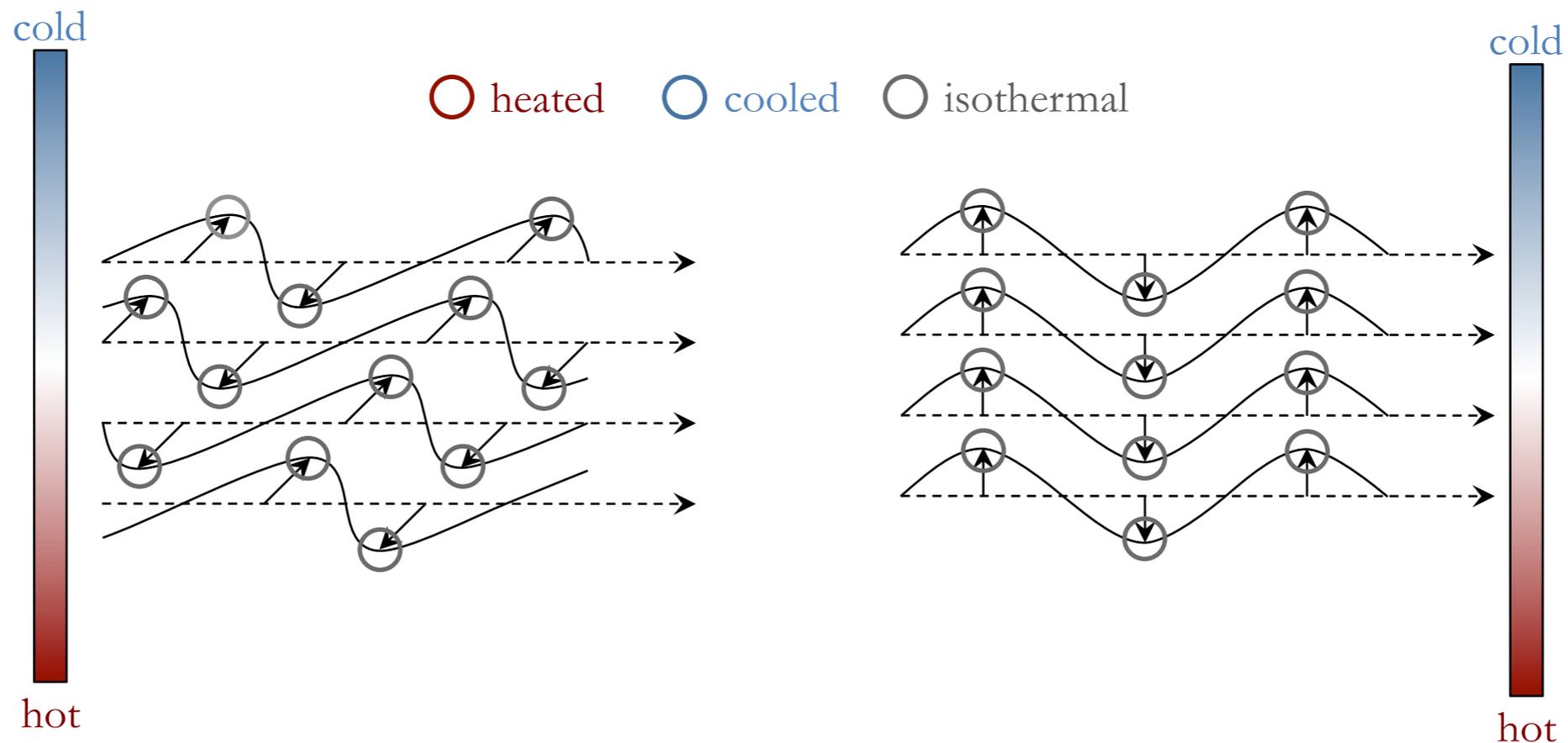


perturb B on which a heat flux is being channeled,
increase/decrease that heat flux

When conduction is rapid...

$$\Delta T \simeq 2\xi_{||} \nabla_{||} T$$

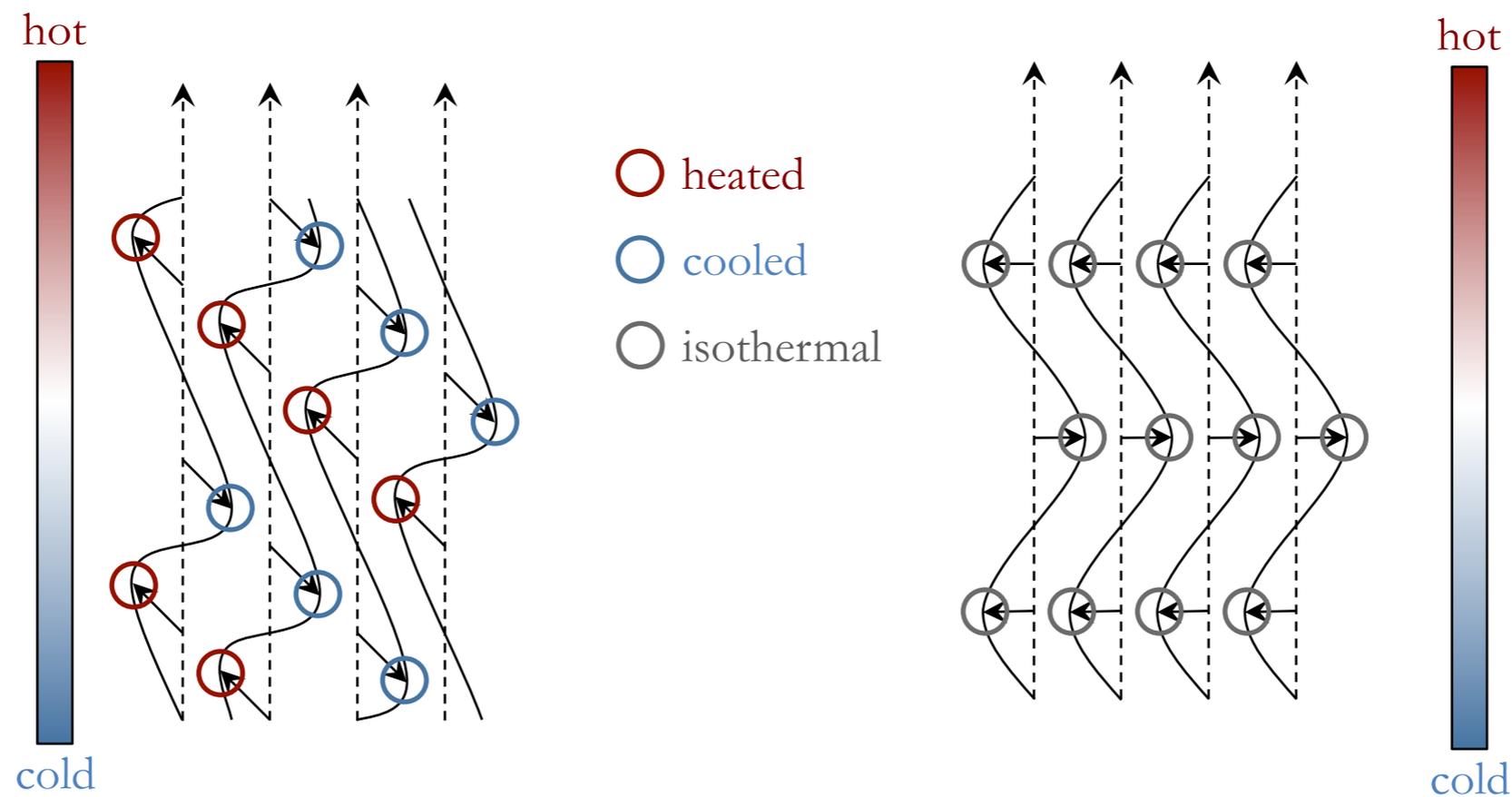
i.e. compressions/rarefactions in ∇T -oriented field lines lead to heating/cooling

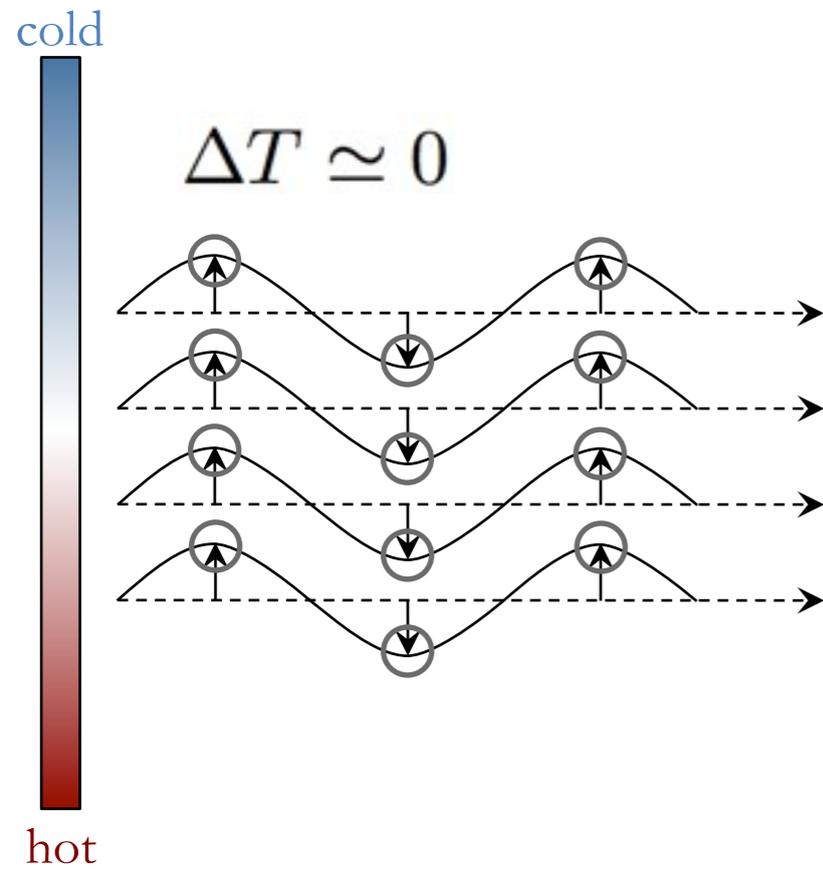


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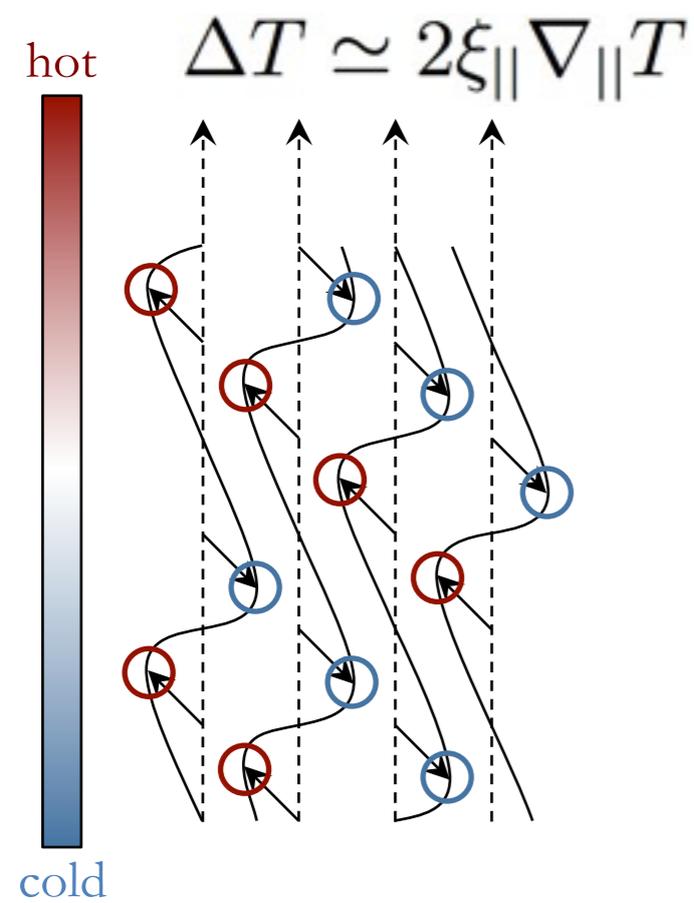
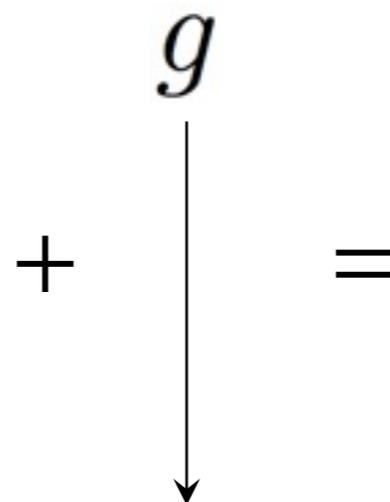
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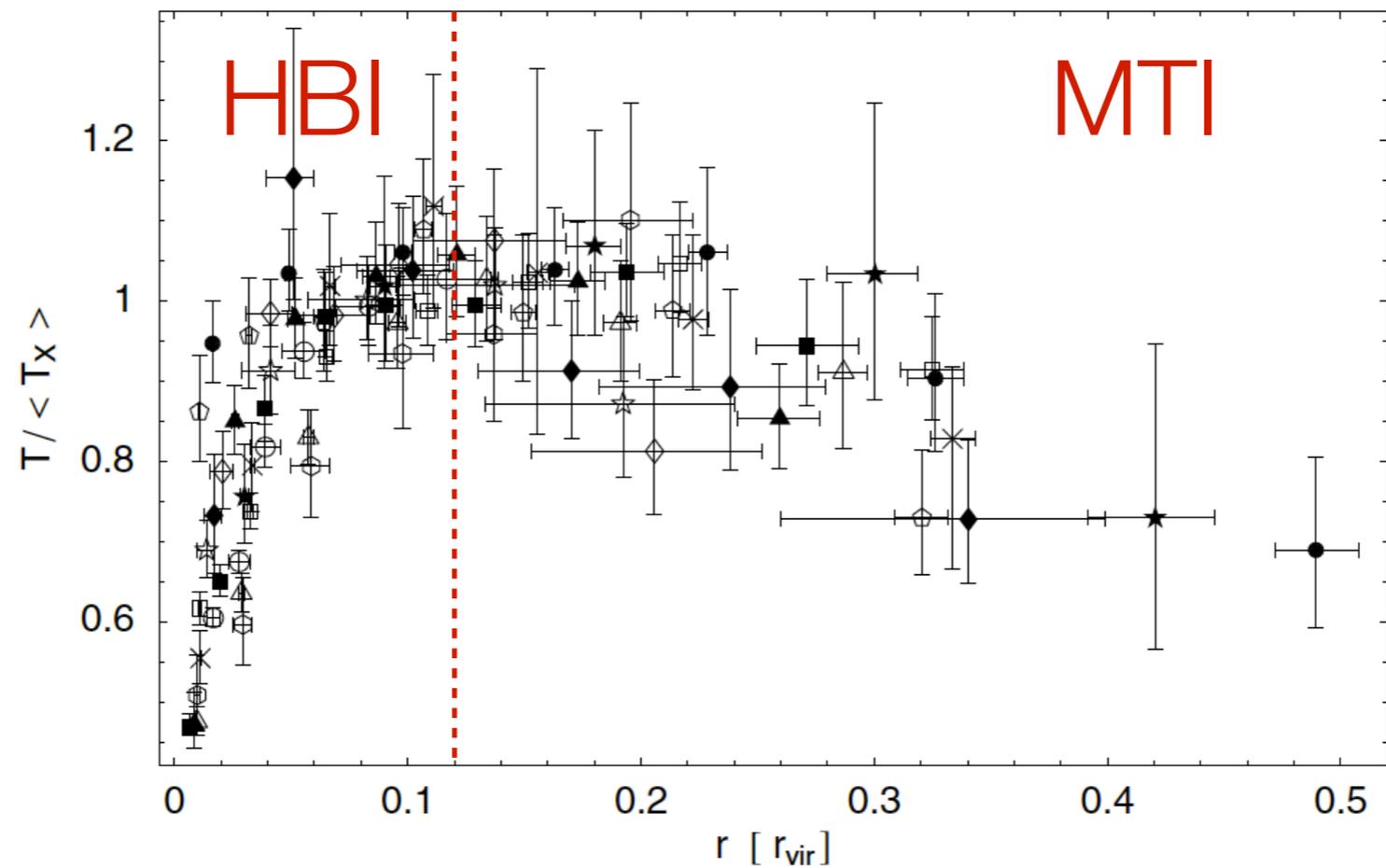




Magneto-Thermal
Instability
(Balbus 2000; 2001)



Heat-flux-driven
Buoyancy
Instability
(Quataert 2008)



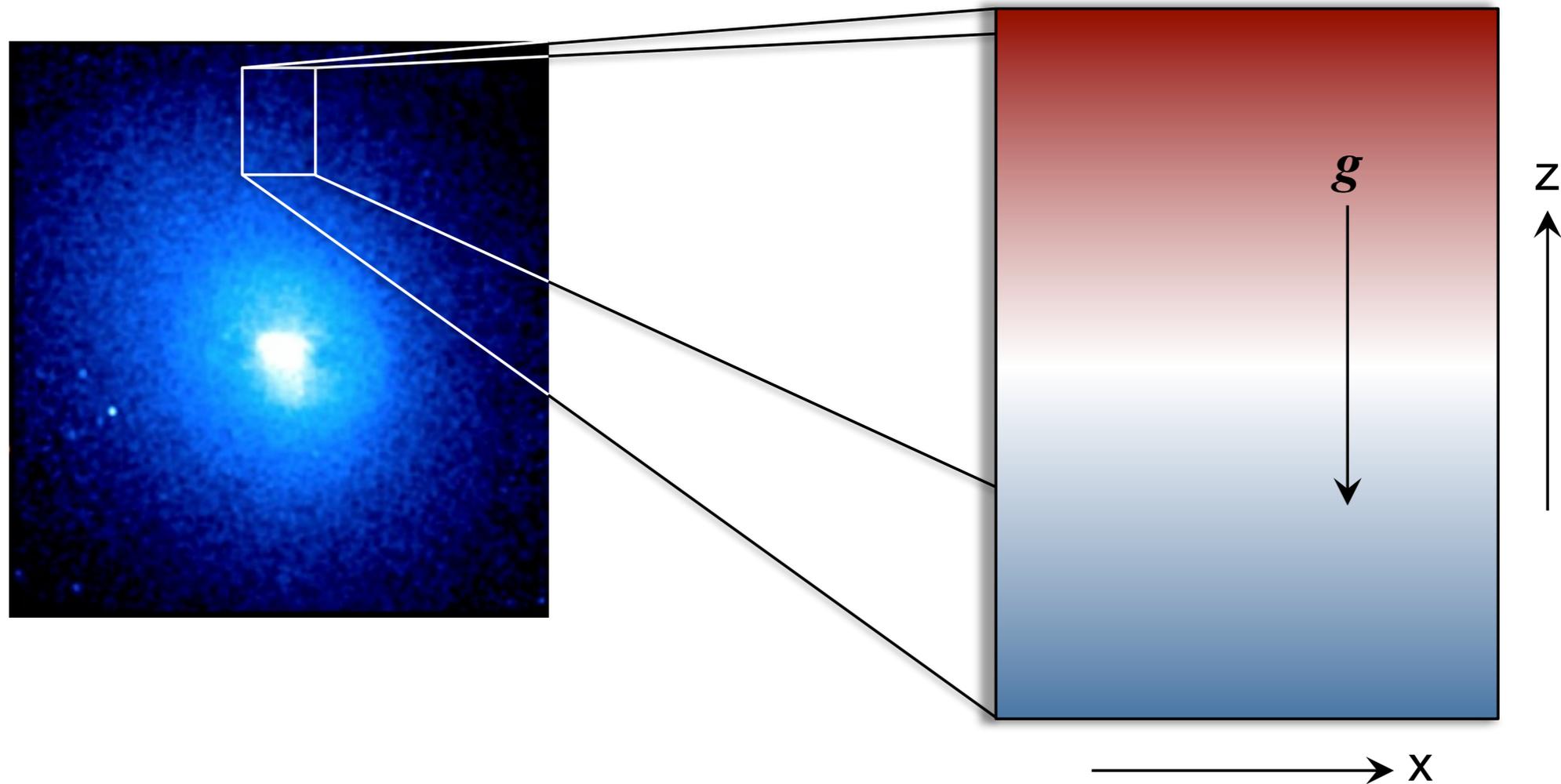
$$\mathbf{g} \cdot \nabla \ln P \rho^{-\gamma} < 0 \longrightarrow \mathbf{g} \cdot \nabla \ln T \neq 0$$

think by analogy:

$$\mathbf{g} \cdot \nabla \ln R^4 \Omega^2 < 0 \longrightarrow \mathbf{g} \cdot \nabla \ln \Omega^2 < 0$$

Balbus (2001)

put a weakly collisional plasma in a
gravitating, thermally stratified atmosphere



$$\mathbf{v} = \delta \mathbf{v} \quad \mathbf{B} = B_{0,x} \hat{\mathbf{x}} + B_{0,z} \hat{\mathbf{z}} + \delta \mathbf{B} \quad p = p_0(z) + \delta p \quad T = T_0(z) + \delta T$$
$$\delta \propto \exp(\gamma t + i \mathbf{k} \cdot \mathbf{r})$$

Dispersion relation (Kunz 2011)

$$-\omega_{\text{visc}} \frac{k_{\perp}^2}{k^2} = \frac{\tilde{\gamma}^2 \left[\tilde{\gamma}^2 (\gamma + \omega_{\text{cond}}) + \gamma g \frac{d \ln \theta}{dz} \frac{k_x^2 + k_y^2}{k^2} + \omega_{\text{cond}} g \frac{d \ln T}{dz} \frac{\mathcal{K}}{k^2} \right]}{\gamma \left[\tilde{\gamma}^2 (\gamma + \omega_{\text{cond}}) + \gamma g \frac{d \ln \theta}{dz} \frac{b_x^2 k_y^2}{k_{\perp}^2} + \omega_{\text{cond}} g \frac{d \ln T}{dz} \frac{b_x^2 k_y^2}{k_{\perp}^2} \right]}$$

$$\tilde{\gamma}^2 \equiv \gamma^2 + (\mathbf{k} \cdot \mathbf{v}_{\text{A}})^2$$

$$\mathcal{K} \equiv (1 - 2b_z^2)(k_x^2 + k_y^2) + 2b_x b_z k_x k_z$$

Dispersion relation (Kunz 2011)

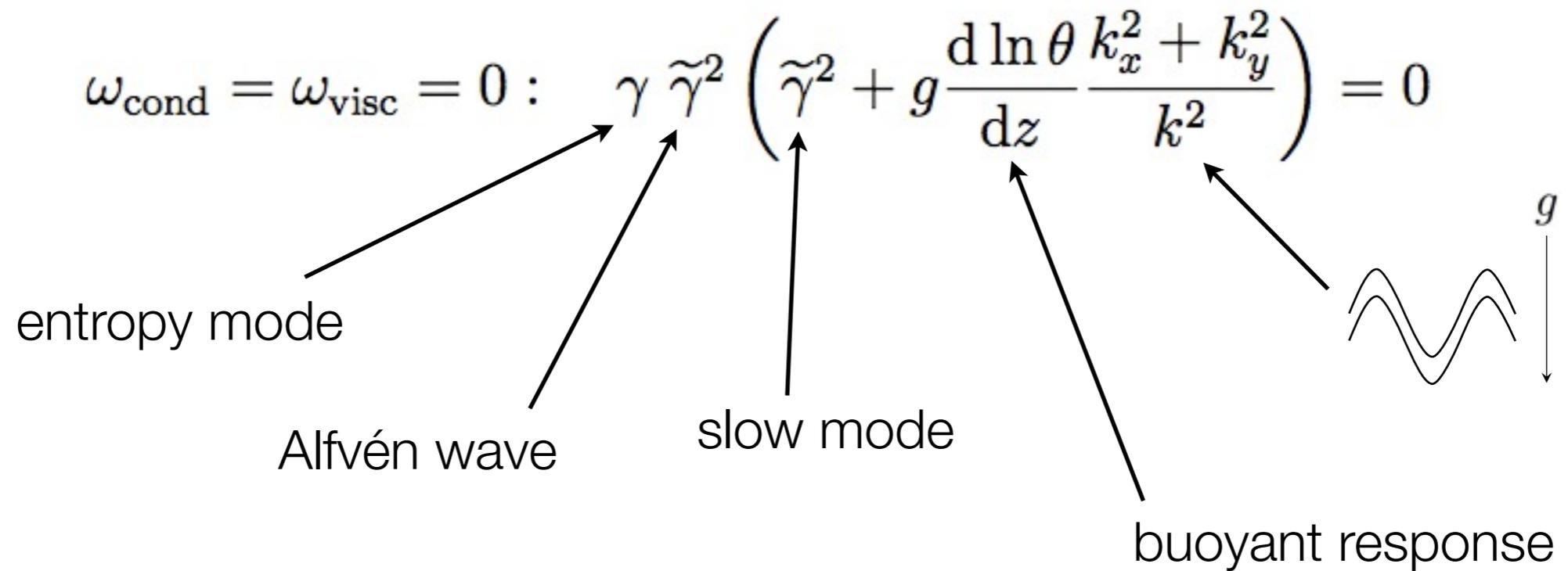
$$\omega_{\text{cond}} = \omega_{\text{visc}} = 0 : \quad \gamma \tilde{\gamma}^2 \left(\tilde{\gamma}^2 + g \frac{d \ln \theta}{dz} \frac{k_x^2 + k_y^2}{k^2} \right) = 0$$

entropy mode

Alfvén wave

slow mode

buoyant response



The diagram illustrates the dispersion relation $\omega_{\text{cond}} = \omega_{\text{visc}} = 0 : \quad \gamma \tilde{\gamma}^2 \left(\tilde{\gamma}^2 + g \frac{d \ln \theta}{dz} \frac{k_x^2 + k_y^2}{k^2} \right) = 0$. Arrows point from labels to specific terms: 'entropy mode' points to γ , 'Alfvén wave' points to $\tilde{\gamma}^2$, 'slow mode' points to the inner $\tilde{\gamma}^2$ in the parentheses, and 'buoyant response' points to the g term. A wavy line with a downward arrow labeled g is also present.

Dispersion relation (Kunz 2011)

$$\omega_{\text{cond}} = \omega_{\text{visc}} = 0 : \quad \gamma \tilde{\gamma}^2 \left(\tilde{\gamma}^2 + g \frac{d \ln \theta}{dz} \frac{k_x^2 + k_y^2}{k^2} \right) = 0$$

$$\omega_{\text{cond}} \gg \omega_{\text{dyn}} \gg \omega_{\text{visc}} : \quad \tilde{\gamma}^2 \left(\tilde{\gamma}^2 + g \frac{d \ln T}{dz} \frac{\mathcal{K}}{k^2} \right) \approx 0 \quad \gamma \approx -\omega_{\text{cond}} \quad \text{entropy mode}$$

Alfvén wave \nearrow
 slow mode \nearrow
 altered buoyant response \nearrow

$\frac{d \ln T}{dz} < 0$ MTI
 $\frac{d \ln T}{dz} > 0$ HBI

Dispersion relation (Kunz 2011)

$$\omega_{\text{cond}} = \omega_{\text{visc}} = 0 : \quad \gamma \tilde{\gamma}^2 \left(\tilde{\gamma}^2 + g \frac{d \ln \theta}{dz} \frac{k_x^2 + k_y^2}{k^2} \right) = 0$$

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$$\omega_{\text{cond}} \gg \omega_{\text{dyn}} \sim \omega_{\text{visc}} : \quad \tilde{\gamma}^2 \left(\tilde{\gamma}^2 + \gamma \omega_{\text{visc}} \frac{k_{\perp}^2}{k^2} + g \frac{d \ln T}{dz} \frac{\mathcal{K}}{k^2} \right) \approx -\gamma \omega_{\text{visc}} g \frac{d \ln T}{dz} \frac{b_x^2 k_y^2}{k^2}$$

Alfvén wave

slow mode

anisotropic

viscous damping

altered

buoyant response

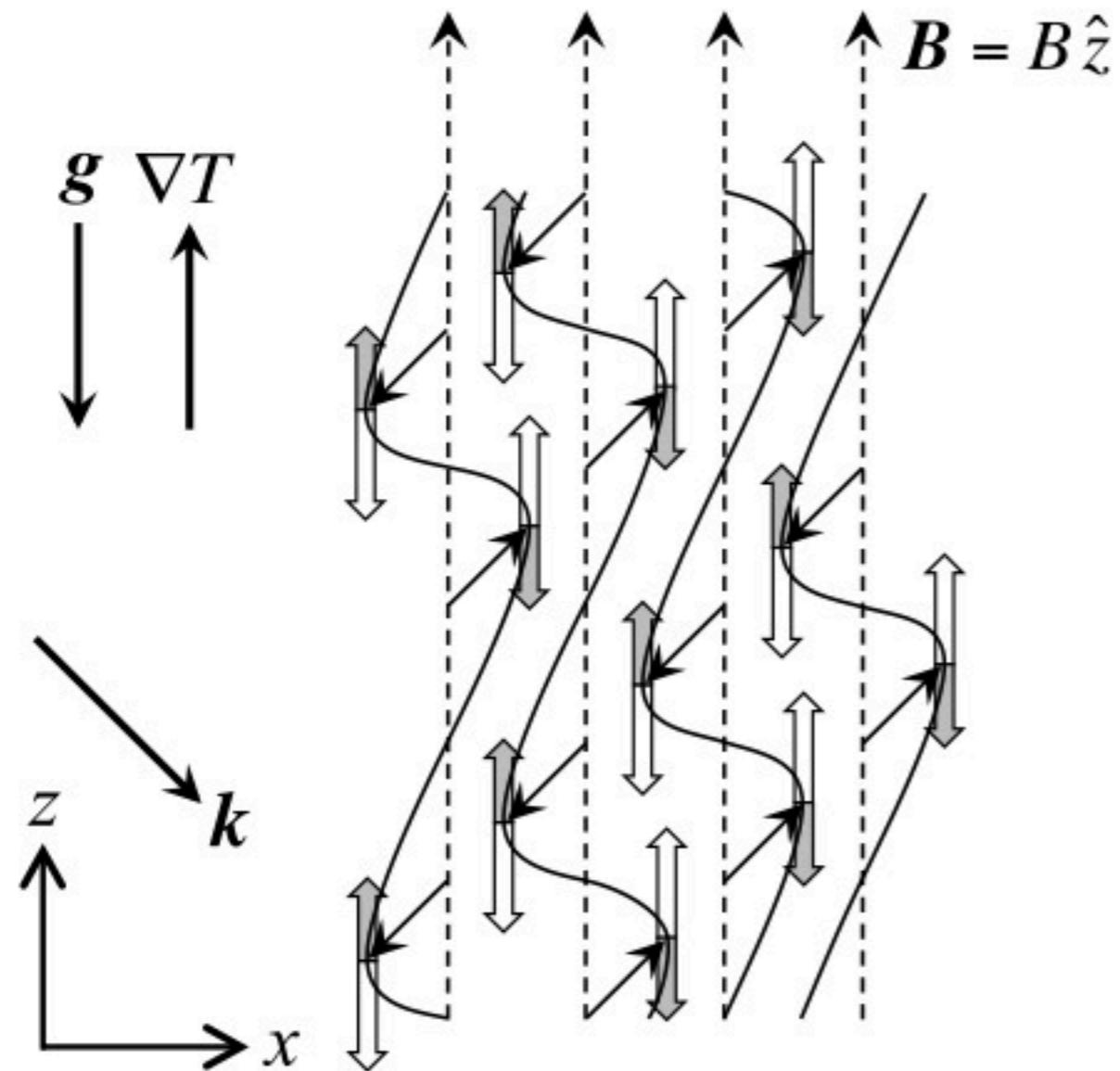
viscous

coupling of
slow & Alfvén
modes

HBI

with Braginskii viscosity

↑ buoyancy force
↑ Braginskii force



$$\Delta T \simeq 2\xi_{||} \nabla_{||} T$$

field-line compressions cause changes in temperature

but motions that cause field-line compressions are viscously damped

Dispersion relation (Kunz 2011)

$$\omega_{\text{cond}} = \omega_{\text{visc}} = 0 : \quad \gamma \tilde{\gamma}^2 \left(\tilde{\gamma}^2 + g \frac{d \ln \theta}{dz} \frac{k_x^2 + k_y^2}{k^2} \right) = 0$$

$$\omega_{\text{cond}} \gg \omega_{\text{dyn}} \gg \omega_{\text{visc}} : \quad \tilde{\gamma}^2 \left(\tilde{\gamma}^2 + g \frac{d \ln T}{dz} \frac{\mathcal{K}}{k^2} \right) \approx 0 \quad \gamma \approx -\omega_{\text{cond}}$$

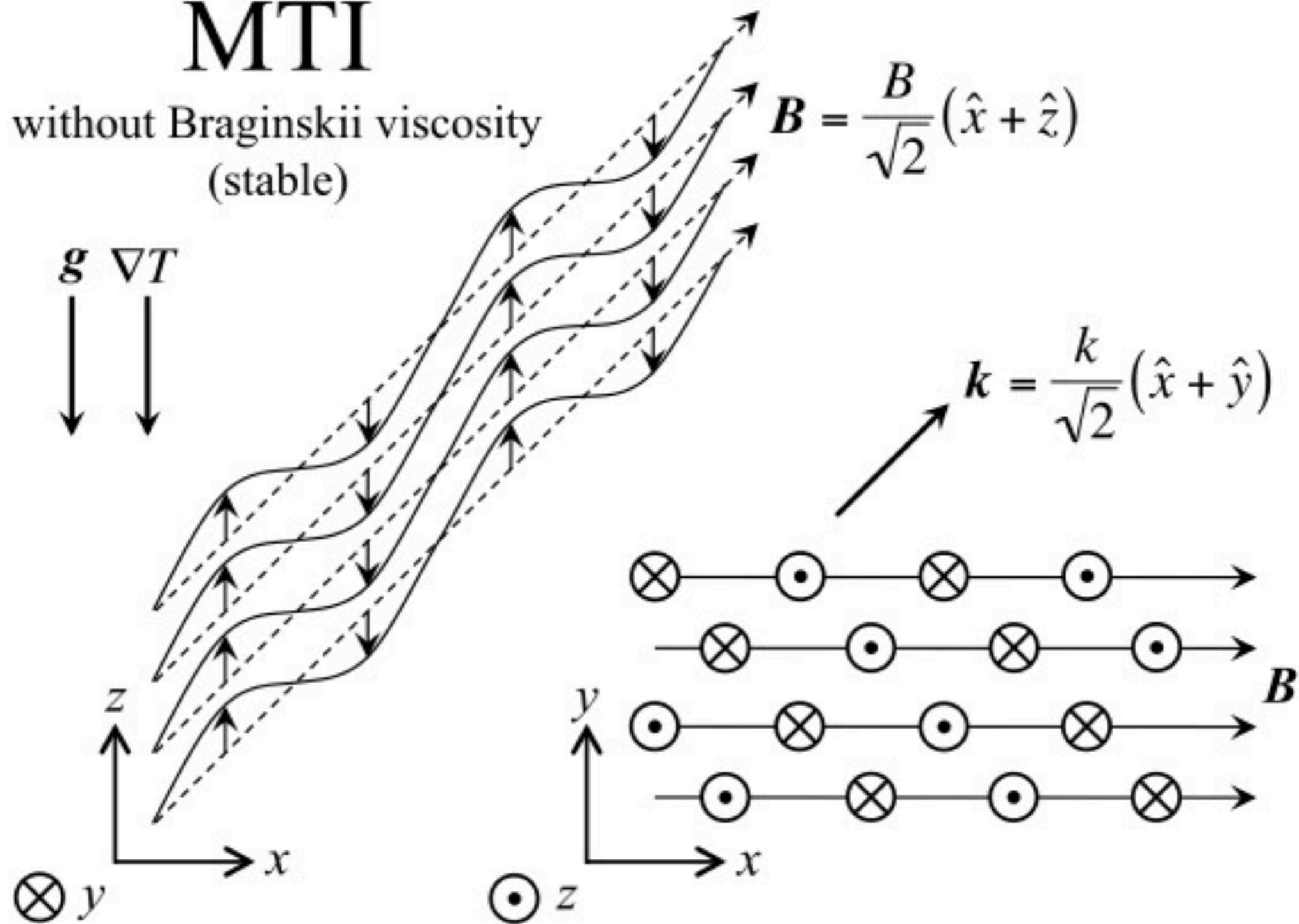
$$\omega_{\text{cond}} \gg \omega_{\text{dyn}} \sim \omega_{\text{visc}} : \quad \tilde{\gamma}^2 \left(\tilde{\gamma}^2 + \gamma \omega_{\text{visc}} \frac{k_{\perp}^2}{k^2} + g \frac{d \ln T}{dz} \frac{\mathcal{K}}{k^2} \right) \approx -\gamma \omega_{\text{visc}} g \frac{d \ln T}{dz} \frac{b_x^2 k_y^2}{k^2}$$

$$\omega_{\text{cond}} \gg \omega_{\text{visc}} \gg \omega_{\text{dyn}} : \quad \tilde{\gamma}^2 \approx -g \frac{d \ln T}{dz} \frac{b_x^2 k_y^2}{k_{\perp}^2}$$

(Alfvénic MTI)

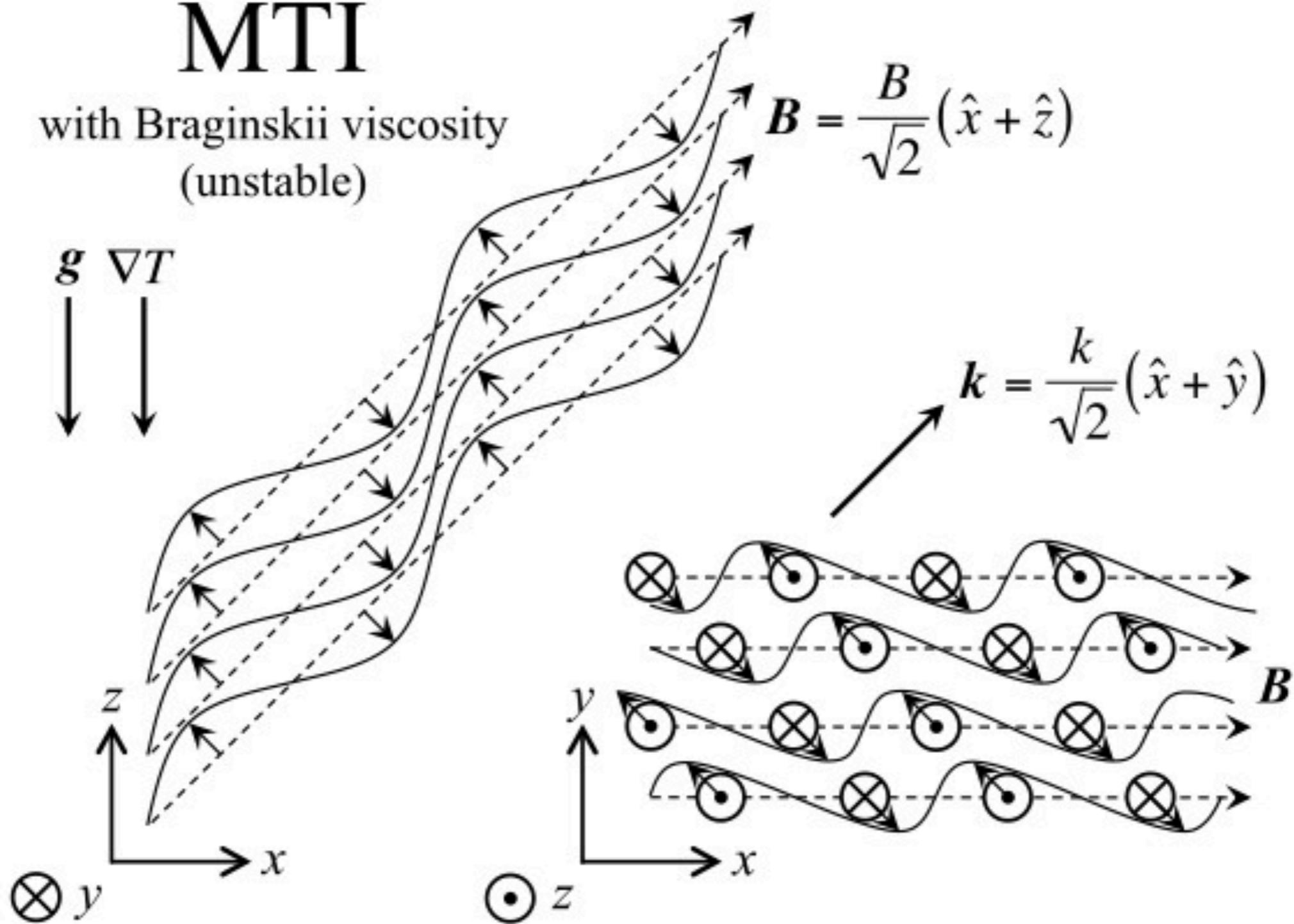
MTI

without Braginskii viscosity
(stable)



MTI

with Braginskii viscosity
(unstable)



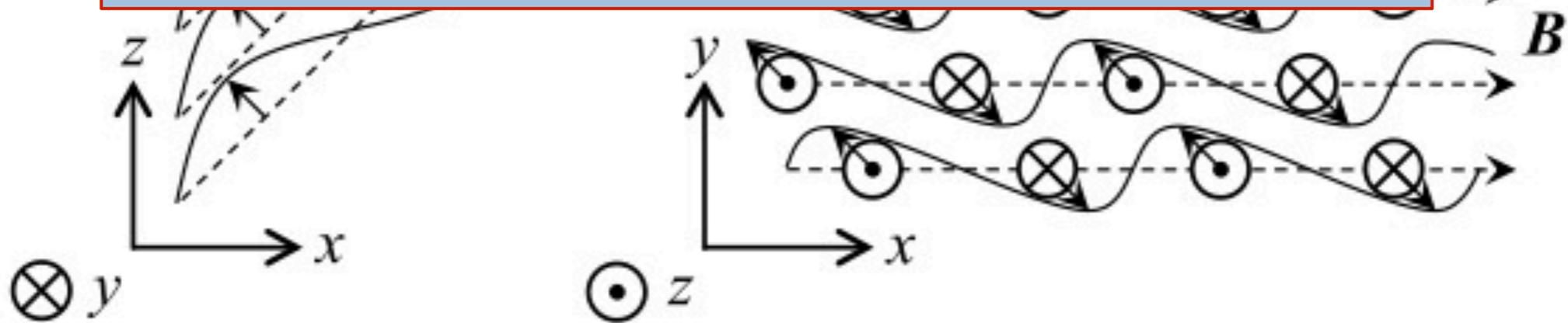
MTI

with Braginskii viscosity
(unstable)

$$\mathbf{B} = \frac{B}{\sqrt{2}}(\hat{x} + \hat{z})$$

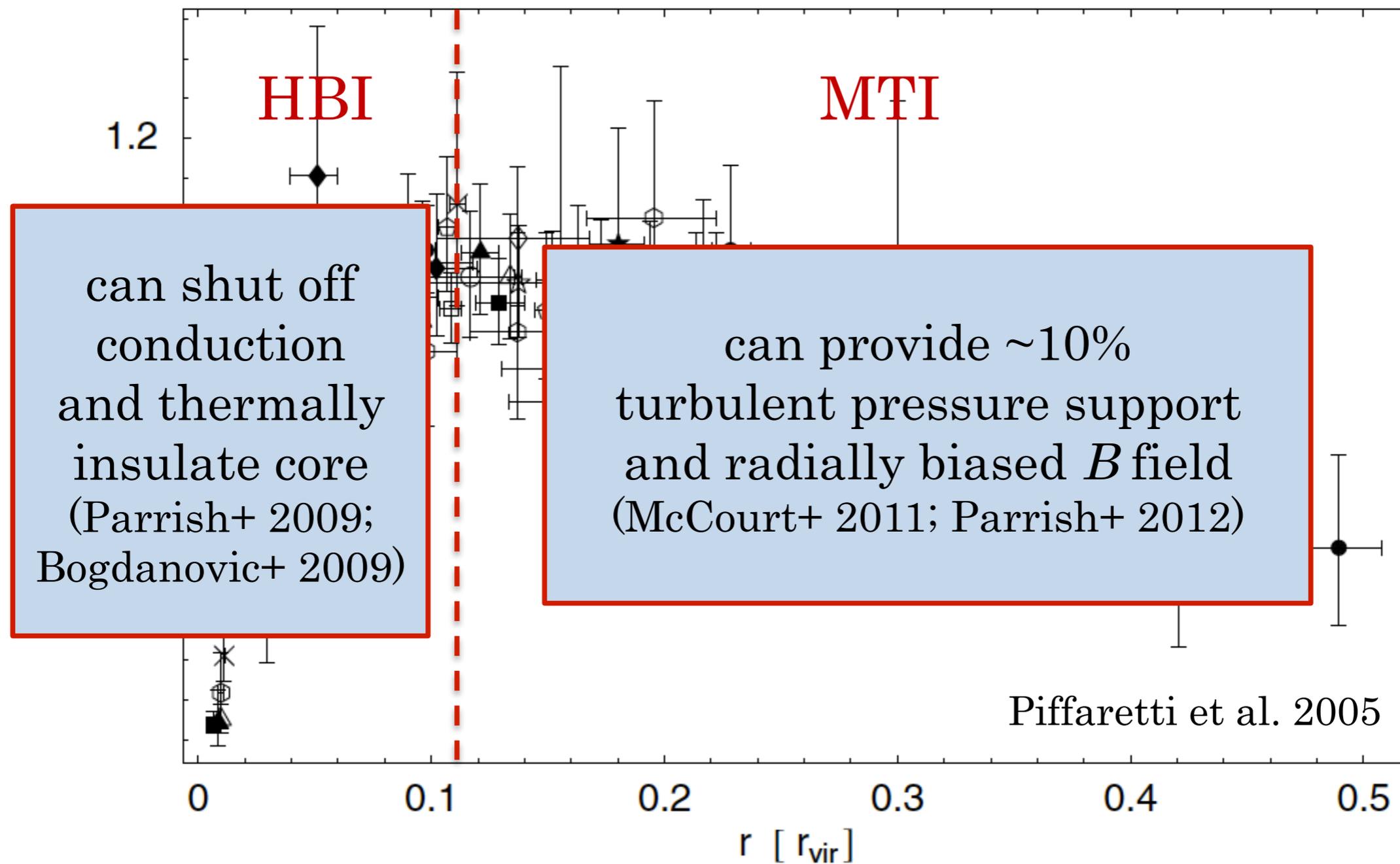
$g \nabla T$

rapid Braginskii viscous damping allows slow-mode perturbations ($\delta n, \delta T \neq 0$) to masquerade as Alfvénic fluctuations (with $\delta \mathbf{B}, \delta \mathbf{v}$ predominantly oriented \perp to $\hat{\mathbf{b}}$)



$$\frac{dT}{dr} > 0$$

$$\frac{dT}{dr} < 0$$

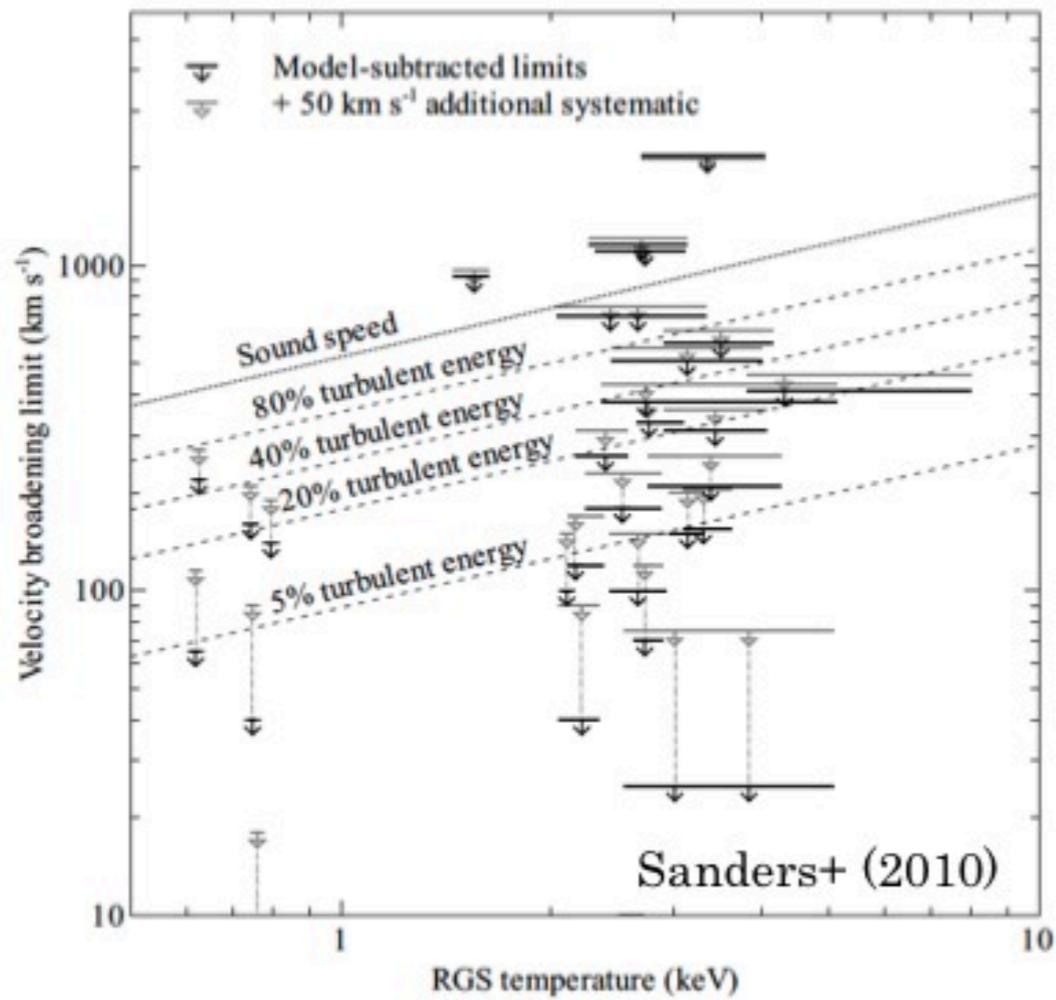


This is all linear theory,
which means a lot...

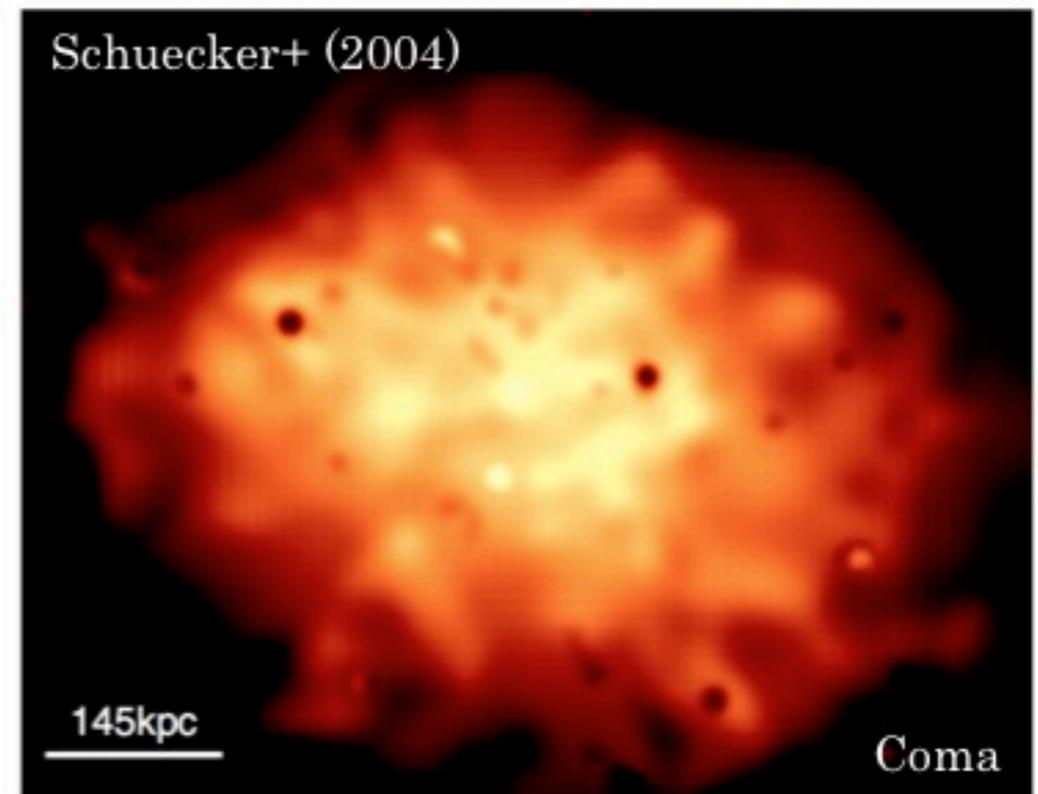
...but only for ~ 100 Myr.

ICM is turbulent

15 clusters with $\mathcal{M}_{\text{turb}} \lesssim 0.2$



$l_{\text{turb}} \sim 10 - 100 \text{ kpc}$



Pressure anisotropy

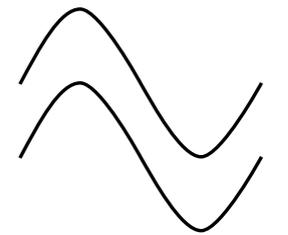
$$\frac{p_{\perp} - p_{\parallel}}{p} \sim \frac{u}{v_{\text{th}}} \frac{\lambda_{\text{mfp}}}{\ell} \sim \text{few} \times 10^{-2}$$

$$\frac{1}{\beta} \sim \text{few} \times 10^{-2}$$

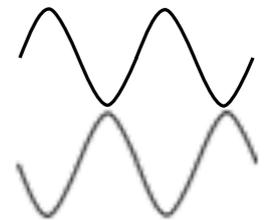
$$\frac{D\mathbf{u}_i}{Dt} = -\frac{1}{m_i n_i} \nabla \cdot \left[\mathbf{I} \left(p_{\perp} + \frac{B^2}{8\pi} \right) - \hat{\mathbf{b}}\hat{\mathbf{b}} \left(p_{\perp} - p_{\parallel} + \frac{B^2}{4\pi} \right) \right] + \mathbf{g}$$

↑
modifies magnetic tension

When $p_{\perp} - p_{\parallel} < -\frac{B^2}{4\pi}$, Alfvén waves are FIREHOSE unstable.
growth time ~ 1 hr, wavelength ~ 10 – 100 npc

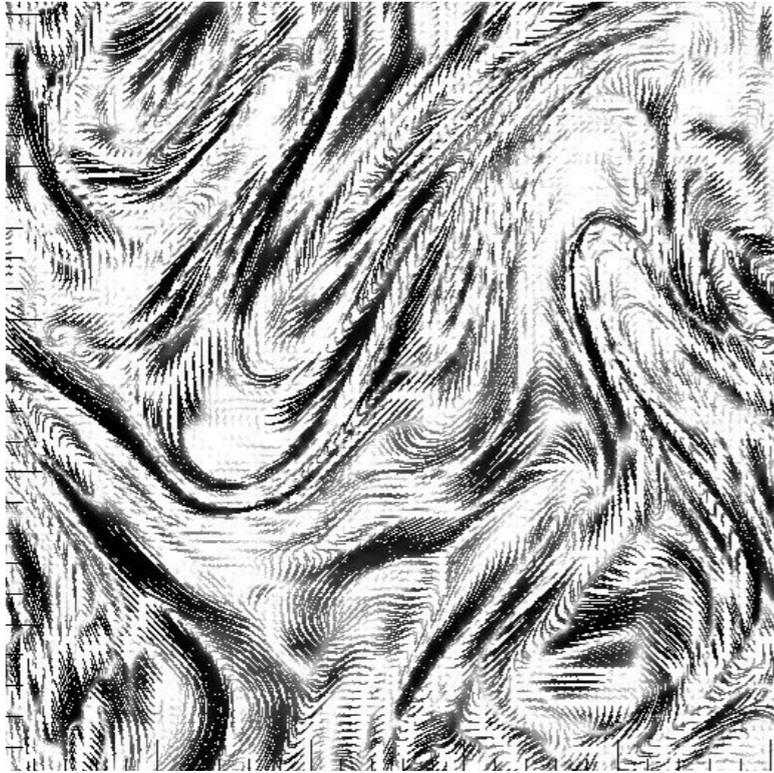


When $p_{\perp} - p_{\parallel} > \frac{B^2}{8\pi}$, slow modes are MIRROR unstable.
growth time ~ 100 hr, wavelength $\sim 10^2$ – 10^3 npc



**TRY CHANGING FIELD STRENGTH
AND EVERYTHING EXPLODES**

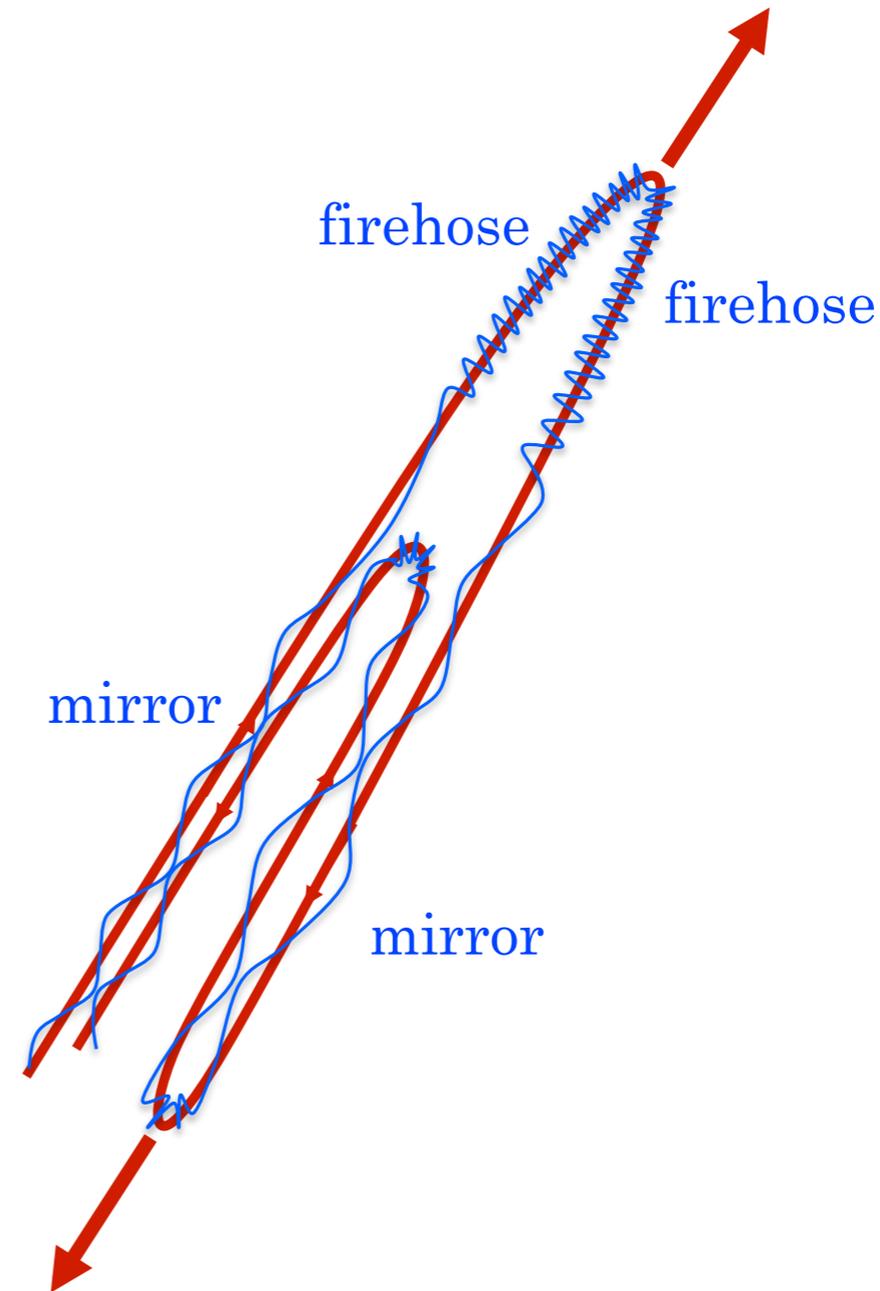
Where do they occur?



Typical structure of magnetic fields
generated by turbulence from
MHD simulations with
(isotropic) $P_m \gg 1$
Schekochihin+ (2004)

$$l_{\perp} \ll l_{\parallel} \sim l_{\text{visc}}$$

if weakly collisional with $P_m \gg 1$:

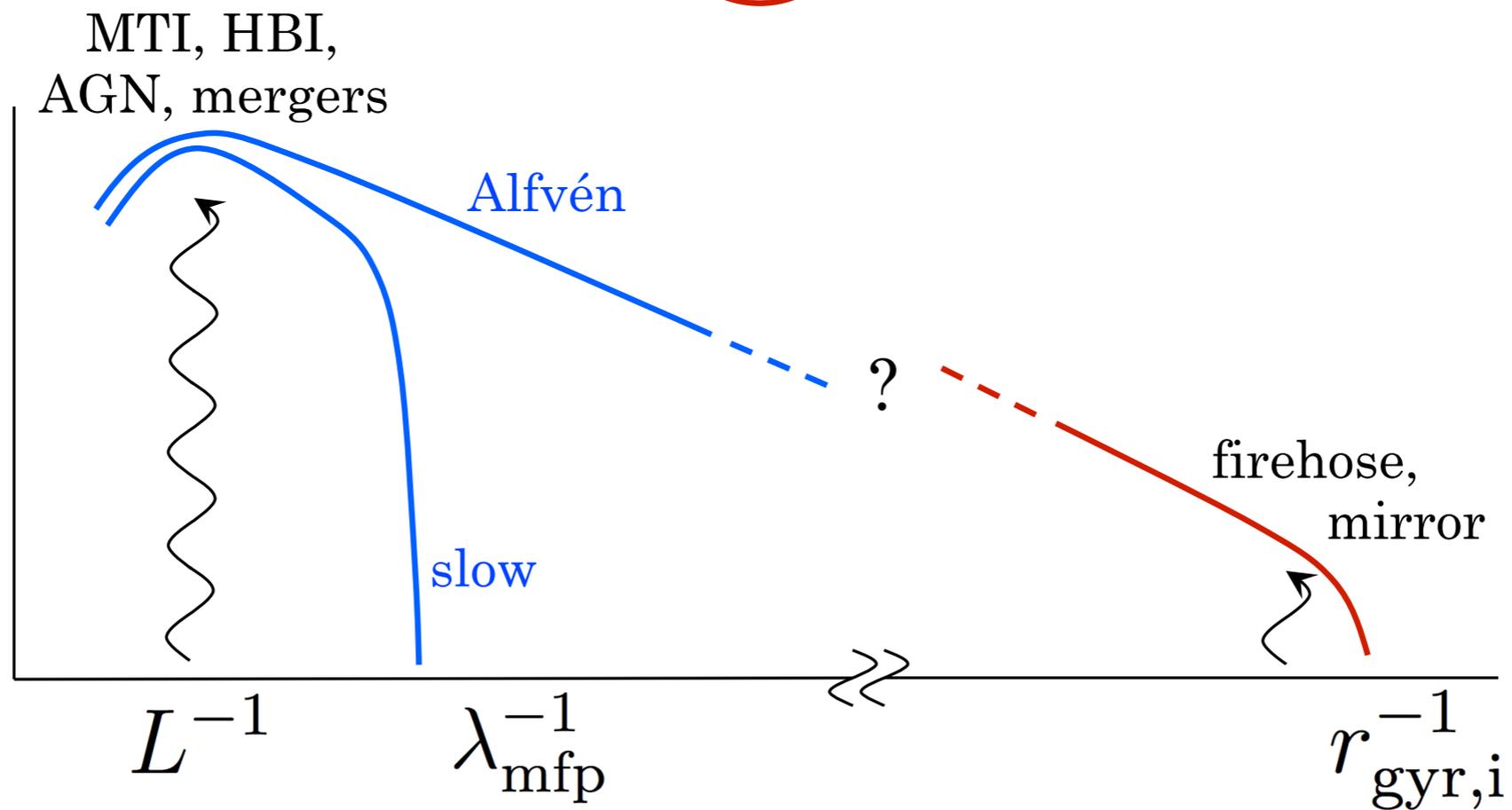


Do microinstabilities occur in ICM?

$$\sim 10^2 - 10^4 (?) \rightarrow \beta \gtrsim \frac{l}{\lambda_{\text{mfp}}} \frac{v_{\text{th}}}{u} ?$$

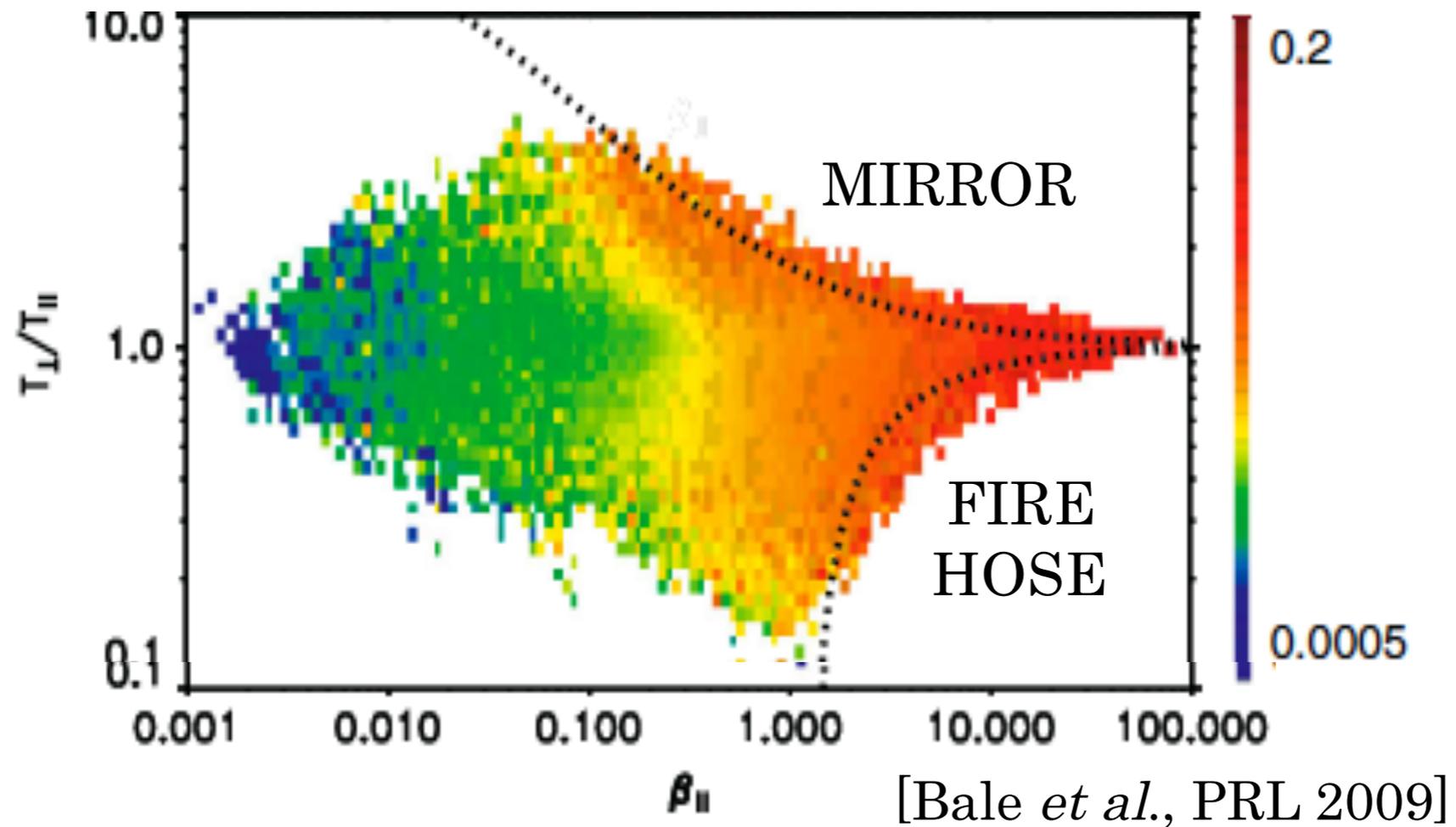
~2-5 (?)

~10-100 (?)



What do these microinstabilities do?

Solar Wind:



perfect opportunity for synergy between space and astrophysics

How do they go marginal?

Option #1:

$$\frac{d \ln B_0}{dt} \quad \text{“+”} \quad \begin{array}{l} \text{firehose} \\ \text{mirror} \end{array} \quad \begin{array}{l} \frac{1}{2} \frac{d}{dt} \frac{\overline{|\delta \mathbf{B}_\perp|^2}}{B_0^2} \\ - \frac{\partial}{\partial t} \left(\frac{\overline{|\delta B_{||}|}}{B_0} \right)^{3/2} \end{array} \quad \Rightarrow \quad \left| \frac{p_\perp - p_{||}}{p} \right| \rightarrow \sim \frac{1}{\beta}$$

How do they go marginal?

Option #1:

rate-of-strain is limited
(in a sense, **more** viscosity)

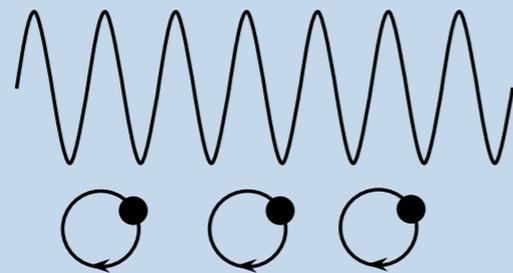
How do they go marginal?

Option #1:

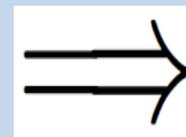
rate-of-strain is limited
(in a sense, **more** viscosity)

Option #2:

$$\frac{d \ln B_0}{dt}$$



break μ



$$\left| \frac{p_{\perp} - p_{\parallel}}{p} \right| \rightarrow \sim \frac{1}{\beta}$$

How do they go marginal?

Option #1:

rate-of-strain is limited
(in a sense, **more** viscosity)

Option #2:

effective collisionality is enhanced
(in a sense, **less** viscosity)

Viscous heating

(done on board — see Kunz et al 2011)

