BASIC PLASMA PHYSICS:
THE COLLISIONLESS LIMIT
AND THE FLUID/ KINETIC DILEMMA

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Plasmas are ubiquitous in astrophysics

More than 99 % of the matter in the known universe exists as plasma. Examples include stars (interior/ coronas), nebulae, interstellar particles, interplanetary medium...

Distant astrophysics allows only to investigate indirect consequences of the plasma phenomena (radiations propagating outside), but space environment, with in-situ measurements, provides the best natural laboratory for Plasma Physics

Neutral gas vs plasma

A plasma is a gas (or a liquid) of charged particles.
I will focus here on gases (excludes the star and planet interiors)
⚠ Our common intuition about gases comes from our terrestrial atmosphere, i.e. from the neutral gases that exist in a thin layer (~100 km) around our planet.

In atmospheres, the thermal particle energy is smaller than the ionization limit: the "normal" state is the neutral state.
To obtain plasmas, external energy must be supplied to ionize the gas, at least partially (cf. ionosphere or lab experiments).

In other places, the reversed condition is valid: the "normal" state is the (fully ionized) plasma state (cf. solar wind).
No need of any additional ionization process in this case.

Collisionless plasmas are ubiquitous too
(but less)

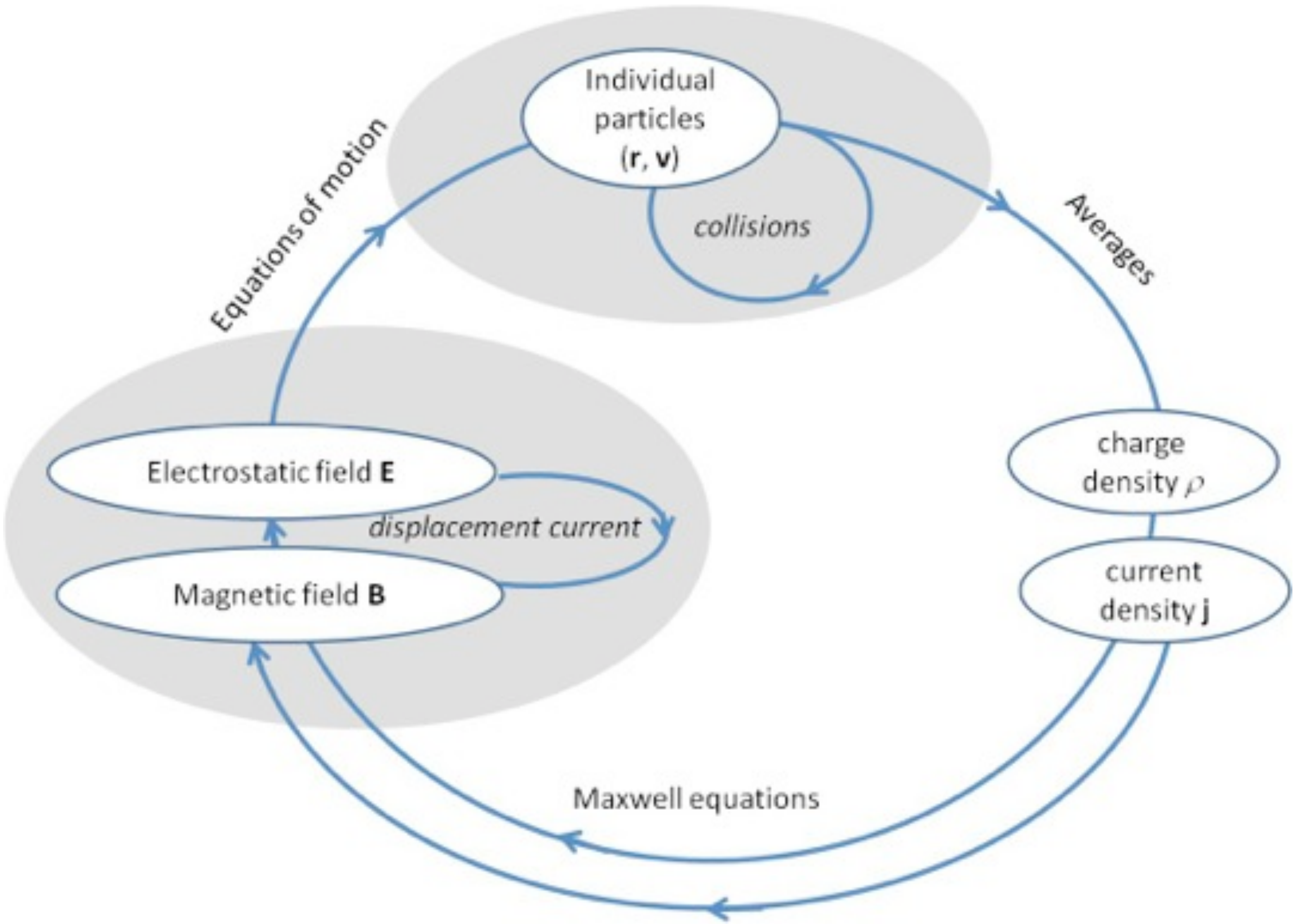
In space plasmas:
Atmospheres are neutral and ionospheres are partially ionized, but full ionization beyond

Solar wind, planetary magnetospheres, etc. are fully ionized and they are mostly "collisionless"
mean free path = a fraction of AU in Solar Wind.

Collisionless conditions are quite frequent also in astrophysics, in all "hot plasmas", whenever they are sufficiently dilute.

Cf. for instance, collisionless shocks for supernovae, etc.

The plasma loop



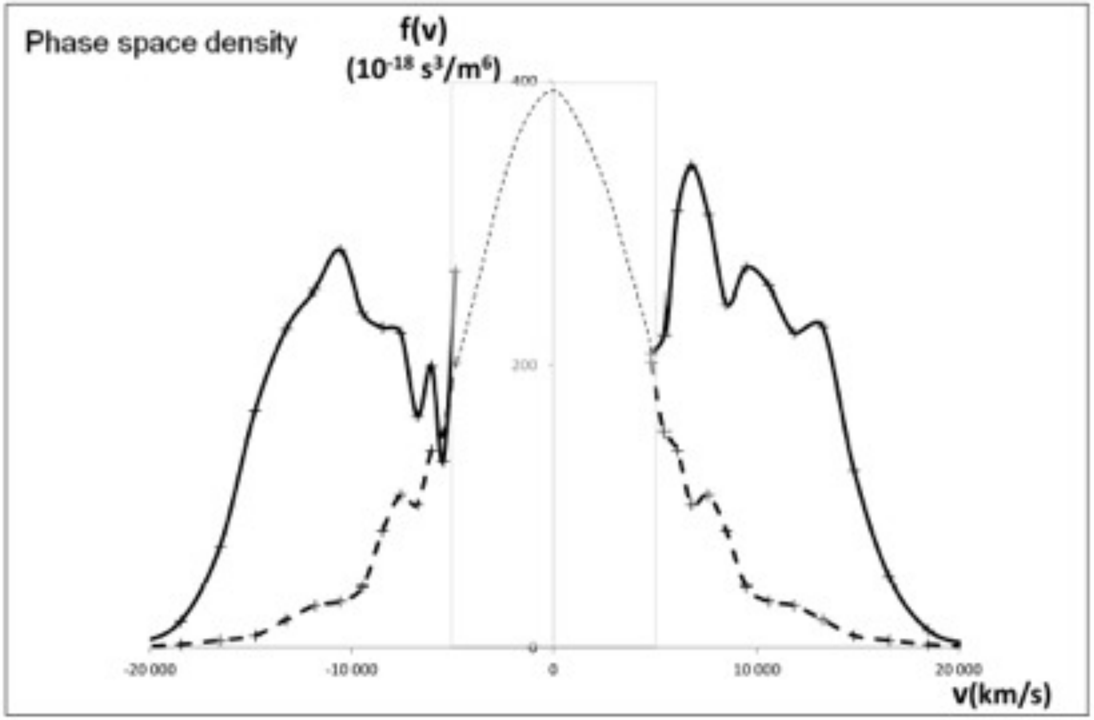
Collisionality: consequences for modeling?

In collisional media:
fluid theories (ρ, $\mathbf{u}$, P) are robust and well established
f($\mathbf{v}$) = Maxwellian
corresponding to the notion of local thermodynamical equilibrium
all the thermodynamical arguments relevant (e.g. entropy).

In collisionless media:
Fluid models like MHD have no universal validity in this case.
Kinetic description of f($\mathbf{v}$) a priori preferable but generally not possible.
Fortunately, fluid models can still be used in many circumstances, with some precautions (cf. closure equation).

Non collisionality in space physics:
a Cluster example

Electron distribution

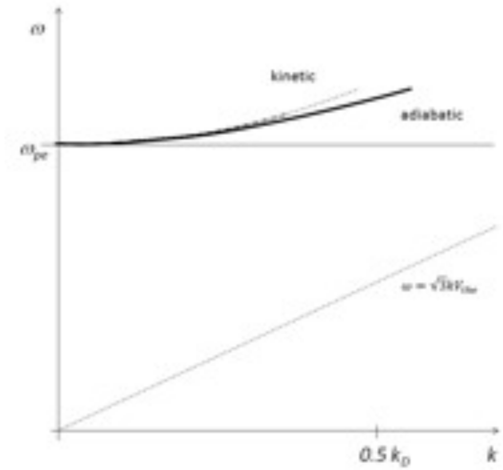


Non isotropic (but gyrotropic)
Non Maxwellian

The form of f(v) is not always important

two examples

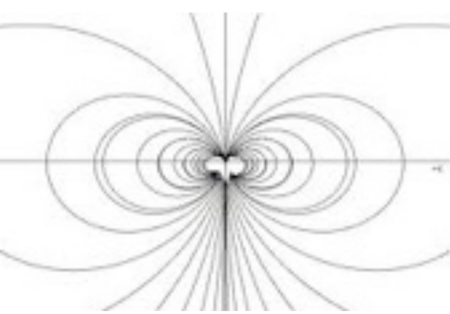
- In these two examples, the fluid result is identical -or close- to the kinetic one



- Langmuir wave

Independent of f(v)
everywhere $v_\phi >> V_{the}$

- Density and temperature distribution in a quasi-stationary flux tube



$$\partial_z \left[T_{\|}\right] = 0$$

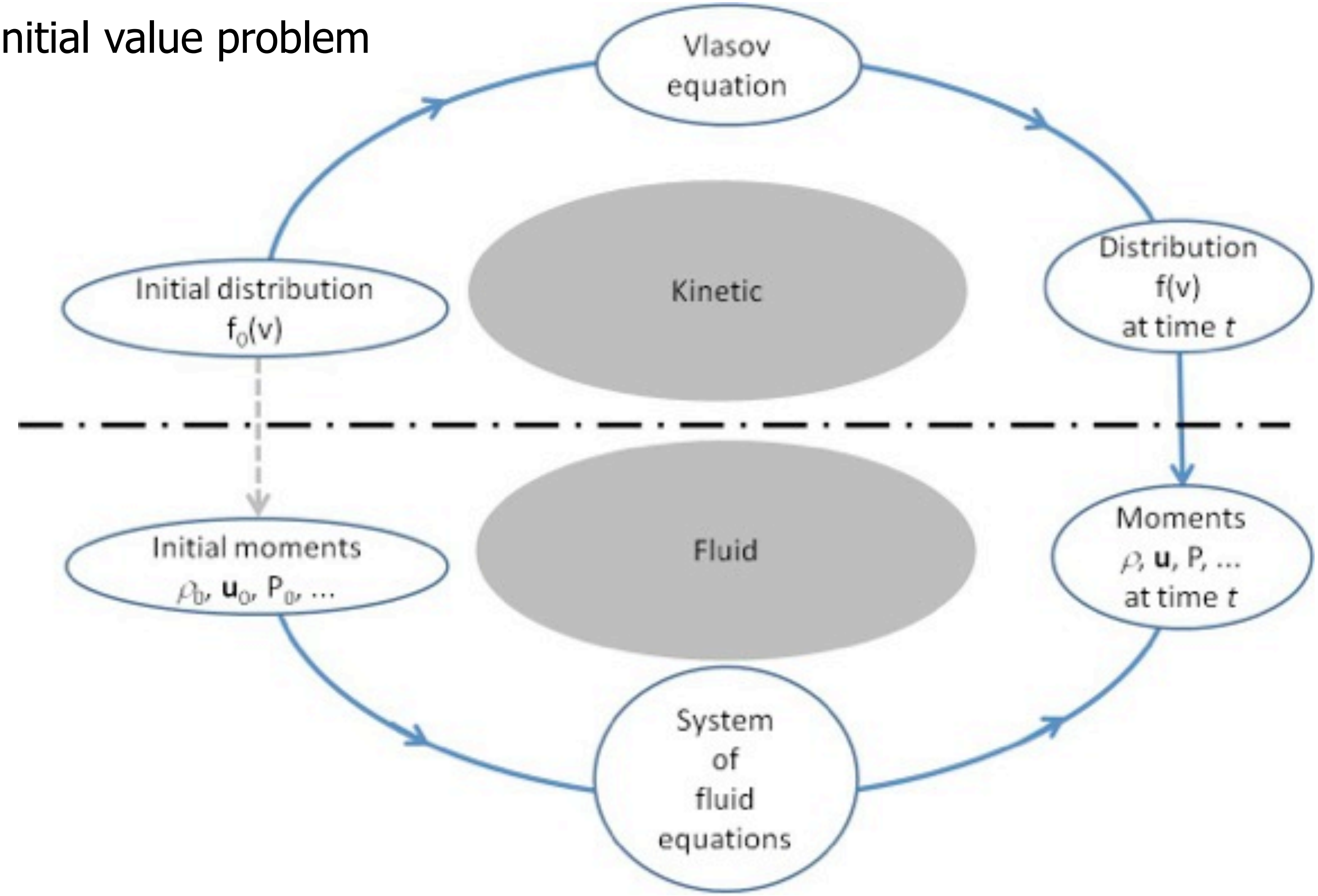
$$\partial_z \left[\left(\frac{1}{T_\perp} - \frac{1}{T_{\|}}\right) B\right] = 0$$

$$\partial_z \left[\frac{T_\perp}{\rho}\right] = 0$$

Exact relations between
the macroscopic parameters,
whatever f(v)
if $\omega / k_{//} << V_{the}$

Fluid/ kinetic

For an initial value problem



Outline1: Kinetic and fluid models: basics

- Notion of collision in a plasma

Collective and collision electric field

The effect of the collision field on a particle

Mean free path and other collisional effects

- Kinetic and fluid descriptions: variables

Macroscopic variables and distribution function

Fluid description is the most affordable

Behind the fluid description: Toy models

- Kinetic and fluid models: equations

Moment equations

Closure equations

Outline2: Solar/stellar wind expansion:

- Necessity of a wind:
  the fluid point of view
  the particle point of view
- Fluid: the Parker model
- Kinetic: the exospheric models

Outline3: Collisionless damping

- A simple example: the Langmuir wave
- Fluid treatment

Closure equation?

- Kinetic treatment

An infinity of modes with exotic distribution functions

Notion of kinetic mode: Landau damping

Outline4: Magnetized plasmas

- MHD

The MHD system

Approximations

Beyond MHD

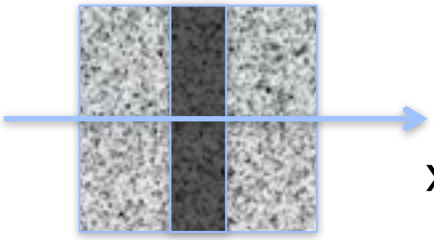
- Ohm's law

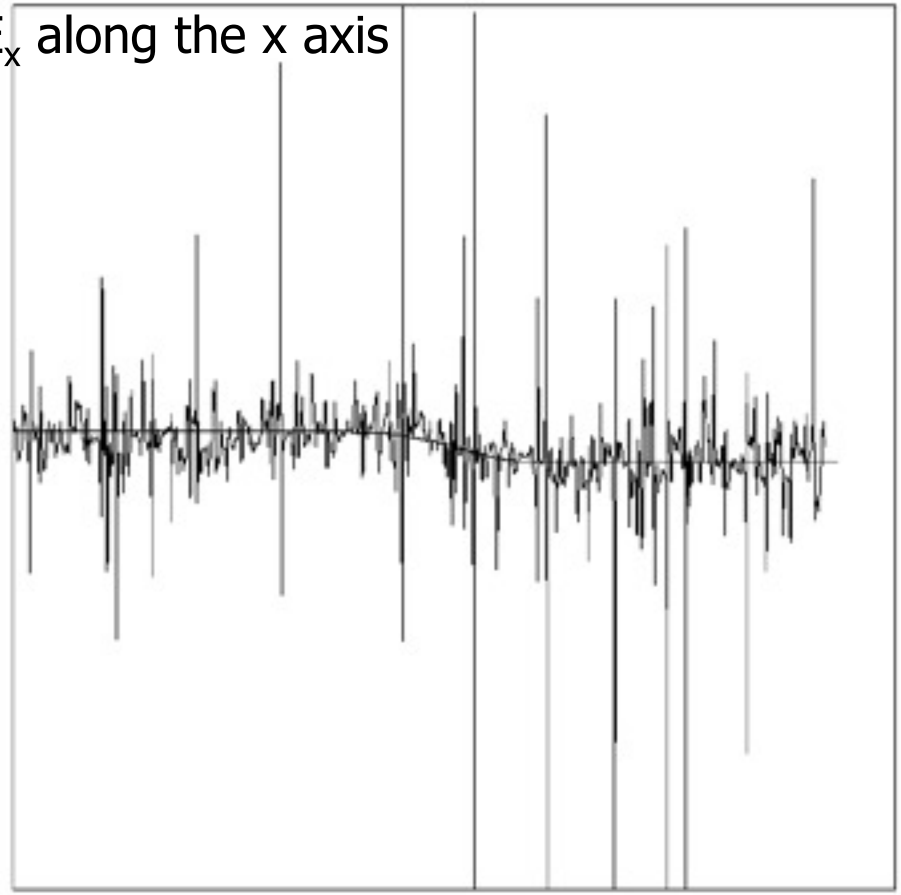
Freezing

Reconnection collisional/ collisionless

Notion of collision in a plasma

1. Collective and collision electric field





Variations at all scales with a decreasing spectrum.

- Strong short peaks due to one single particle.
- Weak large scale variations due to many distant particles.

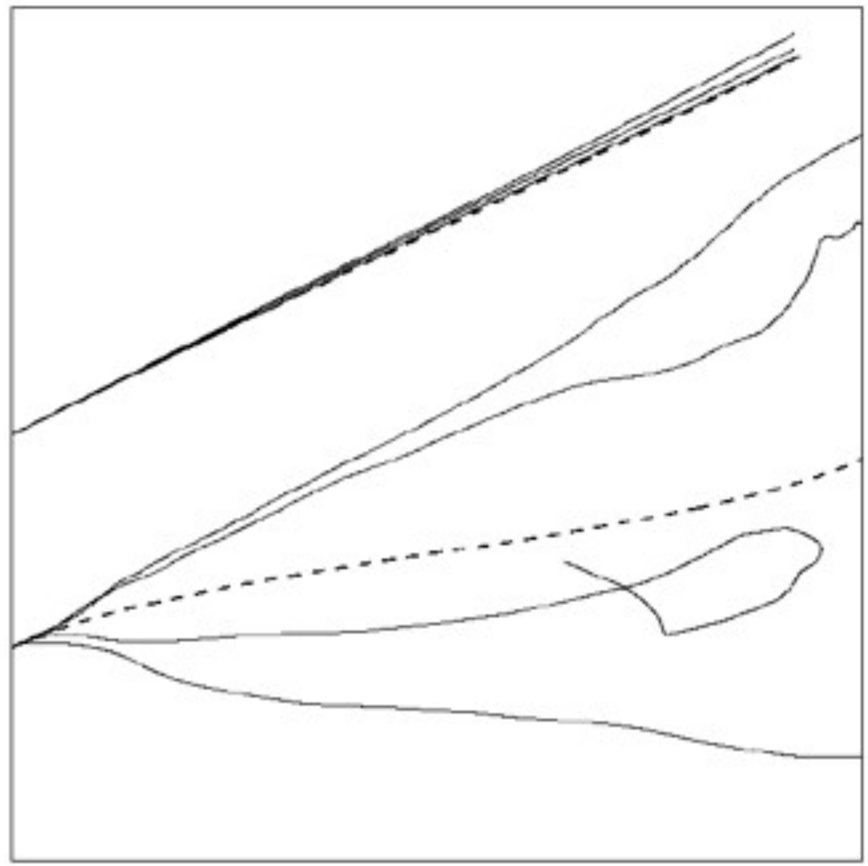
Average value on a large sliding window = collective field
Departure = collision field

Lauching a test particle in this field ➔ deviations
The weak large scale variations are much more efficient than the short peaks

Notion of collision in a plasma

2. The effect of the collision field on a particle

No sharp bend of the trajectory.
The smooth deviation much depends on the test-particle speed.
Only the extremely slow particles would be sensitive to the high electric peaks and give strong "binary" collisions
(diffusive centers are supposed steady)



One never obtains the classical image of hard spheres colliding which prevails in neutral collisions
(because of the long range electrostatic interaction in r^{-2})

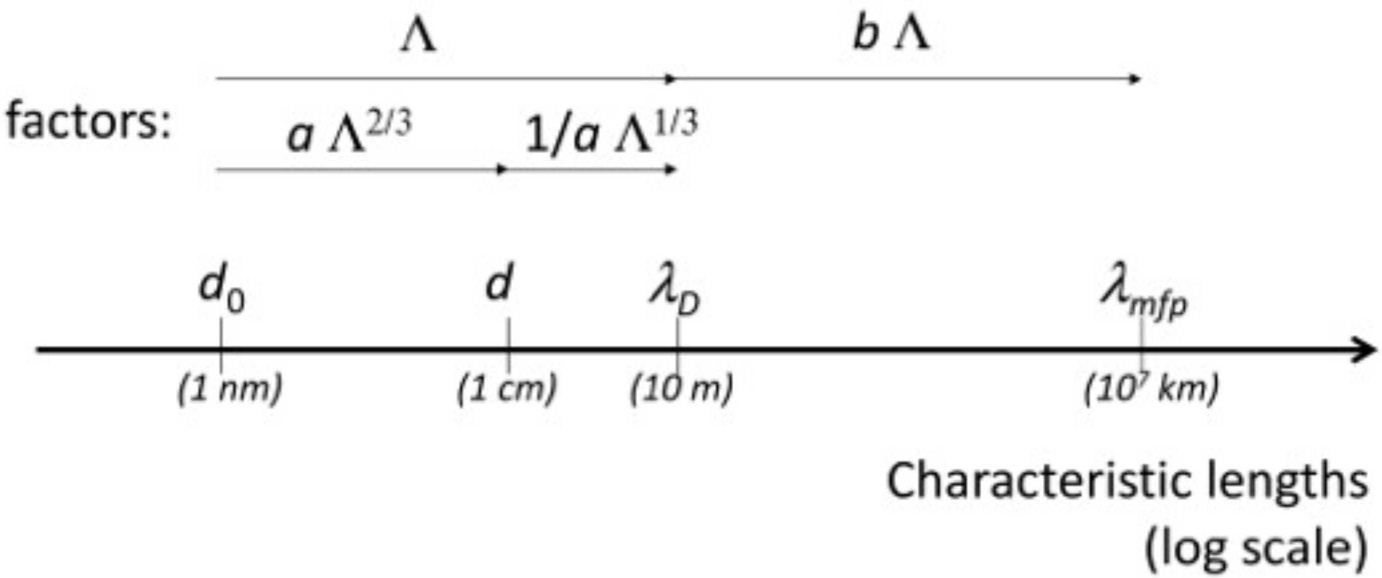
Notion of collision in a plasma

3. Mean free path and other characteristic lengths for collisions

$$\frac{\lambda_{mfp}}{d} = \frac{\pi}{8 \ln \Lambda} \frac{d^2}{d_0^2}$$

Proportional to d_0^{-2} ➔ to v^{-4}

All ratios determined by one single parameter Λ (very large)



d_0 = Landau length = "electrostatic dimension" of the particle = distance for strong collision

$d = n^{-1/3}$ = interparticle distance

$\Lambda = \lambda_D / d_0$ = huge ratio ($\ln \Lambda$ = Coulomb logarithm ≈ 20 in SW)

Notion of collision in a plasma

3. Role of the Debye length

$$\frac{\lambda_{mfp}}{d} = \frac{\pi}{8 \ln \Lambda} \frac{d^2}{d_0^2}$$

with $\Lambda = \lambda_D / d_0$

λ_D = Debye length

Mean free path = length to get $\delta\phi = \pi/2$

Calculation of the mean free path : $\delta\phi^2 = \sum \delta\phi_i^2$

Deviation due to the sum of all the individual deviations, supposed random and steady, integral a priori up to infinite distances.

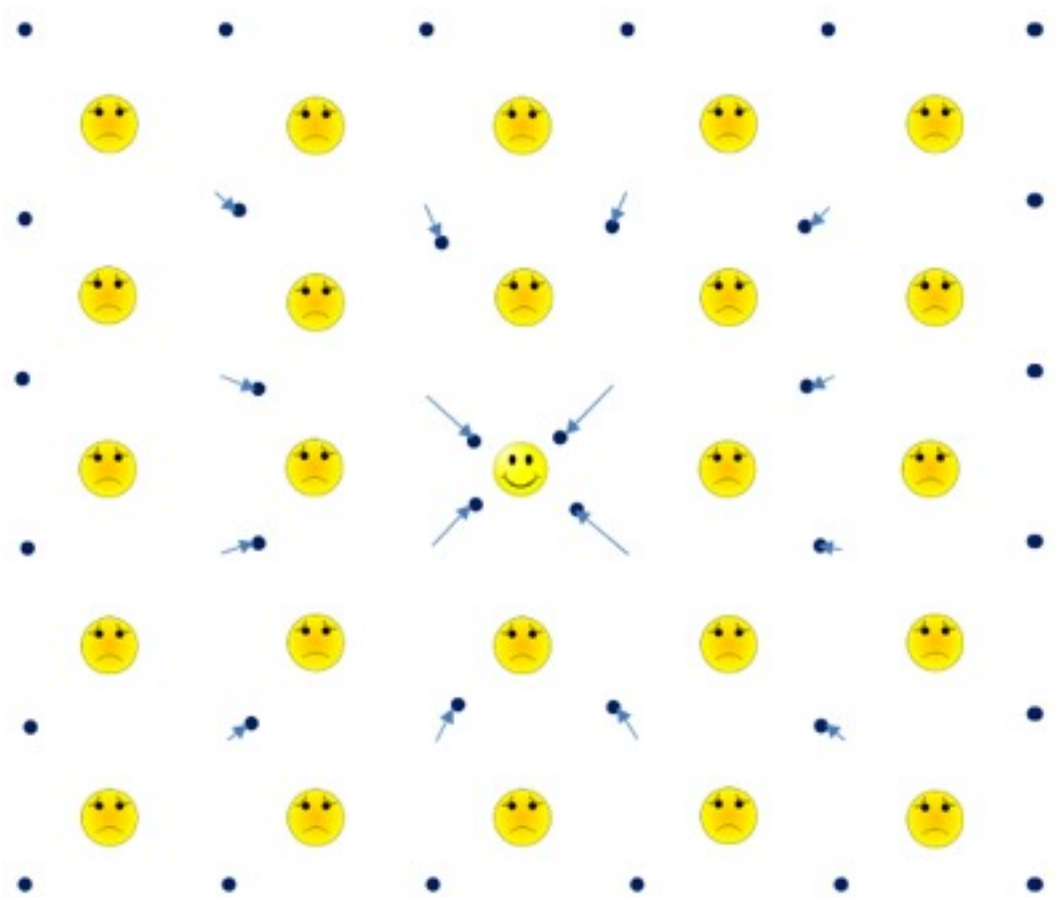
As mentioned, large distance dominant in the sum, but calculation becomes invalid at too large scales because of collective effects.

Reaction to mean charge densities ➔ waves ➔ upper limit for the hypotheses "random and steady"

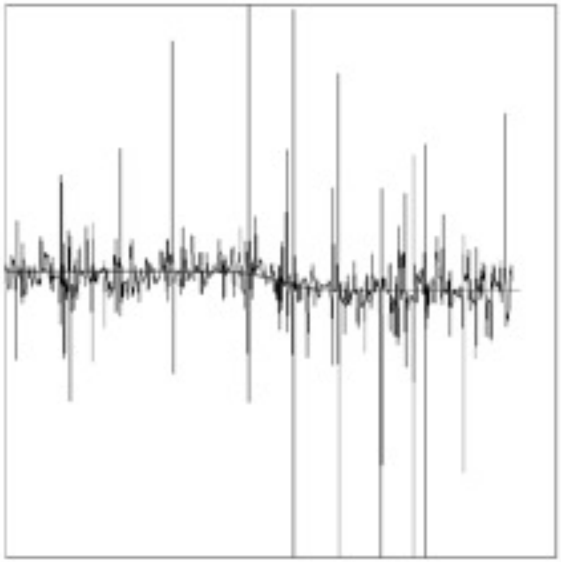
➔ only the particles at $r < \lambda_D$ can actually diffuse the test-particle trajectory

The notion of Debye screening is due to waves. It cannot be viewed as a spatial and stationary effect

If one diffusing ion was "screened" by electrons in a stationary manner, all the other ions could not be screened at the same time by the same electrons...



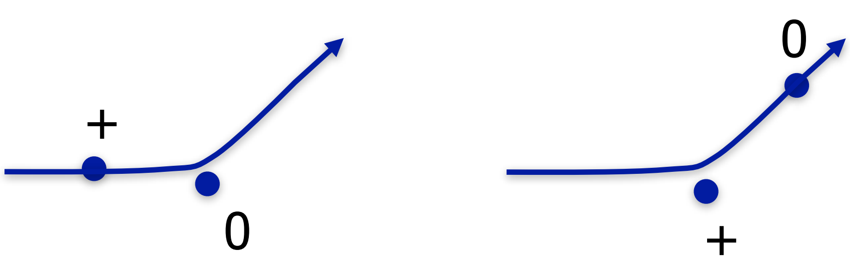
Other collisional effects



Short peaks in the electric field , even strong, are not efficient for deviating a particle.
But other phenomena can be less demanding for the duration of the interaction than the momentum exchange.

It is the case for the charge exchange
(cf. heliosphere, collisionless, but with charge exchange).

The difference between the two phenomena cannot be understood if keeping in mind the classical image of collision with hard speres

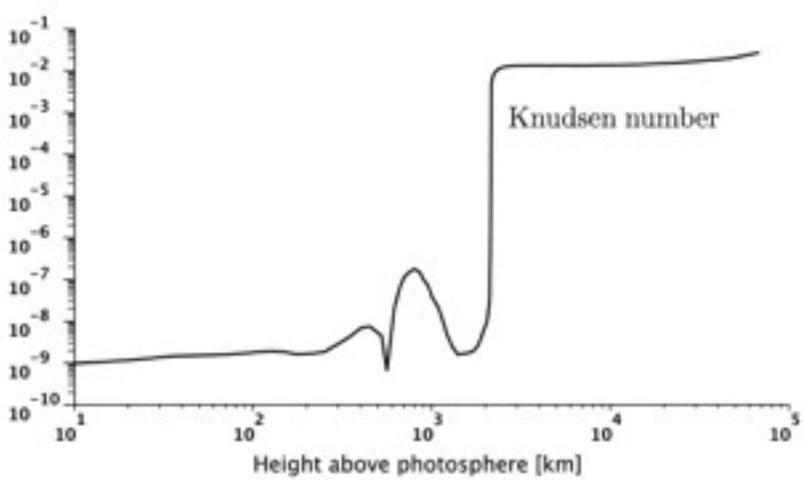


What is called collisionless?

- It is when $K_n = \lambda_{mfp} / L >> 1$, L being the characteristic scale of the phenomena under study.
- Examples in space physics:

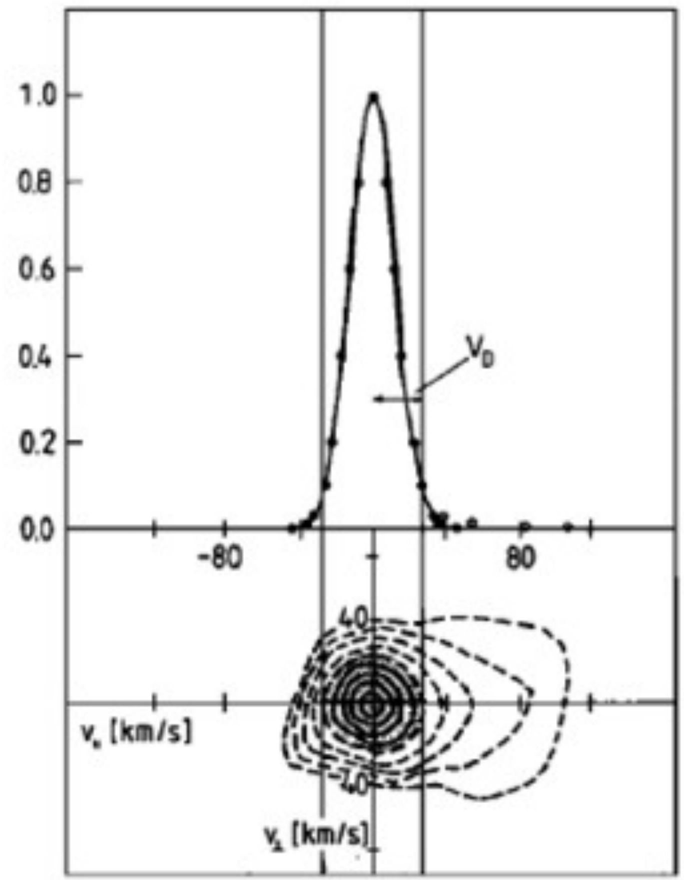
The magnetopause phenomena are unambiguously collisionless since d ≈ 10^3 km while $\lambda_{mfp} \approx 10^7$ km

The variations of temperature at large scale in the solar wind expansion are neither collisionless nor strongly collisional (for e-e collisions):



A measurable effect of collisions in the solar wind

The heart of the proton distribution is isotropized by collisions, but not the higher energies (at 1 AU)



Less than one collision?

- On a distance of λ_{mfp} / 10, one has 0.1 collision
- In a hard sphere view, it means that, in general, a particle has no collision at all on this distance.
- In a plasma, it means that its trajectory has been deviated of $\pi/20$. This is not always negligible.

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Macroscopic variables and distribution function

Fluid description is the most affordable

Behind the fluid description: Toy models

- Kinetic and fluid models: equations

Moment equations

Closure equation

Kinetic and fluid variables

- For each population (at least one for each species, electron and ions), the plasma is described by the fields **E** and **B**, and by:

- **Kinetic**: distribution function f(v)
- **Fluid**: density, fluid velocity, pressure, ρ, **u**, **P**

possibly other macroscopic variables as **Q** (heat flux), etc...

The fluid variables are just the moments of the kinetic ones

$$n = \int f(\mathbf{v}) d^3\mathbf{v}$$ and $\rho = nm$

$$\mathbf{u} = \langle \mathbf{v} \rangle_v = \frac{1}{n} \int \mathbf{v}\, f(\mathbf{v}) d^3\mathbf{v}$$

$$\mathbf{P} = \rho \langle \delta\mathbf{v}\delta\mathbf{v} \rangle_v = m \int \delta\mathbf{v}\delta\mathbf{v}\, f(\mathbf{v}) d^3\mathbf{v}$$

etc.

Except order 1, all moments are centered ($\delta\mathbf{v}$ = **v** - **u**).
The mass density is a scalar, the fluid velocity is a vector, the pressure **P** is a 2d order tensor, **Q** is a third order tensor, etc.

Collisions bring approximate isotropy, which simplifies the tensors

Full thermodynamical equilibrium:
Homogeneous medium or $K_n <<< 1$

$\mathbf{P} = P\,\mathbf{I}$ with $P = nT$ (definition of the temperature here)

$\mathbf{Q} = 0$

$\mathbf{R} = R\,\mathbf{I}$ with $R/m = 3\,(P/m)^2$

Local thermodynamical equilibrium and transport coefficients:
$K_n << 1$

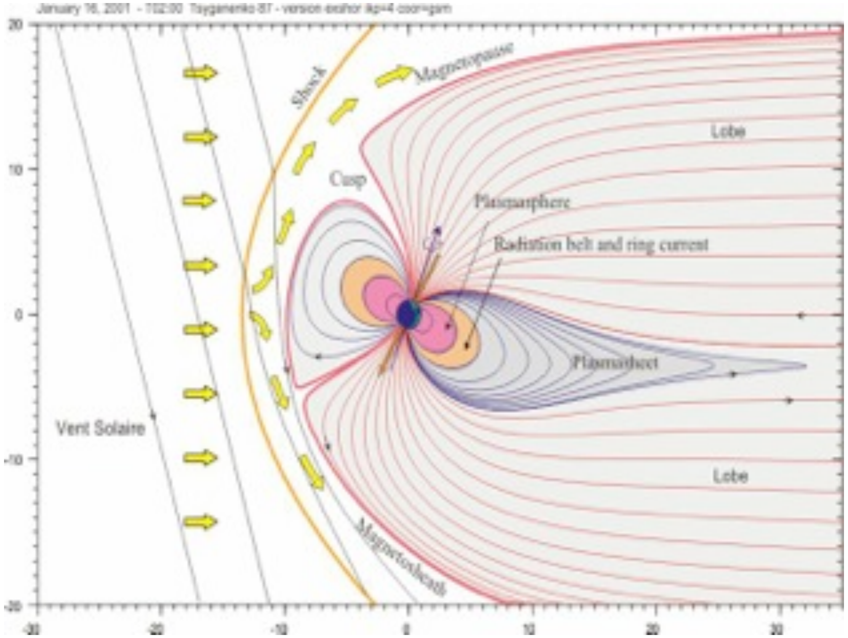
$\mathbf{P} = P\,\mathbf{I} - \sigma$ with $\sigma = f(\nabla \mathbf{u})$

$\mathbf{Q} = \mathbf{q}$ with $\mathbf{q} = f(\nabla T)$

...

In collisionless plasmas, isotropy is never guaranteed, but gyrotropy is generally insured at large scale.

Fluid description is the most affordable



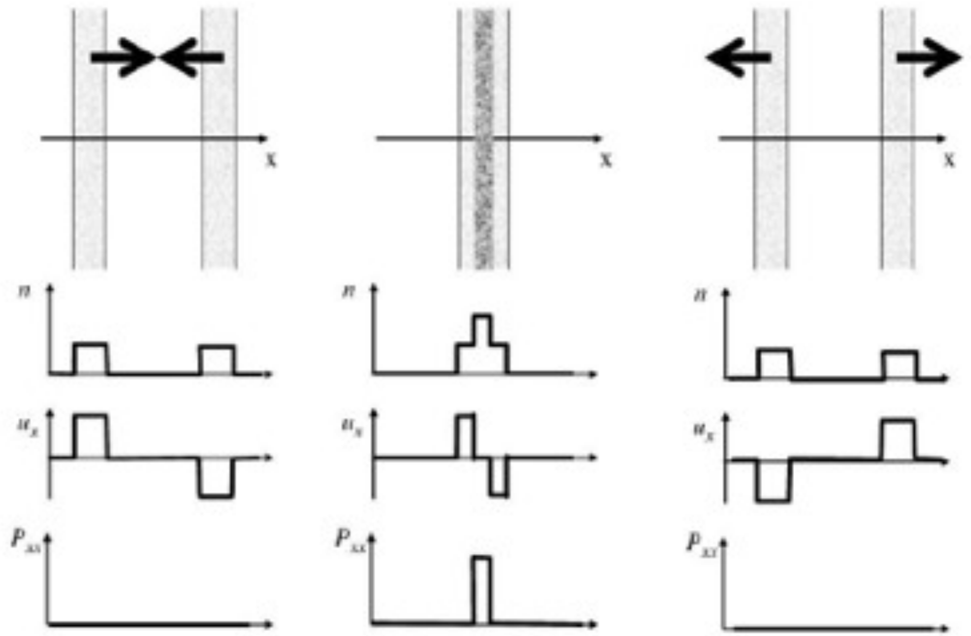
- Describing the solar wind/ magnetosphere interaction:

"The **flow velocity** is unperturbed until it reaches a shock where it is deviated, and where the **pressure** and the **temperature** increase sharply"

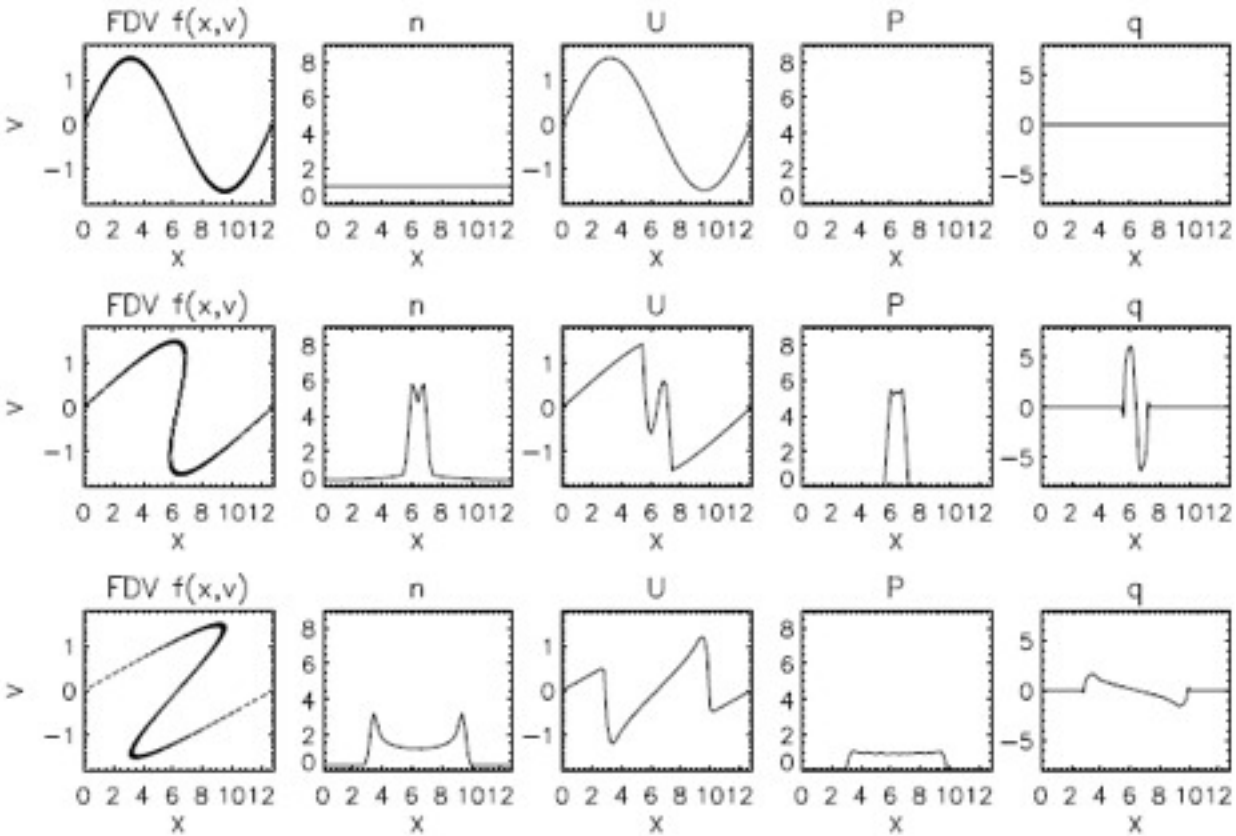
One rarely describes the behavior of the distribution function (neither the individual particle trajectories): only the moments are generally interesting.

Behind the fluid description: Toy models

No collisions,
no interaction



The evolutions of n, u and P respect all the fluid equations, except possibly one (closure equation)



Results of an hybrid simulation code
Same remarks as above

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Closure equations

From kinetic to fluid
(general: with or without collisions)

$$\partial_t f + \mathbf{v}.\nabla f + \mathbf{F}/m.\nabla_v f = C$$

* $\delta \mathbf{v}^p$ and integrated ➔

$$\partial_t \rho + \nabla.(\rho \mathbf{u}) = 0 \quad (2.26)$$

$$\partial_t(\rho \mathbf{u}) + \nabla.(\rho \mathbf{u}\mathbf{u} + \mathbf{P}) = nq(\mathbf{E} + \mathbf{u} \times \mathbf{B}) + \mathcal{C}_1 \quad (2.27)$$

$$\partial_t \mathbf{P} + \nabla.(\mathbf{u}\mathbf{P} + \mathbf{Q}) + \nabla.\left[\mathbf{P}(\mathbf{u}) + (\mathbf{P}(\mathbf{u}))^T\right] = -\{\boldsymbol{\omega}_c \times \mathbf{P}\}^O + \mathcal{C}_2 \quad (2.28)$$

$$\partial_t \mathbf{Q} + \nabla.(\mathbf{u}\mathbf{Q} + \mathbf{R}) + \{D_t \mathbf{u}\, \mathbf{P}\}^O + \nabla.\{\mathbf{Q}(\mathbf{u})\}^O - \nabla.\mathbf{u}\, \mathbf{Q} = \{q/m(\mathbf{E} + \mathbf{u} \times \mathbf{B})\, \mathbf{P} - \boldsymbol{\omega}_c \times \mathbf{Q}\}^O + \mathcal{C}_3 \quad (2.29)$$

...

$$\partial_t \mathcal{M}_p + \nabla.(\mathbf{u}\mathcal{M}_p + \mathcal{M}_{p+1}) + \{D_t \mathbf{u}\, \mathcal{M}_{p-1}\}^O + \nabla.\{\mathcal{M}_p(\mathbf{u})\}^O - \nabla.\mathbf{u}\, \mathcal{M}_p = \{q/m(\mathbf{E} + \mathbf{u} \times \mathbf{B})\, \mathcal{M}_{p-1} - \boldsymbol{\omega}_c \times \mathcal{M}_p\}^O + \mathcal{C}_p \quad (2.30)$$

(if specifying **F** = q [**E** + **v**x**B**])

Moment system

$$\partial_t \rho + \nabla.(\rho \mathbf{u}) = 0 \quad (2.26)$$

$$\partial_t(\rho \mathbf{u}) + \nabla.(\rho \mathbf{u}\mathbf{u} + \mathbf{P}) = nq(\mathbf{E} + \mathbf{u} \times \mathbf{B}) + \mathcal{C}_1 \quad (2.27)$$

$$\partial_t \mathbf{P} + \nabla.(\mathbf{u}\mathbf{P} + \mathbf{Q}) + \nabla.\left[\mathbf{P}(\mathbf{u}) + (\mathbf{P}(\mathbf{u}))^T\right] = -\{\boldsymbol{\omega}_c \times \mathbf{P}\}^O + \mathcal{C}_2 \quad (2.28)$$

$$\partial_t \mathbf{Q} + \nabla.(\mathbf{u}\mathbf{Q} + \mathbf{R}) + \{D_t \mathbf{u}\, \mathbf{P}\}^O + \nabla.\{\mathbf{Q}(\mathbf{u})\}^O - \nabla.\mathbf{u}\, \mathbf{Q} = \{q/m(\mathbf{E} + \mathbf{u} \times \mathbf{B})\, \mathbf{P} - \boldsymbol{\omega}_c \times \mathbf{Q}\}^O + \mathcal{C}_3 \quad (2.29)$$

...

$$\partial_t \mathcal{M}_p + \nabla.(\mathbf{u}\mathcal{M}_p + \mathcal{M}_{p+1}) + \{D_t \mathbf{u}\, \mathcal{M}_{p-1}\}^O + \nabla.\{\mathcal{M}_p(\mathbf{u})\}^O - \nabla.\mathbf{u}\, \mathcal{M}_p = \{q/m(\mathbf{E} + \mathbf{u} \times \mathbf{B})\, \mathcal{M}_{p-1} - \boldsymbol{\omega}_c \times \mathcal{M}_p\}^O + \mathcal{C}_p \quad (2.30)$$

- The system includes all the well-known conservation laws (continuity, etc.)
- Each equation is valid as generally as the original kinetic equation (if kept under this complete tensorial form, without symmetry assumption)
- The tensorial complexity increases with the order of the moments
- $\partial_t \mathcal{M}_p$ is related to $\nabla.\mathcal{M}_{p+1}$ ➔ cannot be closed at a finite order without approximation

➔ Necessity of a closure equation (not universal)

Closure equations

- When strongly collisional ($K_n << 1$), Chapman Enskog expansion in K_n ➔ closure at order 3:
- $\mathbf{q} = -\kappa \nabla T$ ($\mathbf{q}$ = vector: $q_i = 1/2\, Q_{ijj}$ = sufficient in this quasi-isotropic case)
- Not valid in collisionless or weakly collisional media. See solar corona heating: even the sign is wrong: the source of heat is below and the temperature increases upward.
- In collisionless media ($K_n >> 1$), closure equations can be found, with limited ranges of validity. All come from symmetry properties of the distribution functions.

Adiabatic closure

- In a magnetized plasma, the most common closure is the adiabatic closure **Q** = 0.
  It is valid for fast propagating structures ($\omega/k_{//} >> V_{th}$).
  It means that, if f(v) is even, it remains even in these conditions.

- Consequences for the pressure: CGL laws, from (Chew, Goldberger, Law, 1956):

$$D_t(p_\perp/\rho B) = 0$$

$$D_t(p_{||}\, p_\perp^2/\rho^5) = 0$$

In isotropic conditions, the second law writes:
$D_t\,(p^3/\rho^5) = 0$ or $D_t\,(p/\rho^{5/3}) = 0$, as well known.

Closure for quasi stationary conditions

Opposite conditions with respect to the adiabatic o
$\omega/k_{//}$ <<

Also leads to simple relations for the variations of the parallel and perpendic
pressures (along

$$\partial_z \left[T_{\|}\right] = 0$$

$$\partial_z \left[\left(\frac{1}{T_\perp} - \frac{1}{T_{\|}}\right) B\right] = 0$$

$$\partial_z \left[\frac{T_\perp}{\rho}\right] = 0$$

In the isotropic case, it simply leads to T = cst (isothermal)

Other closures

Specific closures can be calculated, which intend to mimic correctly some phenomena that are known either experimentally or theoretically:

1. Simple laws with a few free parameters to be chosen in an empirical way. It is the case in particular for the popular "polytropic" laws:

$$D_t\,(P_{//}/\rho^{\gamma_{//}}) = 0$$

$$D_t\,(P_{\perp}/\rho^{\gamma_{\perp}}) = 0$$

The indices $\gamma_{//}$ and $\gamma_{\perp}$ ar the free parameters to be chosen

($\gamma_{//}=\gamma_{\perp} = 5/3$ ➔ isotropic+adiabatic)

2. More sophisticated laws elaborated to mimic phenomena that can be calculated completely in the kinetic formalism. It is the case of the Landau-fluid models, which can reproduce the Landau effect (collisionless damping) in the linear -or weakly nonlinear- limit (see Thierry Passot's talk).

Outline2:
Solar/stellar wind expansion:

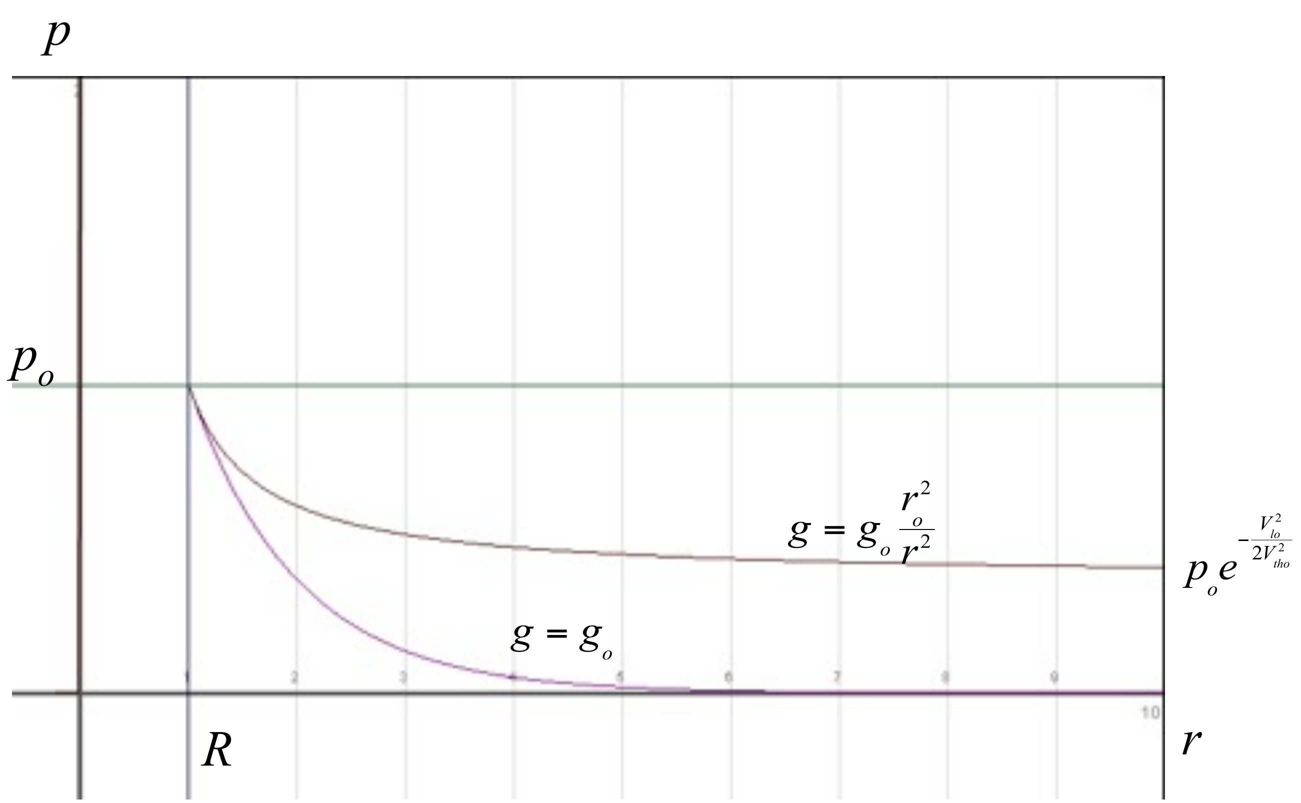
- Necessity of a wind:
  the fluid point of view
  the particle point of view

- Fluid: the Parker model

- Kinetic: the exospheric models

Necessity of a wind : Fluid

Why not a hydrostatic equilibrium? 1. Isothermal



Non null pressure at infinity
Impossible to confine the atmosphere in vacuum
(gravity non sufficient)

Fluid: Why not a hydrostatic equilibrium? 2. Polytropic

Why not a hydrostatic equilibrium? 2. Polytropic (with $\gamma \neq 1$)

Pressure at infinity $$p_\infty = \left(p_o^{\frac{\gamma-1}{\gamma}} - \frac{\gamma-1}{\gamma} \frac{g_o R}{p_o^{1/\gamma} / \rho_o} \right)^{\frac{\gamma}{\gamma-1}}$$

➔ Can be negative if $V_{tho}^2 < \frac{\gamma - 1}{\gamma} g_o R$ $\Leftrightarrow$ $V_{tho}^2 < \frac{\gamma-1}{\gamma} V_{lo}^2$

➔ Possible confinement if $\gamma > 1$ and V_{tho} small enough

Opposite conclusion ➔ Crucial importance of the closure in any fluid modeling

Necessity of a wind: individual particle argument

Without collision, each particle follows: $$\frac{v^2}{2} + \ddot{O} = \frac{v_o^2}{2}$$

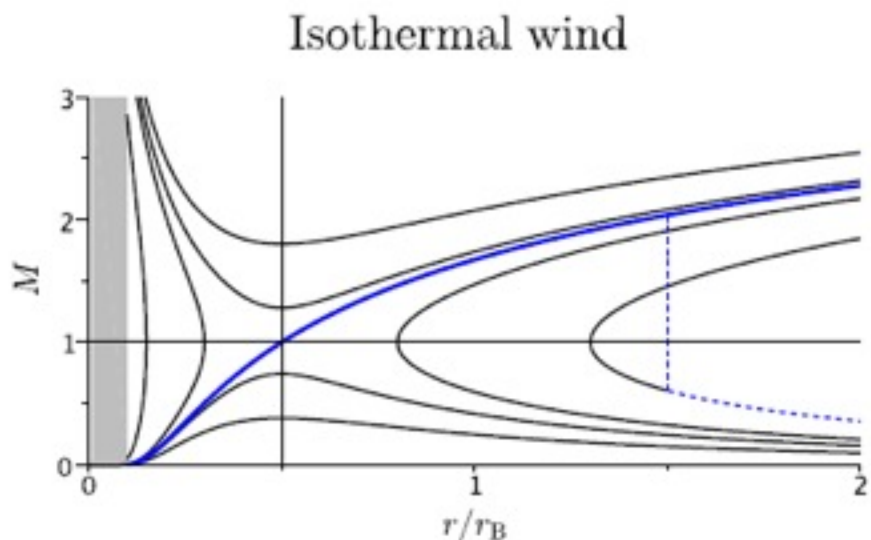
$$v_\infty^2 > 0 \quad \Rightarrow \quad \text{Escape for} \quad v_o^2 > V_{lo}^2 = 2\ddot{O}_\infty = 2g_oR$$

V_{l0} is the "*escape velocity*" at the lower boundary of the model

Always particles, at least in the tail of the distribution, with $v_0 > V_{l0}$ ➔ wind.

Fluid: The Parker model

Solutions with non null velocities (isothermal but not hydrostatic)



Stable solution = transonic

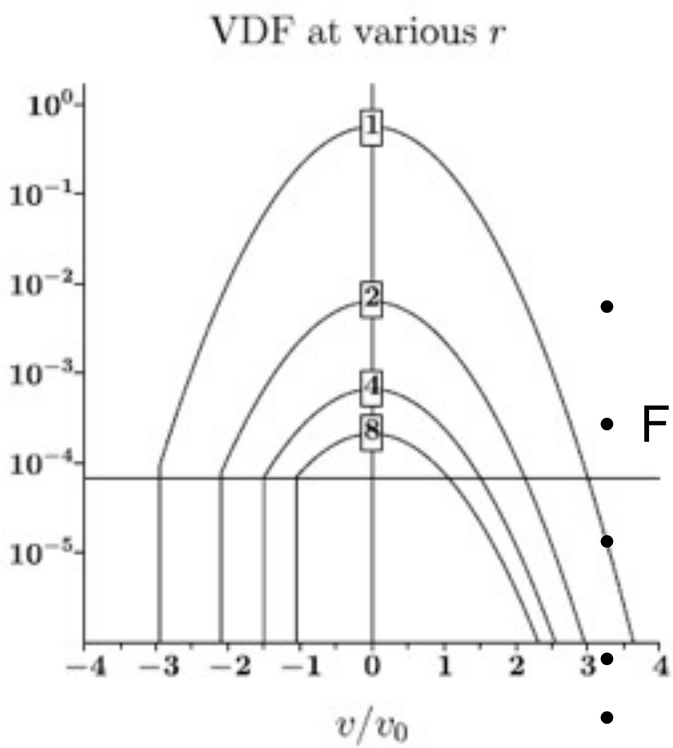
Pressure tends to zero. Velocity increases

Model very simple and rustic

Gives a velocity at 1 UA of 400 km/s instead of 800 km/s for the fast wind

NB. Accretion = reverse problem (same equations)

Kinetic: The exospheric models



Gives the variations of *n*, *u*, *P* with altitude.
Should be closer to reality.

Drawbacks:

- Initial distribution unknown (Maxwellian, kappa,.. ?)
- Final velocity depends on such free parameters (difficult to obtain 800 km/s anyway)
- Electric field ➔ trapped particles difficult to take into account
- No such cut off observed

Figure 9.8 Velocity distribution function at various distances from the star center. The fraction of particles with negative velocities decreases with distance and vanishes at $r = \infty$. Velocities are normalized by the thermal velocity $v_0 = (2T_0/m)^{1/2}$, where T_0 is the temperature of the untruncated Maxwellian distribution.

Modeling of the solar wind expansion: moral

Simple fluid isothermal model (Parker) ➔ approximate results quite correct
A priori difficult to do better with fluid models (as long as arbitrary closures)

Kinetic methods seem to be the only way to better capture the physics.
But many unknown parameters in the kinetic treatment, which still make it not very robust nowadays.

Outline3: Collisionless damping

- A simple example: the Langmuir wave
- Fluid treatment

Closure equation?

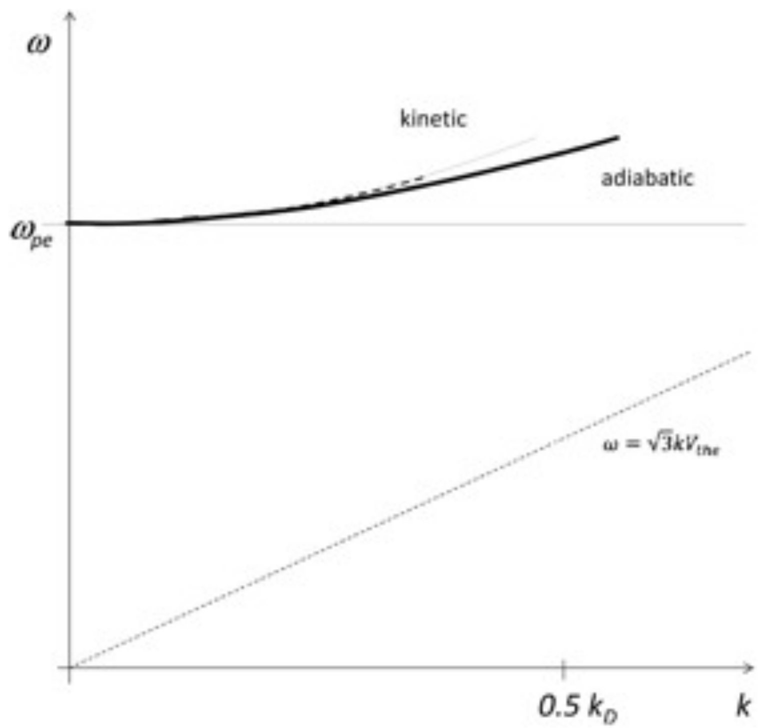
- Kinetic treatment

An infinity of modes with exotic distribution functions

Notion of kinetic mode: Landau damping

A simple example : the Langmuir wave

Langmuir wave =1-D electrostatic electron oscillations
High frequency : only the electrons move
they move under the effect of the electric field
Their density creates the electric field



Fluid treatment

$$D_t n_e + \partial_x (n_e u_e) = 0$$
$$n_e m_e D_t u_e + \partial_x P_e = -n_e e E$$
$$\partial_x E = -n_e e/\varepsilon_0$$

linear .+ Fourier ➔

$$\begin{aligned} u_{e1} &= v_\varphi \frac{n_{e1}}{n_{e0}} \\ v_\varphi u_{e1} &= \frac{P_{e1}}{m_e n_{e0}} + \frac{e}{m_e}\frac{E_1}{ik} \\ E_1 &= -\frac{1}{ik}\frac{n_{e0} e}{\varepsilon_0}\frac{n_{e1}}{n_{e0}} \end{aligned}$$

Adding an adiabatic closure equation $P_{e1} = 3 m_e V_{the}^2 n_1$ leads to the dispersion equation of the Langmuir mode

$$\omega^2 = \omega_{pe}^2 + \gamma k^2 V_{the}^2$$

with $\omega_{pe}^2 = \dfrac{n_{e0} e^2}{m_e \varepsilon_0}$

The 3 above equations are "exact" from a kinetic point of view.
If the kinetic dispersion departs from this one,
it is because the adiabatic closure becomes invalid.

Kinetic treatment

Instead of starting from the fluid equations (which demands a closure),

$$D_t n_e + \partial_x (n_e u_e) = 0$$
$$n_e m_e D_t u_e + \partial_x P_e = -n_e e E$$

let us start with the initial Vlasov equation

$$\partial_t f + v\, \partial_x f - e/m\, E\, f_0' = 0$$

Adding the Gauss equation as before and performing the same work (linearization, Fourierization) leads to:

$$(v - v_\varphi) f_1 = a_1 \qquad \text{with} \qquad a_1(v) = f_0' \frac{e}{m} \frac{E_1}{ik} = f_0' \frac{\omega_{pe}^2}{k^2} \frac{n_{e1}}{n_0}$$

This is not directly a dispersion relation since the LHS contains f_1 and the RHS contains n_1, which is the integral of f_1.

Preliminary remark: number of solutions expected

Fluid treatment:
two first order differential equations /t ➔ **two** modes
Similarly, compressible gas dynamics:
two first order differential equations /t ➔ **two** modes
MHD:
six first order differential equations /t ➔ **six** modes,
etc...

Vlasov:
one first order differential equations /t for each value of v ➔ infinite number of modes expected.

Is there one or two mode(s) among this infinity which can be called Langmuir?
Close to the fluid one?

Kinetic treatment: resolution

$$(v - v_\varphi) f_1 = a_1$$

Where a_1 is proportional to n_1 i.e. to the integral of f_1.

To obtain an equation in n_1, can we write $f_1 = \dfrac{a_1(v)}{v - v_\varphi}$ and integrate?

No.

Because of the resonant denominator, n_1 cannot be calculated in this way.

The corresponding integral is not defined.

In the theory of distributions, the value of f_1 is:

$$f_1 = p.v. \left[\frac{a_1(v)}{v - v_\varphi} \right] + \delta(v - v_\varphi) b_1$$

The two terms correspond, in Fourier space, to the usual result for a first order differential equation:

the general solution is a particular one + the general solution with RHS = 0

Kinetic treatment: resolution2

$$f_1 = p.v.\left[\frac{a_1(v)}{v - v_\varphi}\right] + \delta(v - v_\varphi) b_1$$

$$a_1 = \alpha \int f_1 \, dv$$

v_φ can be chosen arbitrarily thanks to b_1, which is itself arbitrary.

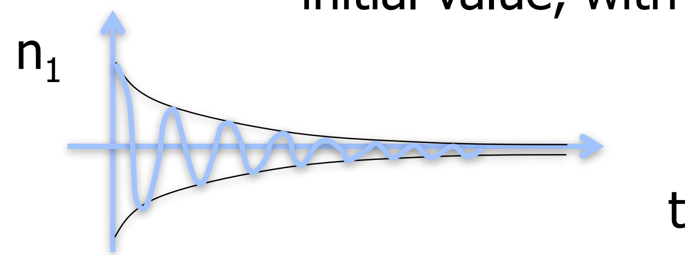
➔ Infinity of solutions as expected

But all these solutions are exotic since they involve singular distributions: the Dirac part and the principal value part.

Is it possible to build a linear superposition of these modes which would involve a regular distribution function?

Kinetic treatment: resolution3

One can build density perturbations which are wave packets decreasing from their initial value, with any damping rate γ.



The corresponding distribution functions have the value, at time t = 0:

$$f_1(v) = -\frac{\widehat{n}_{e1}}{2i\pi}\frac{1}{v - v_\varphi}\left[D_p - i\pi\frac{\omega_{pe}^2}{k^2}\frac{f_0'(v)}{n_0}\right]$$

D_p is a real integral corresponding to the principal value part.

All of them have a complex pole in v = v_φ except one:

$$D_p - i\pi\frac{\omega_{pe}^2}{k^2}\frac{f_0'(v_\varphi)}{n_0} = 0$$

Comments on the kinetic Landau solution

The only solution with no pole is called the kinetic mode.
It has an imaginary part: Landau damping

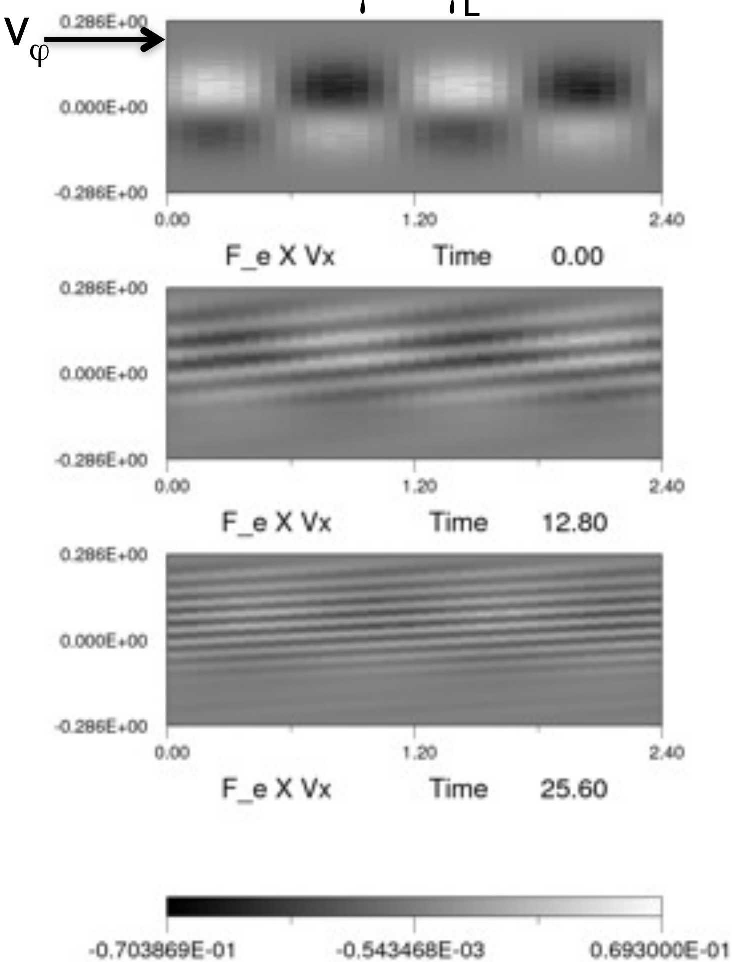
$$D_p - i\pi \frac{\omega_{pe}^2}{k^2} \frac{f_0'(v_\varphi)}{n_0} = 0$$

- D_p is a real function of v_φ whose expansion for $v_\varphi >> V_{the}$ corresponds exactly to the adiabatic fluid solution
- The second term comes from the Dirac part and provides the Landau damping
- The damping depends on f_0 via its derivative in $v = v_\varphi$
- The damping is small whenever f_0' is small, i.e. generally when v_φ is in the far tail of the distribution function
- A distribution with a complex pole is not singular for v real, but it corresponds to a special signature, which cannot exist as long as the system is not specially prepared for that (cf. simulation).

Phase space: $\delta f\ (x, v)$

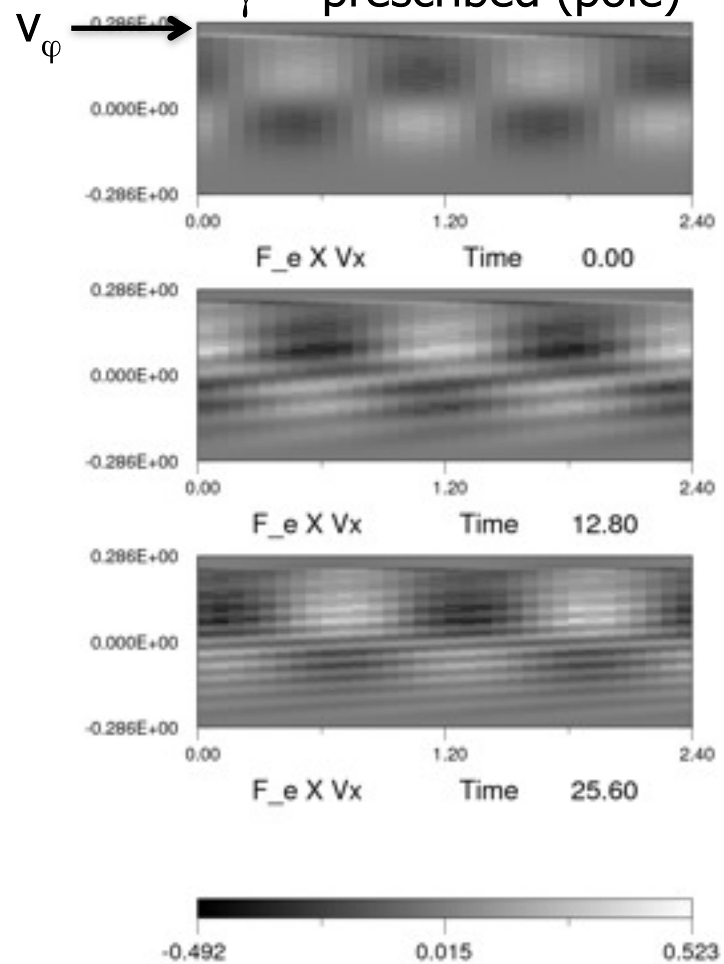
Landau

$\gamma = \gamma_L$



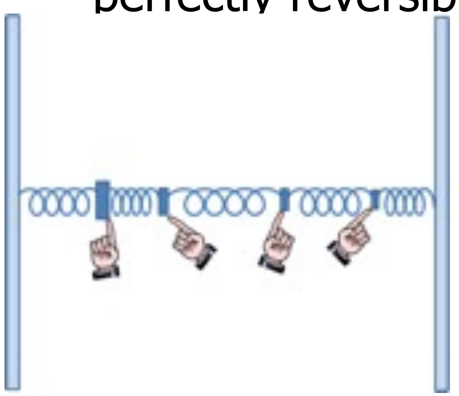
Non Landau

γ = prescribed (pole)



What is specific in the kinetic mode calculation?

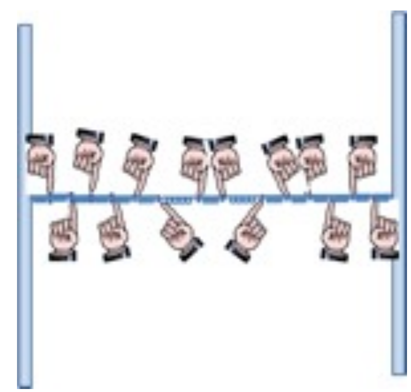
- Why are these mathematical delicate treatments necessary for kinetic modes and not for fluid ones or for any usual mechanical problem?
- It is just because of the existence of an infinity of resonances (any value of v_φ is resonant with one velocity $v = v_\varphi$).
- Exciting one single monochromatic mode would demand an infinite accuracy in the initialization. The kinetic mode retained is the only regular, which does not demand such an accuracy. It can be described by a « coarse-grained » distribution function. (This incomplete description can be quantified by an « entropy », which increases in association to the damping, while the complete distribution function is perfectly reversible)



Infinity of oscillators

➔

Infinity of fingers



Outline4: Magnetized plasmas

- MHD

The MHD system

Approximations

Beyond MHD

- Ohm's law

Freezing

Reconnection collisional/ collisionless

Magnetized plasmas

- They are ubiquitous…

(once again)

MHD system

$$
\begin{aligned}
D_t\rho + \rho\nabla.\mathbf{u} &= 0 \\
\rho D_t\mathbf{u} + \nabla P &= \mathbf{j}\times\mathbf{B} + \delta\mathbf{F} \\
\rho^\gamma D_t(P/\rho^\gamma) + (\gamma-1)\nabla.\mathbf{q} &= 0 \\
\partial_t\mathbf{B} + \nabla\times\mathbf{E} &= 0 \\
\nabla.\mathbf{B} &= 0
\end{aligned}
$$

with

$$
\begin{aligned}
\mathbf{j} &= \nabla\times\mathbf{B}/\mu_0 \\
\mathbf{E} &= -\mathbf{u}\times\mathbf{B} + \delta\mathbf{E}
\end{aligned}
$$

Ideal MHD:

$\delta\mathbf{E} = 0$ (ideal Ohm's law)

$\mathbf{q} = 0$ (adiabatic)

$\delta\mathbf{F} = 0$ (only Laplace force)

Dissipative MHD:

$\delta\mathbf{E} = \eta\,\mathbf{j}$ (resistive Ohm's law)

$\mathbf{q} = -\kappa\,\nabla T$ (thermal conductivity)

$\delta\mathbf{F} = \nu\,\nabla^2 u$ (viscosity)

Approximations to get the MHD system

- Mono-fluid (no distinction ion-electron)

  $\rho \approx \Sigma\rho_i$, $P \approx \Sigma P_i + P_e$. Valid if one single ion population or if all ion populations have the same velocity
- Scalar pressure P. Special condition of isotropy. At **large scale**, gyrotropy is general, but one should still distinguish $T_{//}$ and $T_{\perp}$ in a collisionless plasma.
- Closure equation supposed on the global **q**. Generally not justified except if **q** is carried by only one population or if all populations are adiabatic $\mathbf{q}_s = 0$.
- No ρ_e **E** in the force terms: quasi-neutrality. Valid whenever $\omega << \omega_{pe}$ and $k << k_{De}$
- No ∂_t **E** (displacement current) in Faraday eq. Valid whenever $\omega/k << c$
- Ohm's law simplified. Should be: $\mathbf{E} = -\mathbf{u}_e \times \mathbf{B} - \frac{1}{n_e e}[\rho_e d_t \mathbf{u}_e + \nabla.\mathbf{P}_e] + \eta\mathbf{j}$

Valid at **large scale** (with respect to d_i and R_{Li})

Ohm's law

$$\mathbf{E} = -\mathbf{u}_e \times \mathbf{B} - \frac{1}{n_e e}\left[\rho_e d_t \mathbf{u}_e + \nabla.\mathbf{P}_e\right] + \eta\mathbf{j}$$

$\mathbf{E} = -\mathbf{u} \times \mathbf{B}$ (ideal) instead of:

with $\mathbf{u}_e = \mathbf{u} - \mathbf{j}/ne$ and $\mathbf{j} = \nabla \times \mathbf{B}/\mu_0$

Difference between $\mathbf{u}_e$ and $\mathbf{u}$ ➔ Hall effect.

Negligible for $kd_i \ll 1$ (ion inertial length)

Term in $d_t \mathbf{u}_e$: electron inertia

Negligible for $kd_e \ll 1$ (electron inertial length)

Term in $\nabla.\mathbf{P}_e$: electron pressure

Negligible for $kR_{Le} \ll (m_e/m_i)^{1/2}$ (R_{Le} = electron Larmor radius)

Term in $\eta\mathbf{j}$: resistivity

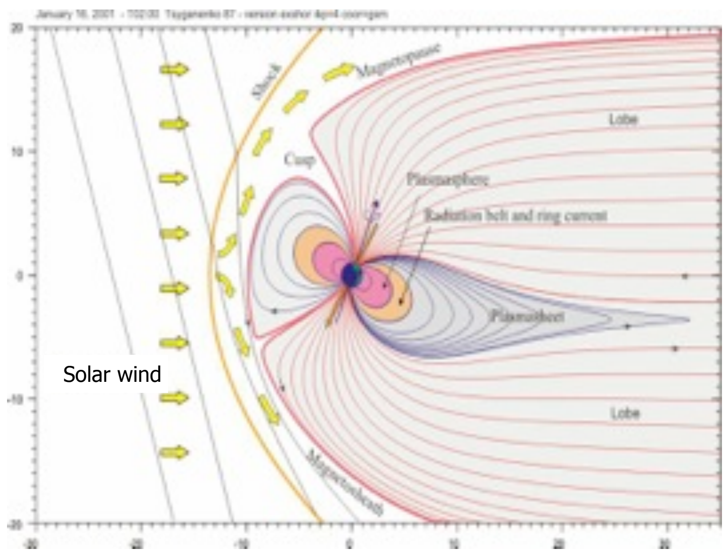
Negligible for $\omega \ll \omega_\eta$ (resistive time^{-1})

Freezing/ defreezing/ reconnection

- Ideal Ohm's law ➔ $E_{//} = 0$ ➔ ideal field line motion at $\mathbf{E}\times\mathbf{B}/B^2$

➔ Each field line keeps its identity in this motion ➔ no change of magnetic connections

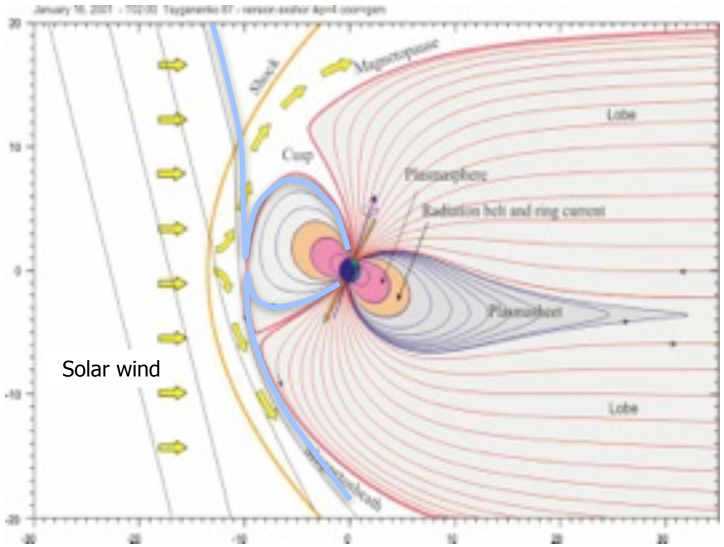
➔ May lead to the formation of thin layers when two magnetized plasmas of different origins meet



Freezing/ defreezing/ reconnection

- If layers thin enough: large scales conditions of validity for MHD violated
  ➔ possibility of reconnection

$$\mathbf{E} = -\mathbf{u}_e \times \mathbf{B} - \frac{1}{n_e e}\left[\rho_e d_t \mathbf{u}_e + \nabla.\mathbf{P}_e\right] + \eta \mathbf{j}$$



Consequences of reconnection

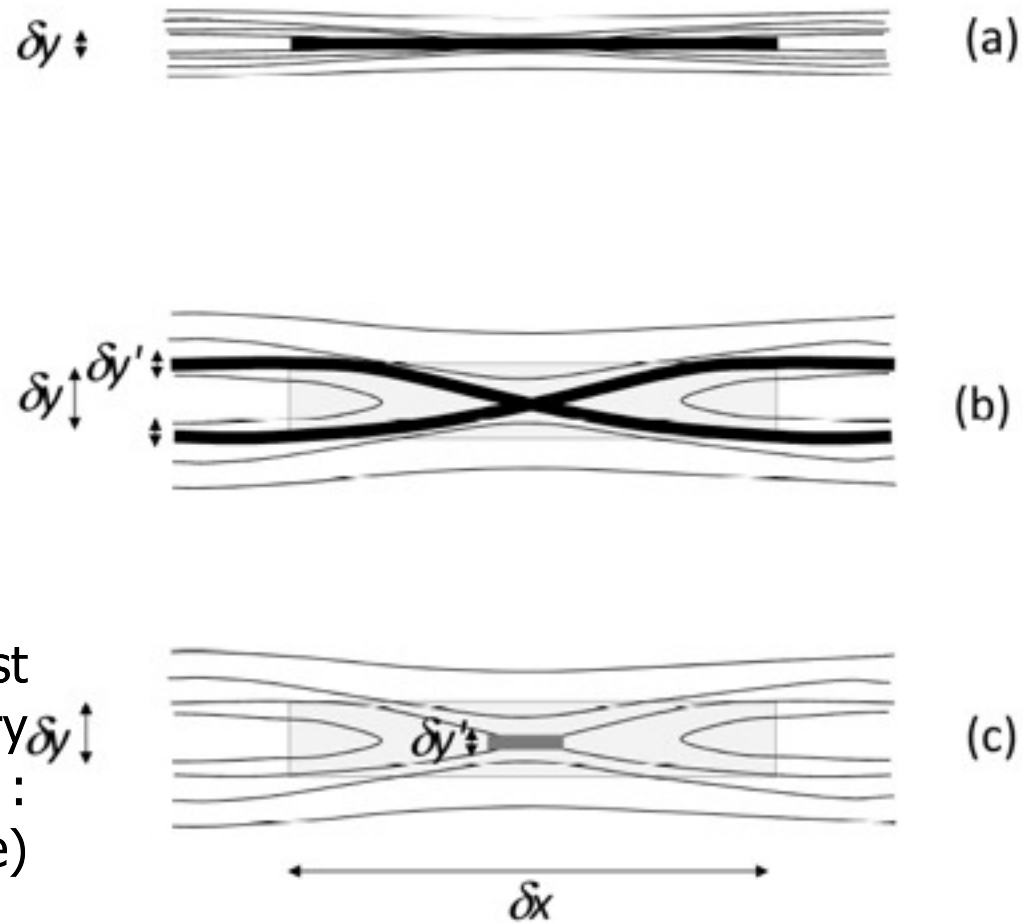
- Change of magnetic connections ➔ penetration through the boundaries and possible changes of magnetic topology
- Acceleration of the flow from $u_1 = \varepsilon V_{A1}$ to $u_2 \approx V_{A1}$
- Heating : flux of thermal energy ≈ flux of bulk kinetic energy
- Acceleration of energetic particles: not with simple geometries

Zooming on the X line (if 2-D)

Sweet Parker : too slow
(small exhaust)

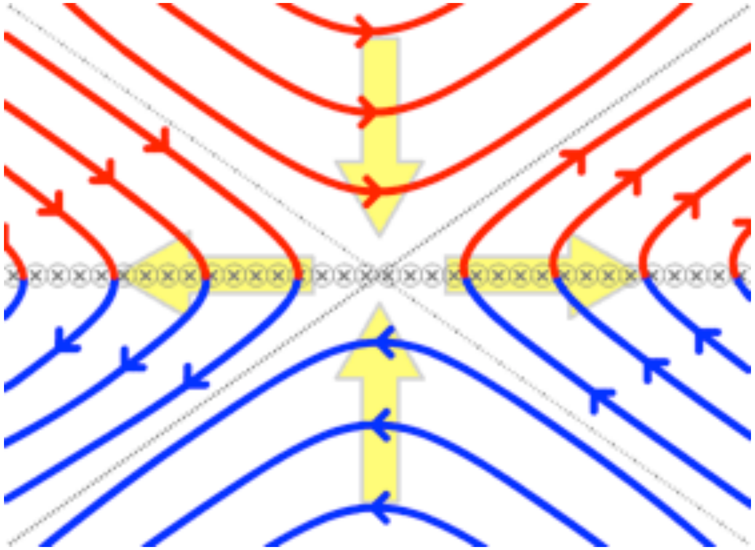
Petsheck : fast
but non stationary

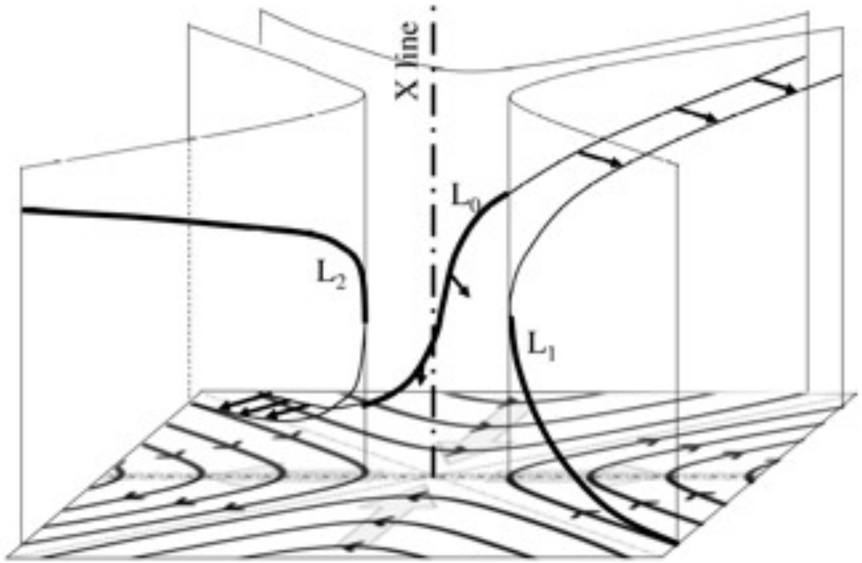
Collisionless : fast
and stationary
(2 small scales in the physics :
i and e)



Remark : the apparent "breaking" of a field line

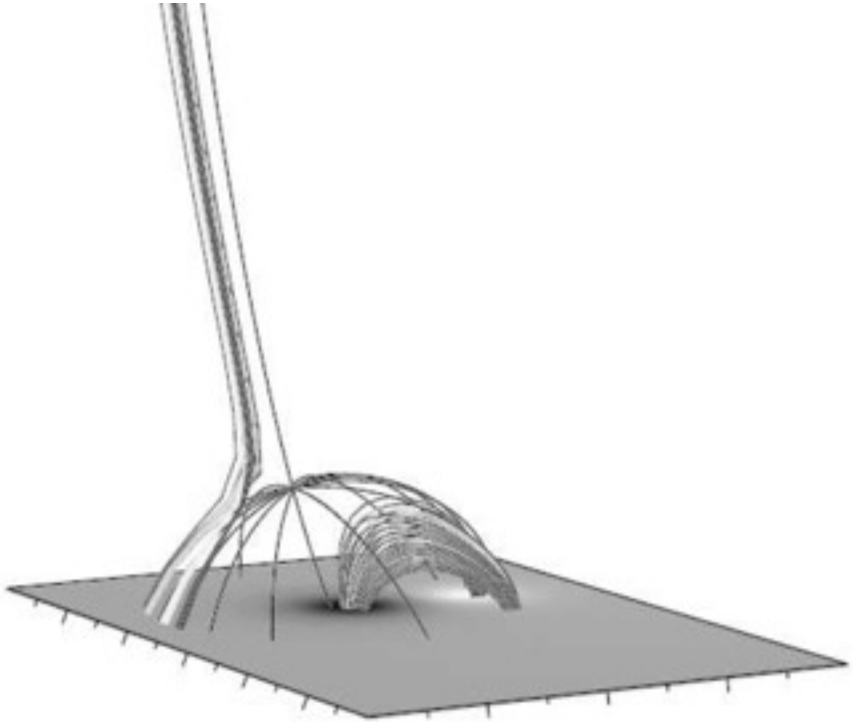
- It is anedotic and not to be considered as a definition of reconnection. As soon as no strict null point (guide field), nothing breaks during reconnection.





3-D reconnection

Solar physics :
encounter of 2 coronal loops ➔ intrinsically 3-D geometries.
Collisionless physics less developed in this case

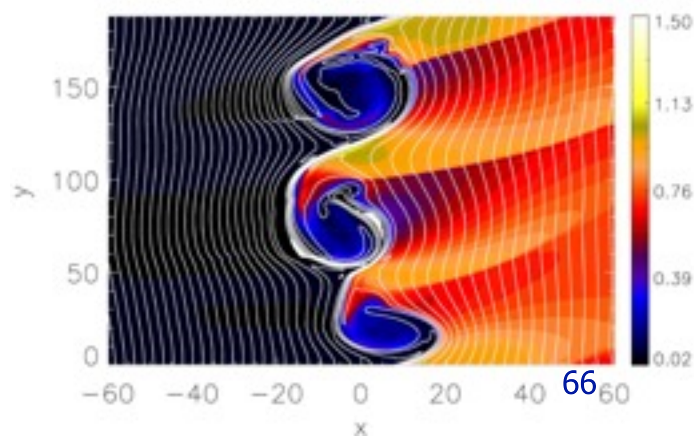


Triggering of reconnection

- Can be due to an instability of the current layers (tearing mode). In resistive MHD :

$$\gamma_{max} = V_A \lambda_\eta^{1/2} \delta y^{-3/2}$$

- Can be large even if η is small if δy is sufficiently small, for instance if it depends on η as $\eta^{1/2}$ (SP)
- Larger in Hall-MHD or in any model with collisionless terms in Ohm's law
- Can be a secondary instability
- of KH or RT (thinning layers)



Conclusion

After all these examples of the kinetic/ fluid duality, brief conclusions concerning MHD :

3 kinds of approximations and 3 kinds of remedies :

1. Ideal/ resistive Ohm's law not sufficient (small scales): not a serious problem. Use a generalized Ohm's law.
2. Mono fluid hypothesis not justified (high frequency): not too serious either. Use bi-fluid theory.
3. Problem with the closure equation ($\omega/k = V_{th}$): more difficult. If specific full kinetic treatment untractable, see Landau fluid models (cf. Thierry Passot).

The end

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