Radiative processes in high energy astrophysical plasmas

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Why radiation?

✓ Why is radiation so important to understand?
  ✓ Light is a tracer of the emitting media
    ✓ Geometry, evolution, energy, magnetic field...
  ✓ Light can influence the source properties
    ✓ cooling/heating, radiation pressure...
  ✓ Light can be modified after its emission

✓ Why are high energy plasmas so important to study?
  ✓ High energy particles are the most efficient emitters
  ✓ They emit over an extremely wide range of frequencies
✓ Goals of this course:
  ✓ Review the main high energy processes for continuum emission
    ✓ Assumptions, approximations, properties
  ✓ Show how they can be used to constrain the physics of astrophysical sources

✓ Main books/reviews:
  ✓ Aharonian F. A., 2004, Very High energy cosmic gamma radiation, World scientific publishing
Contents

✓ Introduction

✓ I. Emission of charged particles

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✓ III. Compton scattering
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Introduction
Non black-body radiation

✓ Black-Body radiation is simple ideal limit
  ✓ independent of internal processes, geometry...
  ✓ Simple law

✓ No black body radiation if:
  ✓ Optically thin media
    ✓ <= finite source size and finite interaction cross sections
    ✓ => absorption/emission/reflection features
  ✓ Matter not at thermal equilibrium
  ✓ <= low density, high energy plasmas

✓ Then, radiation properties depend on
  ✓ The particle distributions
  ✓ The microphysics: what processes?
Radiation processes

✓ Lines (bound-bound):
  ✓ Atomic, molecular transitions (radio to X-rays)
  ✓ Nuclear transitions (γ-rays)

✓ Edges (bound-free):
  ✓ ionization/recombination

✓ Nuclear reactions, decay and annihilations:
  ✓ bosons, pions...
  ✓ electron-positron, photon-photon

✓ Collective processes
  ✓ Faraday rotation, Cherenkov radiation...

✓ Free-free radiation of charged particles in vacuum:
  ✓ Cyclo-synchrotron radiation
  ✓ Compton scattering
  ✓ (Bremsstrahlung radiation)
I. Radiation of relativistic particles
Emission from non-relativistic particles

- Electro-magnetic field created by charges in motion:
  - Charged particles in uniform motion do not emit light
  - Only charged, accelerated particles can emit light
    - Liénard-Wiechert potentials (1898): \((A, \Phi) \Rightarrow E-B\)
    - Power: \(P \propto E^2\)
    - Spectrum: \(P_\nu \propto \left| \text{FFT}(E) \right|^2\)

- Emission of low-energy particles:
  - Total power: \(P_e = \frac{2q^2a^2}{3c}\)
  - Dipolar field perpendicular to acceleration: \(\frac{\partial P_e}{\partial \Omega} = \frac{q^2a^2}{4\pi c} \sin^2 \theta\)
  - Polarization depends on the direction: \(\vec{E} \propto \vec{n} \times (\vec{n} \times \vec{a})\)
  - Is also the emission in the particle rest frame...
Emission of relativistic particles

**Change of frame**

✓ Relativistic particles:

  ✓ Velocity \( \beta = v/c \)
  ✓ Lorentz factor \( \gamma = 1/(1 - \beta^2)^{1/2} \)
  ✓ Energy \( E = \gamma mc^2 \)  \( E_K = (\gamma - 1)mc^2 \)
  ✓ Momentum \( p = (\gamma^2 - 1)^{1/2} = \beta \gamma \)

✓ Let’s consider a particle

  ✓ moving at velocity \( \beta \) as seen in the observer frame K in the parallel direction
  ✓ emitting radiation in its rest frame K’

✓ Total emitted power:

  ✓ Is a Lorentz invariant: \( P_e = P'_e = \frac{2q^2 a'^2}{3c} \)
  ✓ Acceleration is not: \( a'_\perp = \gamma^2 a_\perp \)  \( a'_\parallel = \gamma^3 a_\parallel \)
  ✓ Emission of relativistic particles strongly enhanced: \( P_e = \frac{2q^2}{3c} \gamma^4 \left( \gamma^2 a_\parallel^2 + a_\perp^2 \right) \)
  ✓ High energy sources are amongst the most luminous sources...
Emission of relativistic particles

Relativistic beaming

✓ Angles: an example
  ✓ Emitting body moving at velocity $v$
  ✓ Photon emitted perpendicular to motion in the body frame
  ✓ Photon
    ✓ must fly at $c$ in all frames
    ✓ observed with parallel velocity $v$
    ✓ observed with perpendicular velocity $c/\gamma$
    ✓ observed with an angle $\sin \theta = 1/\gamma$

✓ Angular distribution of emission:

$$ \frac{\partial P_r}{\partial \Omega} = \frac{1}{\gamma^4 (1 - \beta \cos \theta)^4} \frac{\partial P'_e}{\partial \Omega'} $$

✓ beaming
✓ enhancement
Emission of relativistic particles

Relativistic beaming

✓ The dipolar emission of charge particles:
  ✓ The angular distribution depends on the (a,v) angle

✓ Beaming is still present and $\theta \approx 1/\gamma$
Cyclo-Synchrotron radiation

- Emitted power
- Spectrum
- Radiation from many particles
- Polarization
- Self-absorption
Synchrotron in astrophysics

✓ Applications to astrophysical sources started in the mid 20th
✓ Most observations in radio but also at all wavelengths
Cyclo-synchrotron radiation

✓ Radiation from particles gyrating the magnetic field lines
✓ Relativistic gyrofrequency \( \nu_B = \frac{qB}{2\pi mc} \quad \nu_{B,r} = \frac{1}{\gamma} \frac{qB}{2\pi mc} \)
✓ Assumptions:
  ✓ Classical limit: \( h\nu_B << mc^2 \quad B < B_c = 12 \times 10^{12} \text{ G} \)
  ✓ Otherwise: quantization of energies, Larmor radii...
  ✓ Observable cyclotron scattering features in accreting neutron stars...
  ✓ B uniform at the Larmor scale (parallel and perp)
    ✓ ! no strong B curvature (pulsars and rapidly rotating neutron stars)
    ✓ ! no small scale turbulence (at the Larmor scale)
    ✓ ! no large losses (\( t_{\text{cool}} >> 1/\nu_{B,r} \))
✓ Emission/Absorption
Emitted Power and cooling time

✓ Circular motion: \( a = a_\perp = \frac{\nu_B}{2\pi\gamma} v_\perp \)

✓ Emission of an accelerated particle: \( P = \frac{2q^2}{3c^3} \gamma^4 a_\perp^2 = 2c\sigma_T U_B p_\perp^2 \)
  ✓ Power goes as \( p^2 \)
  ✓ Power goes as \( B^2 \)

✓ Isotropic distribution of pitch angles: \( P = \frac{4}{3} c\sigma_T U_B p^2 \)

✓ Cooling time: \( t_{\text{cool}} = \frac{\gamma m c^2}{P_e} \approx \frac{25\text{yr}}{B^2\gamma} \)
  ✓ ISM (\( B=1\mu G \)): \( t_{\text{cool}} > t_{\text{universe}} \) as far as \( \gamma < 10^3 \)
  ✓ AGN jets (\( B=10\mu G, \gamma=10^4 \)): \( t_{\text{cool}} = 10^7\text{yr} \) (=travel time!)

✓ Maximal loss limit \( t_{\text{cool}} > > \frac{1}{\nu_{B,r}} \quad \gamma^2 B < \frac{2q}{r_0^2} \)
Cyclo-synchrotron radiation

**Emission Spectrum**

Very low energy particles:

- Sinus modulation of the electric field at \( \nu_B \):

\[
\nu = \nu_B \sin^2 \theta
\]

- Spectrum = one cyclotron line at \( \nu_B = \nu_{B,r} \)
Cyclo-synchrotron radiation

Emission Spectrum

Mid-relativistic particles:

- Moderate beaming
- Asymmetrical modulation of the electric field at $\nu_{B,r}$:
- Spectrum = many harmonic lines at $k\nu_{B,r} = k\nu_B/\gamma$

![Graph showing emission spectrum with logarithmic scale for $\nu/\nu_e$ vs. $P_0 = 2\sigma c U_b \rho^2 \sin^2 \alpha$]
Cyclo-synchrotron radiation ●●●●●●○○○○○○○○○

Emission Spectrum

Ultra-relativistic particles:

✓ Strong beaming: \( \delta \theta = 1/\gamma \)

✓ Pulsed modulation of the electric field at \( v_B \):

\[ \delta t \approx \frac{1}{(\nu B, r \gamma^3)} \]

✓ Spectrum = continuum up to

\[ \nu_c = \frac{3}{2} \gamma^2 \nu_B \sin \alpha \]
Emission Spectrum

\[ \frac{\partial^2 P}{\partial (\nu/\nu_B) \partial \Omega} = 4 \sigma_T c U_B \beta^2 \frac{\nu^2}{\nu_B^2} \sum_{n=1}^{\infty} \left[ n^2 \frac{(\cos \theta - \beta_\parallel)^2}{(1 - \beta_\parallel \cos \theta)^2} \frac{J_n^2(x)}{x^2} + J_n'(x) \right] \delta \left[ \frac{\nu}{\nu_B} (1 - \beta_\parallel \cos \theta) - \frac{n}{\gamma} \right] \]

\[ x = (\nu/\nu_B) \beta_\perp \sin \theta \]

- Exact spectrum depends on:
  - the particle energy
  - the pitch angle \( \alpha \): \( \cos \alpha = \beta_\parallel / \beta \)
  - the observation angle \( \theta \) with respect to \( B \)

- Line broadening results from integration over:
  - Observation direction
  - Particle pitch angle
  - Particle energy
Cyclo-synchrotron radiation

Spectrum of Relativistic Particles

- High energy plasmas have a continuous spectrum:
  \[
  \frac{\partial P}{\partial (\nu/\nu_B)} = 12\sqrt{3}\sigma T c U_B G \left( \frac{\nu}{2\nu_*} \right)
  \]

- Peaks at the critical frequency:
  \[
  \nu_* = \frac{3}{2} \nu_B \gamma^2 \propto B \gamma^2
  \]

- Most of the emission is at \( \nu_* \)
  - AGN (\( B=10\mu G, \gamma=10^4 \)): 10cm
  - Crab nebula (\( B=0.1mG, \gamma=10^7 \)): 10 keV

- Highest possible energy of photons:
  \[
  \gamma^2 B < \frac{2q}{\nu_0^2} \quad h\nu = 3m_e c^2 / \alpha_f \approx 70\text{MeV}
  \]
Spectrum of many particles

- Emission integrated over the particle distribution: \( \dot{j}_\nu = \int P_\nu(\nu, \gamma) f(\gamma) d\gamma \)

- Thermal distribution: \( f(\gamma) \propto \gamma^2 e^{-\gamma/\theta} \)
  - Same as for a single particle for the mean energy.

- Power-law distribution: \( f(\gamma) \propto \gamma^{-s} \)
  - Integrated emission:
    \[
    \dot{j}_\nu \propto \int G(3\nu/2\nu_B \gamma^2) \gamma^{-s} d\gamma \\
    \propto \nu^{-(s-1)/2} \int x^{(s-3)/2} G(x) dx \\
    \propto \nu^{-\alpha}
    \]
  - Power-law spectrum with
    - slope: \( \alpha = \frac{s-1}{2} \)
    - minimal energy: \( \nu_{\text{min}} \propto B\gamma_{\text{min}}^2 \)
    - maximal energy: \( \nu_{\text{max}} \propto B\gamma_{\text{max}}^2 \)
The Crab Nebula

- Pulsar wind nebula
  - Outflow of high energy electrons
  - Magnetized medium (0.1 mG)
- Synchrotron emission from radio to \( \gamma \)-rays
- Broken power-law distribution
  - Two slopes: \( s_1 \), \( s_2 \)
  - Two breaks: \( \gamma_1 \), \( \gamma_2 \)
Cyclo-synchrotron radiation

Polarization

✓ One single particle produces a coherent EM fluctuation
  ✓ Intrinsically polarized: elliptically
  ✓ Depends on p, α and θ

✓ Turbulent magnetic field: no net polarization

✓ Ordered magnetic field:
  ✓ Ensemble of particles with random pitch angles => partially linearly polarized
  ✓ Polarization angle perpendicular to observed B: $P_\perp >> P_\parallel$
  ✓ High polarization degree: $\Pi(\nu, p) = \frac{P_\perp - P_\parallel}{P_\perp + P_\parallel}$
    ✓ depends on frequency and particle energy
    ✓ averaged over all frequencies: $\Pi=75\%$
    ✓ In average over PL distribution of particles: $\Pi=(s+1)/(s+7/3)$
Cyclo-synchrotron radiation ●●●●●●●○○○

Polarization

✓ Such high polarization is characteristic of synchrotron radiation
✓ Measure of the direction gives the direction of B

Crab nebula

M87
Synchrotron self-absorption

\[ j_\nu = n \frac{\partial P}{\partial \nu \partial \Omega} \]

absorption coefficient:

\[ \alpha_\nu(p, \nu) = \frac{c^2}{2h\nu^3} \frac{1}{p\gamma} \left[ \gamma p j_\nu \right]_{\gamma + h\nu/mc^2} \]

\[ \approx \frac{1}{2m\nu^2} \frac{1}{p\gamma} \partial_\gamma (\gamma p j_\nu) \]

✓ Absorption decreases with frequency
  ✓ high energy photons are weakly absorbed
  ✓ low energy photons are highly absorbed

Cyclo-synchrotron radiation ●●●●●●●●●●●●●○○

Synchrotron self-absorption

Spontaneous emission

True absorption

Stimulated emission (negative absorption)
Cyclo-synchrotron radiation

Synchrotron self-absorption

 ✓ Radiation transfer:
   ✓ Equation for specific intensity $I_\nu$: \[ \frac{\partial I_\nu}{\partial l} = j_\nu - \alpha_\nu I_\nu \]
   ✓ Solution for a uniform layer of thickness $L$: \[ I_\nu = \frac{j_\nu}{\alpha_\nu} \left(1 - e^{-\alpha_\nu L}\right) \]

 ✓ $\tau_\nu=\alpha_\nu L$ is the optical depth at energy $h\nu$
   ✓ When $\tau_\nu<<1$: thin spectrum
   ✓ When $\tau_\nu>>1$: thick spectrum
   ✓ transition for: $\tau_\nu = \alpha_\nu L \approx 1$

 ✓ The transition energy increases with optical depth, i.e. with:
   ✓ Physical thickness of the layer
   ✓ Density of the medium
Self-absorbed Spectra

✓ Thermal distribution

Rayleigh-Jeans $F_\nu \propto \nu^2$

Tends to BB radiation in the thick part

✓ Power-law distribution

$F_\nu \propto \nu^{5/2}$

Does not tend to BB radiation in the thick part
Compton Scattering

- Thomson/Klein-Nishina regimes
- Spectrum, angular distribution
- Particle cooling
- Multiple scattering
Compton scattering

In the particle rest frame

- Scattering of light by free electrons
- Result described by 6 quantities
- 4 Conservation laws:
  - Energy: $h\nu_0 + mc^2 = h\nu + \gamma mc^2$
  - Momentum: $\frac{h\nu_0}{c} \vec{n}_0 = \frac{h\nu}{c} \vec{n} + mc\vec{p}$
- 1 symmetry
- One quantity is left undetermined, e.g. the scattering angle $\theta$
- Direction/energy relation: $\frac{h\nu}{h\nu_0} = \frac{1}{1 + \frac{h\nu_0}{mc^2} (1 - \cos \theta)}$
- Photon loose energy in the particle rest frame
- Two regimes:
  - The Thomson regime ($h\nu_0 < mc^2$): coherent scattering: $h\nu = h\nu_0$
  - The Klein-Nishina regime ($h\nu_0 > mc^2$): incoherent scattering: $h\nu < h\nu_0$
Compton scattering

Thomson scattering

✓ Scattering of linearly polarized waves:
  ✓ Harmonic motion of the particle
  ✓ Emission of light in all directions

✓ Scattering of unpolarized waves:
  ✓ Average of linearly polarized waves with random directions

✓ Scattered power: \( \frac{\partial P}{\partial \Omega} = \frac{\partial \sigma}{\partial \Omega} F \)

✓ Thomson cross section: \( \frac{\partial \sigma}{\partial \Omega} = \frac{3}{8 \pi} \sigma_T \frac{1 + \cos^2 \theta}{2} \)

✓ Total cross section: \( \sigma_T = 6.65 \times 10^{-25} \text{cm}^2 \)

✓ Partially polarized: \( \Pi = \frac{1 - \cos^2 \theta}{1 + \cos^2 \theta} \)

✓ Spectrum: a line at the incident frequency
Klein Nishina scattering

✓ Requires quantum mechanics but still analytical formulae

Total cross section:

\[ \sigma = \sigma_T \frac{3}{4} \left[ \frac{1 + \omega_0}{\omega_0^3} \left( \frac{2\omega_0(1 + \omega_0)}{1 + 2\omega_0} - \ln \left(1 + 2\omega_0\right)\right) + \frac{\ln \left(1 + 2\omega_0\right)}{2\omega_0} - \frac{1 + 3\omega_0}{(1 + 2\omega_0)^2} \right] \]

Differential cross sections:

\[ \frac{d\sigma}{d\Omega} = \sigma_T \frac{3}{16\pi} \left( \frac{h\nu}{h\nu_0} \right)^2 \left( \frac{h\nu_0}{h\nu} + \frac{h\nu}{h\nu_0} - \sin^2 \theta \right) \]

Angular distribution:

Spectrum:

\( h\nu = mc^2 \)

Compton scattering ●●●●●●●●●●

Klein-Nishina scattering

Thomson Regime

Klein-Nishina Regime

\( \sigma \approx \sigma_T \)
In the source frame

✓ In the particle frame: one incident parameter \( (h\nu_0) \)

✓ In the source frame: new dependance on
  ✓ The particle energy
  ✓ The collision angle

✓ Photon can now gain/loose energy

✓ An example:
  ✓ Head-on collision:
    ✓ Cold photon \( h\nu_0 \) and relativistic electron \( \gamma_0 \gg 1 \)
    ✓ Photon energy in the particle frame: \( \nu'_0 = 2\gamma_0\nu_0 \)
  ✓ Thomson backward scattering: \( h\nu'_0 \ll mc^2 \)
    ✓ Emitted photon energy in the electron frame: \( \nu' = \nu'_0 \)
    ✓ Photon energy in the source frame: \( \nu = 2\gamma_0\nu' \)
  ✓ In the end: \( \nu = 4\gamma_0^2\nu_0 \)
  ✓ Compton up-scattering

✓ Often: isotropy assumption and average over angles
Compton scattering

In the source frame

- For isotropic distributions:
- The Thomson limit: $\gamma_0 h\nu_0 << mc^2$
- The scattered spectrum:
- Average energy of scattered photons
  - Down scattering: $(\gamma_0 - 1)mc^2 < h\nu_0$
  - Up-scattering: $(\gamma_0 - 1)mc^2 > h\nu_0$
  - Amplification factor:
    $$A = \frac{<h\nu>}{h\nu_0} \approx \gamma^2$$
- Scattering by relativistic plasmas produces high energy radiation
- Particle cooling:
  $$\frac{\partial E_p}{\partial t} = \frac{4}{3} c\sigma_T p^2 U_{ph}$$
**Blazar Spectra**

- $E > \text{TeV}$! Comptonization?
- Model = Synchrotron Self-Compton (SSC) + Doppler boosting
  - Seed photons = synchrotron photons
  - The same particle emit though synchrotron and scatter these photons
- Here:
  - $A=10^8 \Rightarrow \gamma=10^4$
  - $h\nu_0 = 0.1 \text{ keV} \Rightarrow \text{KN regime}$
  - Synchrotron peaks at $B\gamma^2$, Compton amplifies with $A=\gamma^2 \Rightarrow B!$
Compton scattering ●●●●●●●○○○○○

Single scattering by many electrons

✓ Emission integrated over the particle distribution

✓ Thermal distribution: \( f(\gamma) \propto \gamma^2 e^{-\gamma/\theta} \)
  ✓ Same as for a single particle for the mean energy..

✓ Power-law distribution: \( f(\gamma) \propto \gamma^{-s} \)
  ✓ Power-law spectrum with
    ✓ slope: \( \alpha = \frac{s - 1}{2} \)
    ✓ minimal energy: \( h\nu_0 \gamma^2_{\text{min}} \)
    ✓ maximal energy: \( h\nu_0 \gamma^2_{\text{max}} \)

✓ Scattered photons distributions should also be integrated over the source seed photons
Multiple Scattering

✓ Photons can undergo successive scattering events

✓ Medium of finite size L: Thomson optical depth: $\tau_T = n_e \sigma_T L$

✓ Competition scattering/escape:
  ✓ $\tau$ (or $\tau^2$) = Mean number of scattering before escape
  ✓ $\tau<1$: single scattering
  ✓ $\tau>1$: multiple scattering

✓ $y$ parameter = $<\text{photon energy change}>$ before escape
  ✓ $y = <\text{Energy change per scattering}> \times <\text{scattering number}>$
  ✓ For mono-energetic particles: $y = \tau \gamma^2$
  ✓ For thermal distributions: $y = 4\tau \theta (1+4\theta)$
The SZ effect

Compton scattering

Cold photons

\( k_B T_{ph} = 2.7 \text{ K} \)

Hot electrons

\( k_B T_e = 1-10 \text{ keV} \)  \( (\theta_e = k_B T_e / m_e c^2 \approx 10^{-2}, p \approx \beta \approx 0.1) \)

\( \tau = N_e \sigma_T L \approx 10^{-2} \)

- Typical distortion whose amplitude gives: \( y \approx \tau \theta \approx 10^{-4} \)
- Bremsstrahlung gives \( T \)
- \( \Rightarrow \) density...
Compton scattering

Compton orders

\[ \frac{h\nu_0}{mc^2} = 10^{-7} \]
\[ \gamma_0 = 10 \]
\[ A = 100 \]

bumpy spectrum

cutoff at the particle energy

\[ \tau \Rightarrow \text{spectrum hardness} \]
Compton scattering

Compton regimes

Sub-relativistic particles: \( \frac{h \nu_0}{mc^2} < p < 1 \)
- Thomson regime \( h \nu_0 \gamma_0 \ll mc^2 \)
- Inefficient scattering: \( A = 1 \)

Relativistic particles: \( \gamma_0 \gg 1 \)
- Thomson regime: \( h \nu_0 \gamma_0 \ll 1 \)
- Efficient scattering: \( A \gg 1 \)

Ultra-relativistic particles: \( \gamma_0 \gg 1 \)
- KN regime: \( h \nu_0 \gamma_0 > mc^2 \)
- Efficient scattering: \( A \gg 1 \)

Power-law spectrum
- cutoff at the particle energy
- Slope = \( \ln(\tau)/\ln(A) \)

\( \Rightarrow X\text{-ray binaries} \)

One-bump spectrum
- single scattering

\( \Rightarrow \text{AGN/Blazars} \)
Compton scattering

X-ray binaries

✓ Soft states:
  ✓ Multi-color black body at 1 keV from the accretion disk
  ✓ Non-thermal comptonization in a hot corona ($\tau=1$)

✓ Hard states:
  ✓ Soft photons from the accretion disk or synchrotron
  ✓ Inefficient thermal Comptonization in a hot corona (100 keV, $\tau=0.01$)

✓ What heating acceleration mechanism?

Zdziarsky 2003
Bremsstrahlung radiation
Bremsstrahlung

✓ Radiation of charged particles accelerated by the Coulomb field of other charges

✓ Astrophysical sources:
  ✓ Some modes of hot accretion disks
  ✓ Hot gas of intra-cluster medium (1-10 keV)
  ✓ ...

...
Easy bremsstrahlung

✓ Assumptions:
  ✓ Classical physics
  ✓ Sub-relativistic particles
  ✓ Far collision (small deviation, no recombination)
  ✓ Small energy change ($\Delta v \ll v$ i.e. $h\nu \ll mv^2/2$)

✓ Single event
  ✓ No $p^+/p^+$, $e^-/e^-$, $e^+/e^+$ Bremsstrahlung
  ✓ $p^+/e$, $iZ^+/e$ Bremsstrahlung
  ✓ Approximation: static heavy $iZ^+$

✓ Approximated motion:
  ✓ Typical collision time: $\tau \approx 2b/v$
  ✓ Typical velocity change: $\Delta v \approx \tau a \approx \tau Z e^2/(mb^2) \approx 2Ze^2/(mvb)$
  ✓ $\tau$ and $\Delta v$ characterize the motion = enough to compute the spectrum
Easy bremsstrahlung

✓ Spectrum: \( \frac{\partial P}{\partial \nu} \propto \left| FFT(\tilde{E}) \right|^2 \propto \left| FFT(\tilde{a}) \right|^2 \)

✓ Fourier transform: \( TF[\dot{\nu}] = \int_{-\infty}^{+\infty} \dot{\nu}(t) e^{-2i\pi \nu t} dt \approx \begin{cases} 0 & \text{if } \nu \tau \gg 1 \\ \Delta \nu & \text{if } \nu \tau \ll 1 \end{cases} \)

✓ One single particle produces a flat spectrum:
\[
\frac{\partial E}{\partial \nu}(v, b) \approx \begin{cases} 0 & \text{if } \nu \tau \gg 1 \\ \frac{16e^6 \nu^2}{3c^3 m^2 v^2 b^2} & \text{if } \nu \tau \ll 1 \end{cases}
\]

✓ Many particles with a range of impact parameters:
  ✓ with \( b_{\min} \) from the small angle approximation
  ✓ with \( b_{\max} \) from \( \nu \tau < 1 \)
\[
\frac{\partial P}{\partial V \partial \nu} = n_i n_e v \int_{b_{\min}}^{b_{\max}} \frac{\partial E}{\partial \nu} 2\pi b db = \frac{32\pi e^6}{3m^2 c^3 v} n_i n_e g_{\text{ff}}(v, \nu)
\]

✓ Produce a flat spectrum that cuts at the electron energy
✓ Total losses: \( P_{\text{cool}} \propto n_i n_e Z^2 v \)
Emission from many electrons

✓ Emission integrated over the particle distribution
✓ Power-law distributions produce power-law spectra
✓ Thermal distributions:
  ✓ Emission coefficient: $j_\nu \propto n_i n_e Z^2 T^{-1/2} e^{-\hbar \nu/k_B T}$
  ✓ Total losses: $P_{cool} \propto n_i n_e Z^2 T^{1/2}$
✓ Relativistic and quantum correction can be added to give more general spectra
✓ In media of finite size: bremsstrahlung self-absorption at low energy
  ✓ c.f. synchrotron
Summary

✓ Particle cooling:
  ✓ Synchrotron: \( P \propto \sigma_T p^2 U_B \)
  ✓ Compton in the Thomson regime: \( P \propto \sigma_T p^2 U_{ph} \)
  ✓ Bremsstrahlung: \( P \propto \sigma_T \alpha_f p U_i \) (with \( U_i = n_i m_e c^2 \))

✓ Photons:
  ✓ Synchrotron:
    ✓ Thin spectrum of 1 particle peaks at \( \nu_c \propto \gamma^2 B \)
    ✓ Thin spectrum of a power-law distribution is a power-law
    ✓ Absorption \( \Rightarrow \) Thick spectrum at low frequency
  ✓ Compton
    ✓ Amplification factor in the Thomson regime: \( A = \gamma^2 \)
    ✓ Mildly relativistic particles: power-law spectrum
    ✓ Comptonization by a relativistic power-law distribution is a PL spectrum
  ✓ Bremsstrahlung
    ✓ Flat spectrum
    ✓ up to the particle energy