Calculations of antihydrogen loss from collisions with H and He

Svante Jonsell (Stockholm)
Edward Armour (Nottingham) & Martin Plummer (Daresbury)
Antihydrogen collisions

- Nuclear annihilation
  \[ \bar{H} + A \rightarrow \pi^+ + \pi^- + \pi^0 + e^+ + Ne^- \]

- Electron-positron annihilation
  \[ \bar{H} + A \rightarrow \bar{p} + Z + 2\gamma/3\gamma \]

- Rearrangement processes
  \[ \bar{H} + A \rightarrow [Z \bar{p} (N - 1)e^-] + Ps \]
  \[ \bar{H} + A \rightarrow [Z \bar{p} Ne^-] + e^+ \]

- Formation of molecule
  \[ \bar{H} + A \rightarrow [\bar{HA}] + \gamma \]

- Elastic scattering
The low-energy cross sections are characterised by the complex scattering length $a = \alpha - i\beta$.

**Elastic scattering:**
$$\sigma^\text{el} = 4\pi(\alpha^2 + \beta^2)$$

**Inelastic processes:**
$$\sigma^\text{inel} = \frac{4\pi\beta}{k_i}$$

Inelastic processes always dominate below a certain collision energy. This energy sets the lower limit for sympathetic cooling of antihydrogen.

The scattering-length approximation is valid up to energies $\sim 10^{-6}$ a.u. or temperatures $\sim 1$ Kelvin.

In this energy range only s-wave scattering contributes.
Energy conservation limits which Ps and Pn states can be formed.

\[ E_{1s}^H + E_{1s}^{\bar{H}} + \varepsilon_i = E_{N}^{Pn} + E_{n}^{Ps} + \varepsilon_f \]

The highest allowed Pn state is \( N = 24 \).

Excited states of Ps require \( N \leq 22 \).
Hydrogen-antihydrogen potential

At the critical distance \( R_c \lesssim 0.744 \) a.u. the potential joins the continuum. (Strasburger, J.Phys. B35, L435 (2002))
Consider a proton scattering on antihydrogen:

- At large internuclear distances the positron is bound to the antiproton.
- In the limit of zero internuclear separation the positron is unbound.
- At some intermediate distance the dipole created by the proton and antiproton ceases to bind the positron. This is the critical internuclear distance.
The system of a ground-state atom colliding with ground-state antihydrogen does not posses a critical distance if the atom before it in the periodic table is able to bind positronium.

Systems without critical distance:

- **Alkalis** no
- **Alkaline earths** Be, Mg, Ca
- **Inert gases** He, Ne, Ar, Kr, Xe
- **other** N, F, Zn


These atoms are likely to have smaller rearrangement cross sections.
Low-energy limit for formation of Ps(1s) with angular momentum $l = 0$, $\sigma_{\text{rearr}} \sqrt{\varepsilon_i}$.

<table>
<thead>
<tr>
<th>Pn state</th>
<th>pl. w. (1)</th>
<th>Kohn (2)</th>
<th>Optical (3)</th>
<th>DWBA (4)</th>
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<tr>
<td>24</td>
<td>0.038</td>
<td>0.21</td>
<td>0.32</td>
<td>0.15</td>
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<td>23</td>
<td>0.022</td>
<td>0.45</td>
<td>0.48</td>
<td>0.24</td>
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<td>22</td>
<td>0.016</td>
<td>0.01</td>
<td>0.14</td>
<td>0.002</td>
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<tr>
<td>21</td>
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<td></td>
<td>0.10</td>
<td>0.02</td>
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<td>20</td>
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<td></td>
<td>0.04</td>
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</tr>
<tr>
<td>19</td>
<td>0.003</td>
<td></td>
<td>0.03</td>
<td>0.003</td>
</tr>
</tbody>
</table>

(3) B. Zygelman et al. PRA 69, 042715 (2004)
Strong nuclear force

- The nucleus and anti-nucleus are attracted by the Coulombic interaction.

- At short distances (~1fm) the strong nuclear force dominates.

- The strong force causes both annihilation and a change in the elastic cross section.
Strong force scattering lengths

For **hydrogen**:

\[ a_{sf} = 0.84 - 0.70i \text{ fm} \]

Determined from the shift and width of the 1S state of protonium.

For **helium**:

\[ a_{sf} = 1.85 - 0.63i \text{ fm} \]

Determined from low-energy annihilation data and the shift and width of the 2P state of protonium \((\text{Gal, Nucl. Phys. A699, 300c (2002)})\).

Scattering lengths are spin-averaged values.
H-Hbar cross sections

energy [atomic units]

cross section [atomic units]

-10  -8  -6  -4  -2

elastc
annihilation
rearrangement N=23
rearrangement N=24
sum
The He-$\Bar{H}$ system is simpler since there is no critical distance.

Rearrangement channels:

- $\Bar{H}(1s) + \text{He}(1s^2) \rightarrow (\text{He}^+\Bar{p})_{\nu,J=0} + \text{Ps}(1s)$
  dominating channel is $\nu = 35$
  with low-energy cross section $\sigma = 0.142/\sqrt{\varepsilon_i}$

- $\Bar{H}(1s) + \text{He}(1s^2) \rightarrow (\text{He}\Bar{p})_{\nu,J=0} + e^+$
  smaller cross sections,
  \[
  \sigma_{\nu=33} = 2.77 \times 10^{-4}/\sqrt{\varepsilon_i} \\
  \sigma_{\nu=32} = 2.96 \times 10^{-4}/\sqrt{\varepsilon_i}
  \]

- $\Bar{H}(1s) + \text{He}(1s^2) \rightarrow (\alpha\Bar{p})_{\nu,J=0} + \text{Ps}^-$
  in progress (not likely to be large)
Helium-anithydrogen potential


No critical distance

Barrier (height $3.2 \times 10^{-4}$ a.u.)
Elastic cross section, partial waves

Resonant enhancement around $E_r = 1.04 \times 10^{-4} \approx 30 \text{ K}$.
Minimum around $E = 2 \times 10^{-5} \approx 6 \text{ K}$.
Helium-antihydrogen

Over $E = 6.1 \times 10^{-5} \approx 19$ K elastic loss dominates. Elastic and inelastic comparable around $E = 2.5 \times 10^{-4} \approx 80$ K.
Weakly bound resonance in outer well

\[ E_r = 1.04 \times 10^{-4} \]
\[ \Gamma = 3.40 \times 10^{-5} \]

Only exists for \( J=4 \).
No enhancement in inner barrier.
Density of background gas

Can be estimated from the lifetime of antiprotons in the trap.

Langevin cross section: \( \sigma = \pi \sqrt{\frac{2\alpha}{E}} \)

\( \alpha \) is the polarizability of the target (H/He/H\(_2\))

Long-range interaction \(-\frac{\alpha}{2r^4}\)

Gives an energy-independent destruction rate, valid at energies \(~\) eV.

Measured antiproton lifetime \(15000 \text{ s} = n_p v \sigma\)

Gives background gas density \(5 \times 10^{10} \text{ m}^{-3}\)
Rate of collisions with H

![Graph showing collision rate vs. Energy (Kelvin)]
Total collision rate with He

ALPHA cites >1000 s lifetime of trapped antihydrogen seems perfectly reasonable
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