Cascade and dissipation of solar wind turbulence from MHD to electron scales

Fouad Sahraoui

G. Belmont, S. Huang, M. Goldstein, J.L. Pinçon, A. Retino, Y. Narita, K. Kiyani

1 LPP, CNRS-Ecole Polytechnique, France
2 NASA/GSFC, Maryland, USA
3 University of Wuhan, China
4 LPC2E, Orléans, France
5 Max Plank Institute, Allemagne
6 Imperial college, UK
Outline

1. Solar wind turbulence vs heating
2. The problem of measuring spatial properties of space plasma turbulence
3. 3D spatial spectra and anisotropies of MHD turbulence in the solar wind (Cluster data)
4. Kinetic scales in the SW: Some hotly debated question vs Cluster observations
   - *Cascade or dissipation below $\rho_i$?*
   - *The scaling: power-law? Exponential? Others?*
   - *The nature of the cascade: KAW? whistler? Others?*
   - *The nature of the dissipation: wave-particle interactions? Current sheets/Reconnection?*
   - *Weak vs strong turbulence? Monofractality vs multifractality?*
5. Conclusions & perspectives (turbulence & the future space missions)
Turbulence in the Universe

It is observed from quantum to cosmological scales!

It controls mass transport, energy transfers & heating, magnetic reconnection in plasmas, …

Performance limited by plasma turbulence

Strong pressure gradients
\( \Rightarrow \) Instabilities

\[ \begin{align*}
N & \sim 10^6 \text{ cm}^3 \\
T_i & \sim 10^{12} \text{ K} \\
B & \sim 10^6 \text{ nT}
\end{align*} \]

M100 galaxy \( 10^{23} \text{ m} \)

Eagle nebula \( 10^{18} \text{ m} \)

\[ \begin{align*}
N & \sim 10 \text{ cm}^3 \\
T_i & \sim 10 \text{ K} \\
B & \sim 10 \text{ nT}
\end{align*} \]

GYRO code

\( \Rightarrow \) Prediction & control of turbulent transport
Near-Earth space plasmas

\[ \beta = \frac{\text{Pression thermique}}{\text{Pression magnétique}} \approx 0.4 \frac{NT}{B^2} \]


[Representative Parameters for Astrophysical Plasmas]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Solar wind at 1 AU(a)</th>
<th>Warm ionized ISM(b)</th>
<th>Accretion flow near Sgr A(c)</th>
<th>Galaxy clusters (core)(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_p ), cm(^{-3} )</td>
<td>30</td>
<td>0.5</td>
<td>( 10^6 )</td>
<td>( 6 \times 10^{-2} )</td>
</tr>
<tr>
<td>( T_e ), K</td>
<td>( T_i^{(e)} )</td>
<td>8000</td>
<td>( 10^{11} )</td>
<td>( 3 \times 10^7 )</td>
</tr>
<tr>
<td>( T_i ), K</td>
<td>( 5 \times 10^{5} )</td>
<td>8000</td>
<td>( \sim 10^{12}(f) )</td>
<td>( ?^{(e)} )</td>
</tr>
<tr>
<td>B, G</td>
<td>( 10^{-4} )</td>
<td>( 10^{-6} )</td>
<td>30</td>
<td>( 7 \times 10^{-6} )</td>
</tr>
</tbody>
</table>

The solar wind

The solar wind plasma is generally:

- Fully ionized (H\(^+\), e\(^-\))
- Non-relativistic (\(V_\text{A} \ll c\)), \(V \sim 350-800\) km/s
- *Collisionless*
Phenomenology of turbulence

\[ \partial_t \mathbf{V} + \mathbf{F}_i = -\mathbf{V} \cdot \nabla \mathbf{V} - \nabla P + \nu \nabla^2 \mathbf{V} \]

\[ E(k) \sim k^{-5/3} \]

- **Hydro**: Scale invariance down to the dissipation scale \( 1/k_d \)
- **Collisionless Plasmas**: 
  - Breaking of the scale invariance at \( \rho_{i,e} d_{i,e} \)
  - Absence of the viscous dissipation scale \( 1/k_d \)

"Big whorls have little whorls
That feed on their velocity,
And little whorls have lesser whorls
And so on to viscosity"

Lewis Fry Richardson (1920)
Solar wind turbulence

Typical power spectrum of magnetic energy at 1 AU

Does the energy cascade or dissipate below the ion scale $\rho_i$?

Leamon et al. 98; Goldstein et al. JGR, 94

Richardson & Paularena, GRL, 1995 (Voyager data)
How to analyse space turbulence?

Turbulence theories generally predict **spatial spectra**: K41 \((k^{-5/3})\); IK \((k^{-3/2})\), Anisotropic MHD turbulence \((k_{\perp}^{-5/3})\), Whistler turbulence \((k^{-7/3})\), ...

Example of measured spectra in the SW

But measurements provide only **temporal spectra** (generally with different power laws at different times).

How to infer **spatial spectra** from **temporal** ones measured in the spacecraft frame? \(B^2 \sim \omega_{sc}^{-\alpha} \Rightarrow B^2 \sim k_{\parallel}^{-\beta} k_{\perp}^{-\gamma}\)?
The Taylor frozen-in flow assumption

In the solar wind (SW) the Taylor’s hypothesis can be valid at MHD scales.

High SW speeds: $V \sim 600\text{km/s} \gg V_{\phi} \sim V_A \sim 50\text{km/s} \Rightarrow$

$$\omega_{\text{spacecraft}} = \omega_{\text{plasma}} + k.V \approx k.V = k_V V$$

$\Rightarrow$Inferring the $k$-spectrum is possible with one spacecraft

But only along one single direction
1. At MHD scales, even if the Taylor assumption is valid, inferring 3D $k$-spectra from an $\omega$-spectrum is impossible.

2. At sub-ion and electron scales, $V\phi$ can be larger than $V_{sw}$ → The Taylor's hypothesis is invalid.

1 & 2 ⇒ Need to use multi-spacecraft measurements and appropriate methods to infer 3D $k$-spectra.
Anisotropy and the critical balance conjecture

The critical balance conjecture [Goldreich & Sridhar, 1995]:
Linear (Alfvén) time ~ nonlinear (turnover) time
\[ \omega \sim k_\parallel V_A \sim k_\perp u_\perp \]
\[ \Rightarrow k_\parallel \sim k_\perp^{2/3} \]


Single satellite analysis $\rightarrow$ use of the Taylor assumption:

$$\omega_{sc} \sim k \cdot V_{sw} \sim k_v V_{sw}$$

$$V//B \rightarrow k_v = k_//$$

Assumes axisymmetry around $B$

$$V \perp B \rightarrow k_v = k_\perp$$

$$\Theta_{BV} \rightarrow 0 \Rightarrow B^2 \sim k_//^{-2} \Rightarrow \text{Partial evidence of the critical balance} \ [\text{Horbury et al., PRL, 2008}]$$

See also Chen et al., PRL, 2010
The ESA/Cluster mission

The first multispacecraft mission: 4 identical satellites

Objectives:

- **3D exploration** of the Earth magnetosphere boundaries (magnetopause, bow shock, magnetotail) & SW

- **Measurements of 3D quantities:** \( J = \nabla \times B \), …

- **Fundamental physics:** turbulence, reconnection, particle acceleration, …

Different orbits and separations (\(10^2\) to \(10^4\) km) depending on the scientific goal
The 4 satellites before launch
The $k$-filtering technique

Interferometric method: it provides, by using a NL filter bank approach, an optimum estimation of the 4D spectral energy density $P(\omega,k)$ from simultaneous multipoints measurements [Pinçon & Lefevre; Sahraoui et al., 03, 04, 06, 10; Narita et al., 03, 06,09]

We use $P(\omega,k)$ to calculate

1. 3D $\omega$-$k$ spectra $\Rightarrow$ plasma mode identification e.g. Alfvén, whistler
2. 3D $k$-spectra (anisotropies, scaling, …)
Measurable spatial scales

Given a spacecraft separation $d$ only one decade of scales $2d < \lambda < 30d$ can be correctly determined

- $\lambda_{\text{min}} \approx 2d$, otherwise spatial aliasing occurs.

- $\lambda_{\text{max}} \approx 30d$, because larger scales are subject to important uncertainties

$\omega_{\text{sat}} \sim kV \Rightarrow f_{\text{max}} \sim k_{\text{max}} V/\lambda_{\text{min}} (V \sim 500\text{km/s})$

- $d \sim 10^4$ km $\Rightarrow$ MHD scales

- $d \sim 10^2$ km $\Rightarrow$ Sub-ion scales

- $d \sim 1$ km $\Rightarrow$ Electron scales *(but not accessible with Cluster: $d > 100$)*
1- MHD scale solar wind turbulence

Position of the Quartet on March 19, 2006

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Color</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster-1</td>
<td>Blue</td>
<td>15.038</td>
<td>-6.569</td>
<td>-9.299</td>
</tr>
<tr>
<td>Cluster-2</td>
<td>Red</td>
<td>15.139</td>
<td>-7.034</td>
<td>-8.672</td>
</tr>
<tr>
<td>Cluster-3</td>
<td>Green</td>
<td>13.979</td>
<td>-7.397</td>
<td>-10.41</td>
</tr>
</tbody>
</table>
Data overview

FGM data (CAA, ESA)

Ion plasma data from CIS (AMDA, CESR)
\( f_1 = 0.23 \text{Hz} \sim 2f_{ci} \)

\( f_2 = 0.9 \text{Hz} \sim 6f_{ci} \)

To compute reduced spectra we integrate over

1. all frequencies \( f_{sc} \):
   \[
   \tilde{P}(\mathbf{k}) = \sum_{k_y,k_z} P(f_{sc}, \mathbf{k})
   \]

2. all \( k_{i,j} \):
   \[
   \tilde{P}(k_x) = \sum_{k_y,k_z} \tilde{P}(k_x, k_y, k_z)
   \]
Anisotropy of MHD turbulence along $B_o$ and $V_{sw}$

Turbulence is not axisymmetric (around $B$) [see also Sahraoui, PRL, 2006]

The anisotropy ($\perp B$) is along $V_{sw} \rightarrow SW$

expansion effect?[Saur & Bieber, JGR, 1999]
Kinetic (sub-ion scale) turbulence in the SW
I- Theoretical predictions on small scale turbulence

\[ \mathbf{E} + \mathbf{V} \times \mathbf{B} = \frac{1}{en} \mathbf{J} \times \mathbf{B} - \frac{\nabla P_e}{en} + \ldots \]

1. Fluid models (Hall-MHD)
   - Whistler turbulence (E-MHD): (Biskamp et al., 99, Galtier, 08)
   - Weak Turbulence of Hall-MHD (Galtier, 06; Sahraoui et al., 07)

2. Gyrokinetic theory: \( k_// \ll k_\perp \) and \( \omega \ll \omega_{ci} \) (Schekochihin et al. 06; Howes et al., 11)
2D PIC simulations gave evidence of a power law dissipation range at $k \rho_e > 1$

Figure 4. Spectrum of magnetic fluctuation $|\delta B|^2 / |B_0|^2$ in the parallel direction $k_{||} \rho_e$. The noise level curve is in red. The power-law best fits are superimposed. (A color version of this figure is available in the online journal.)

Figure 5. Spectrum of magnetic fluctuation $|\delta B|^2 / |B_0|^2$ in the perpendicular direction $k_{\perp} \rho_e$. The noise level curve is in red. The power-law best fits are superimposed.

3D PIC simulations of whistler turbulence: $k^{-4.3}$ at $kd_e > 1$

Chang & Gary, GRL 2011
2- Observations of kinetic SW turbulence

What happens to the energy at, and below, the ion scale $\rho_i$ (not $f_{ci}$): a total dissipation or a new cascade?

Richardson & Paularena, GRL, 1995 (Voyager data)

Leamon et al, 98; Goldstein et al., 94
First evidence of a cascade from MHD to electron scale in the SW

1. Two breakpoints corresponding to $\rho_i$ and $\rho_e$ are observed.

2. A clear evidence of a new inertial range $\sim f^{-2.5}$ below $\rho_i$

3. First evidence of a dissipation range $\sim f^{-4}$ near the electron scale $\rho_e$

Sahraoui et al., PRL, 2009
Similar observations from STAFF-SA data, but the spectra were fit by an exponential model.
The largest survey of Cluster/STAFF-SC data

A better correlation with $\rho_e$ than with $d_e$

[Sahraoui+, 2013; Huang+, 2013]

SW (moderate SNR)

Magnetosheath (high SNR)
Whistler or KAW turbulence?

1. Large (MHD) scales ($L > \rho_i$): strong correlation of $E_y$ and $B_z$ in agreement with $E = -V \times B$

2. Small scales ($L < \rho_i$): steepening of $B^2$ and enhancement of $E^2$ (however, strong noise in $E_y$ for $f > 5$Hz)

$\Rightarrow$ Good agreement with GK theory of Kinetic Alfvén Wave turbulence

Howes et al.
PRL, 11

FGM, STAFF-SC and EFW data

See also Bale et al., PRL, 2005
Theoretical interpretation: KAW turbulence

Linear Maxwell-Vlasov solutions: $\Theta_{kB} \sim 90^\circ$, $\beta_i \sim 2.5$, $T_i/T_e \sim 4$

The Kinetic Alfvén Wave solution extends down to $k \rho_e \sim 1$ with $\omega_r < \omega_{ci}$


$$\omega_r = k_{//} V_A k_{\perp} \rho_i / \sqrt{\beta_i + 2/(1+T_i/T_e)}$$
E/B observations vs observations

\[ \omega_i = k||V_Ak_\perp\rho_i / \sqrt{\beta_i} + 2/(1+T_i/T_e) \]

- Lorentz transform: \( E_{\text{sat}} = E_{\text{plas}} + V \times B \)
- Taylor hypothesis to transform the spectra from \( f \) (Hz) to \( k \rho_i \)

1. Large scale (\( k \rho_i < 1 \)): \( \delta E/\delta B \sim V_A \)

2. Small scale (\( k \rho_i > 1 \)): \( \delta E/\delta B \sim k^{1.1} \Rightarrow \) in agreement with GK theory of KAW turbulence \( \delta E^2 \sim k_\perp^{-1/3} \) & \( \delta B^2 \sim k_\perp^{-7/3} \Rightarrow \delta E/\delta B \sim k \)

3. The departure from linear scaling (\( k \rho_i \approx 10 \)) is due to noise in \( E_y \) data

\[ \Theta_{KB} \approx 90 \]

\[ k_{\rho_p} = 1; k_{\rho_e} = 1 \]

Sahraoui \( et \) al., PRL, 2009
Magnetic compressibility

Additional evidence of KAW at $k\rho_i \gtrsim 1$

Fast magnetosonic

KAW

Cluster/STAFF-SC data

KAW, $\Theta_{kB} = 89.9$


3D k-spectra at sub-proton scales of SW turbulence

Conditions required:

1. Quiet SW: NO electron foreshock effects
2. Shorter Cluster separations (~100km) to analyze sub-proton scales
3. Regular tetrahedron to infer actual 3D $k$-spectra [Sahraoui et al., JGR, 2010]
4. High SNR of the STAFF data to analyse HF (>10Hz) SW turbulence.

20040110, 06h05-06h55
3D $k$-spectra at sub-proton scales

We use the $k$-filtering technique to estimate the 4D spectral energy density $P(\omega, k)$

We use $P(\omega, k)$ to calculate

1. 3D $\omega$-$k$ spectra
2. 3D $k$-spectra (anisotropies, scaling, …)
Turbulence is

- \( \perp \mathbf{B}_0 \) but non axisymmetric
- Quasi-stationary (\( \omega_{\text{plas}} \sim 0 \) although \( \omega_{\text{sat}} \sim 20\omega_{ci} \))
Comparison with the Vlasov theory

Turbulence cascades following the Kinetic Alfvén mode (KAW) as proposed in Sahraoui et al., PRL, 2009

→ Rules out the cyclotron heating

→ Heating by p-Landau and e-Landau resonances

[Sahraoui et al., PRL, 2010]

\[ \beta_i \sim 2 \quad \frac{T_i}{T_e} = 3 \quad 85^\circ < \Theta_{kB} < 89^\circ \]
3D $k$-spectra at sub-ion scales

1. First *direct* evidence of the breakpoint near the proton gyroscale in $k$-space (*no additional assumption, e.g. Taylor hypothesis, is used*)

2. Strong steepening of the spectra below $\rho_i \rightarrow$ A *Transition Range* to dispersive/electron cascade
1. Turbulence
2. e-Acceleration
   & Heating
3. Reconnection

Another interpretation in Meyrand & Galtier, 2010
Dissipation through reconnection/current sheets

Large scale laminar current sheet: reconnection can occur and the can be heated or accelerated (e.g. jets)

[Zhong+, Nature Physics, 2010]
Turbulent current sheets

[96x84][Lazarian & Vishniac, 1999]

2D Hall-MHD simulation of turbulence: evidence of a large number of reconnecting regions

[e.g., Retinò+, Nature Physics, 2007]
Dissipation by wave-particle interaction or via reconnection?

Good correlation between enhanced $T_p$ and threshold of linear kinetic instabilities

Good correlation between enhanced high shear $B$ angles and the threshold of linear instabilities!!

Osman et al., PRLs, 2012a,b
Statistical approach to small scale SW turbulence

Which statistical description applies to sub-proton scale SW turbulence:

1. Weak or strong turbulence?
2. If strong, then is it self-similar/monofractal or intermittent/multifractal?
1. Strong vs Weak Turbulence:

Often it has been argued that small scale/high frequency turbulence in the solar wind is a weak turbulence because \( |\delta B|/B \ll 1 \)

This is wrong!

Because *only* the ratio nonlinear/linear times (or terms) for each physical system can indicate how weak or strong is the turbulence

Let us consider the example of Incompressible MHD
Incompressible Alfvénic Turbulence

\[ \partial_t z^\pm \mp v_A \nabla z^\pm + z^\mp \nabla z^\pm = -\nabla p \]

**Linear term:** \( k_{\parallel} v_A z^+ \)

**Nonlinear term:** \( k_{\perp} u_{\perp} z^+ \)

Ratio of nonlinear to linear terms:

\[ \chi = \frac{k_{\perp} u_{\perp}}{k_{\parallel} v_A} \]

\( \chi \ll 1 \) \Rightarrow Weak turbulence with \( k_{\parallel} v_A \gg k_{\perp} u_{\perp} \)

\( \chi \sim 1 \) \Rightarrow Strong turbulence with \( k_{\parallel} v_A \sim k_{\perp} u_{\perp} \) (or \( \omega \sim \omega_{NL} \Rightarrow Critical balance conjecture \))

For anisotropy \( k_{\perp} \gg k_{\parallel} \) we have STRONG turbulence \( (\chi \sim 1) \) even when \( \frac{u_{\perp}}{v_A} \sim \frac{\delta B}{B_0} \ll 1 \)
⇒ *One has to give up using mere criteria, e.g.* $|\delta B|/B << 1$, *to discriminate within the data between weak/strong turbulence theories*

Other alternatives?

1. Estimation of the linear/nonlinear times of the turbulence from the data

   ⇒ *But it is difficult because this generally requires to know accurately the nature of the turbulence and its spatial scales (|| and \(\perp\))*

2. Estimating phase coherence *directly* from the measured Fourier phases of the turbulence from the data using, e.g., *Surrogate data* [Hada et al., 2003; Sahraoui, PRE, 2008; Sahraoui & Fauvarque, in prep.]
2. Monfractality vs multifractality in the dispersive range:

- $n_e \sim 4 \text{ cm}^{-3}$
- ion $\beta \sim 2$
- $V_A \sim 50 \text{ km s}^{-1}$
- $T_i \sim 103 \text{ eV}$
- $|B| \sim 4 \text{ nT}$

[Kiyani et al., PRL, 2009]
Evidence of monofractality (self-similarity) at sub-proton scales, while MHD-scales are multifractal (intermittent) [See also Alexandrova et al., ApJ, 2008]
Conclusions

The Cluster data helps understanding crucial problems of astrophysical turbulence:

- Its nature and anisotropies in $k$-space at MHD and sub-ion scales
- Its cascade and dissipation down to the electron gyroscale $\rho_e \Rightarrow$ electron heating and/or acceleration by turbulence
- Strong evidences of KAW turbulence ($\omega \ll \omega_{ci}$, $k// \ll k_\perp$) $\Rightarrow$ Heating by e-p-Landau dampings (no cyclotron heating)
- Importance of kinetic physics in SW turbulence
⇒ Need of multi-scale measurements with appropriate spacecraft separations

Narita et al. PRL, 2010

MMS 2014

Sahraoui et al. PRL, 2010
La turbulence et les futures missions spatiales

Magnetospheric Multiscale
A Solar-Terrestrial Probe

4 NASA satellites, launch 2014
Higher resolution instrumentations
Small separations (~10km)
Equatorial orbits
Solar Orbiter

Exploring the Sun-Heliosphere Connection

Launch 2017

Distance: 0.28 AU

In-situ measurements & remote sensing
Launch 2019
Distance: ~0.03 AU
In-situ measurements & remote sensing

Solar Probe
Plus

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland 20771
TOR
Solving energy dissipation problem at kinetic scales in the solar wind

Mission proposed to ESA (2012)
Currently under review with SNBS
Recently proposed to CNES